Automata Exercises

Tasks for 2.12.2014

Task 1 Show that the following language of palindromes of odd length with middle symbol c,
$$L = \{wcw^R \mid w \in \{a,b\}^*\}$$
is not regular. You may use that the language $\{a^n c a^n \mid n \in \mathbb{N}\}$ is not regular and that regular languages are closed under intersection.

Task 2 Give a context-free grammar for the language over $\Sigma = \{(,)\}$ of well-formed (properly nested) parenthesis. Note that a word belongs to this language if the number of closed parenthesis never (in any prefix) exceeds the number of open ones and at the end of the word these numbers are the same.

Task 3 Show that the language L of palindromes over alphabet $\Sigma = \{a,b\}$ is context-free. Note that

$$L = \{w \in \{a,b\}^* \mid w = w^R\}.$$

Task 4 Give a context-free grammar for the language

$$L = \{uawb \mid u,w \in \{a,b\}^*, |u| = |w|\}.$$

Task 5 Construct a context-free grammar for the language

$$L = \{a^i b^j c^k \mid i = j \ \ or \ \ j = k\}.$$

Notice that the language is a union of two context-free languages.

Task 6 Show that context-free languages are not closed under intersection, i.e., give an example of two context-free languages whose intersection is not context-free.

Task 7 Show that the language

$$L = \{w \in \{a,b,c\}^* \mid \#_a(w) = \#_b(w) = \#_c(w)\}$$

is not context free. You may use Theorem CF4 as well as that $\{a^n b^n c^n \mid n \in \mathbb{N}\}$ is not context free.