

Weak bisimulation for action-type coalgebras

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Outline

- Introduction and motivation
 - * coalgebras (action-type)
 - * LTS
 - * bisimulation, strong and weak
 - * coalgebraic vs. concrete bisimulation
- Weak bisimulation for coalgebras
- Examples and correspondence results
 - * LTS
 - * generative probabilistic systems
- Conclusions

Action-type coalgebras

A coalgebra (on \mathbf{Set} , of type \mathcal{F}) is a pair

$$\langle S, \alpha : S \rightarrow \mathcal{F}S \rangle$$

for \mathcal{F} a \mathbf{Set} endofunctor

Action-type coalgebras

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- \mathcal{F} - bifunctor, A - fixed set
 $\Rightarrow \mathcal{F}_A$ - Set endofunctor,

$$\mathcal{F}_A(S) = \mathcal{F}(A, S),$$

$$\mathcal{F}_A f = \mathcal{F}\langle \text{id}_A, f \rangle, \quad f : S \rightarrow T$$

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- * an action-type coalgebra of type \mathcal{F}_A is a triple

$$\langle S, A, \alpha: S \rightarrow \mathcal{F}_A(S) \rangle$$

Coalgebraic bisimulation

A **bisimulation** between two \mathcal{F}_A -coalgebras $\langle S, A, \alpha \rangle$ and $\langle T, A, \beta \rangle$ is a relation

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$$\begin{array}{ccccc} S & \xleftarrow{\pi_1} & R & \xrightarrow{\pi_2} & T \\ \alpha \downarrow & & \downarrow \gamma & & \downarrow \beta \\ \mathcal{F}_A S & \xleftarrow{\mathcal{F}_A \pi_1} & \mathcal{F}_A R & \xrightarrow{\mathcal{F}_A \pi_2} & \mathcal{F}_A T \end{array}$$

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$s \sim t$ - bisimilarity, as usual ...

Labelled transition systems

LTS is a triple $\langle S, A, \rightarrow \subseteq S \times A \times S \rangle$

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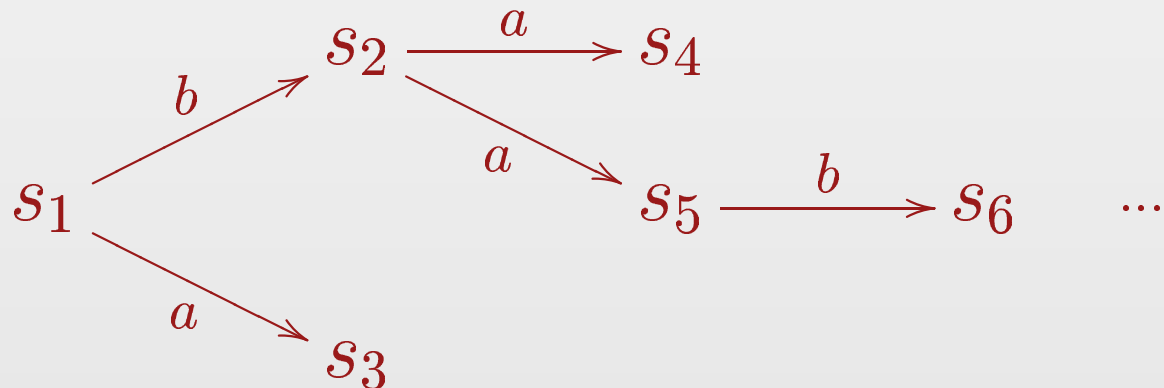
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Note: LTS $\langle S, A, \rightarrow \rangle = \mathcal{L}_A$ coalgebra $\langle S, A, \alpha \rangle$ for the bifunctor $\mathcal{L} = \mathcal{P}(\mathcal{I} \times \mathcal{I})$ with

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\sim_{LTS} - strong bisimilarity

Characterizing bisimulation

* $R \subseteq S \times T$, \mathcal{F} - functor, $\equiv_{\mathcal{F},R}$ - lifting

$$x \equiv_{\mathcal{F},R} y \iff \exists z \in \mathcal{F}R: \mathcal{F}\pi_1(z) = x, \mathcal{F}\pi_2(z) = y$$

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- if \mathcal{F} w.p. **total** pullbacks and R - equivalence, then $\equiv_{\mathcal{F},R}$ is the pullback of

$$\mathcal{F}S \xrightarrow{\mathcal{F}c} \mathcal{F}(S/R) \xleftarrow{\mathcal{F}c} \mathcal{F}S$$

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\approx_{LTS} - weak bisimilarity for LTS

Weak bisimulation - LTS

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by

$$\langle a_1 \dots a_k, s' \rangle \in \alpha'(s) \iff s \xrightarrow{a_1} \circ \dots \circ \xrightarrow{a_k} s'$$

and

$$\langle a_1 \dots a_k, s' \rangle \in \alpha''(s) \iff \langle w, s' \rangle \in \alpha'(s)$$

for some $w \in \tau^* a_1 \tau^* \dots \tau^* a_k \tau^*$

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- weak boils down to strong !

Weak bisimulation - \mathcal{F}_A coalgebras

two stages approach:

1. transform any \mathcal{F}_A coalgebra into \mathcal{G}_{A^*} coalgebra, faithfully.
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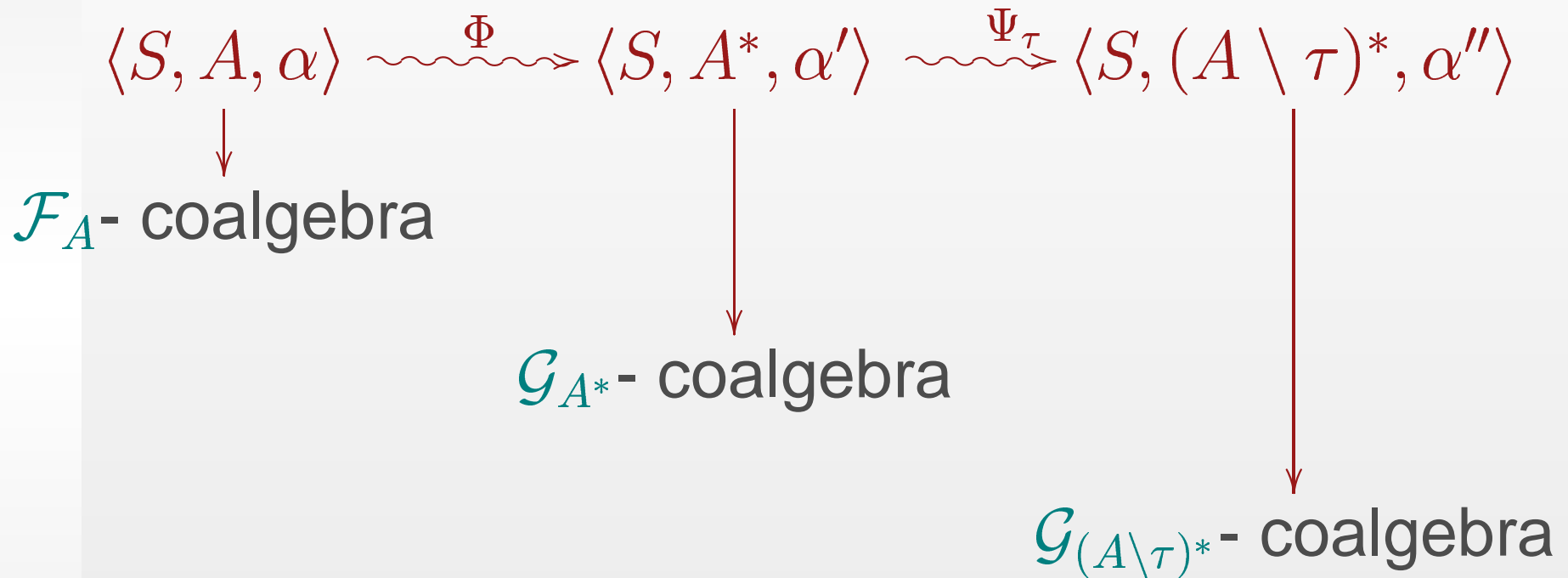
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weak bisimulation = bisimulation for the "double-arrow" coalgebra

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$$* \quad \Phi(\langle S, A, \alpha \rangle) = \langle S, A^*, \alpha' \rangle$$

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”double-arrow coalgebra”: $\Psi_\tau \circ \Phi(\langle S, A, \alpha \rangle)$

Weak bisimilarity, properties

Given $\Phi : \mathcal{F} \xrightarrow{*} \mathcal{G}$ and $\tau \subseteq A$

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Properties:

- $\sim \subseteq \approx_{\tau}$
- $\sim = \approx_{\emptyset}$
- $\tau_1 \subseteq \tau_2 \Rightarrow \approx_{\tau_1} \subseteq \approx_{\tau_2}$

Generative probabilistic systems

Generative system is a triple

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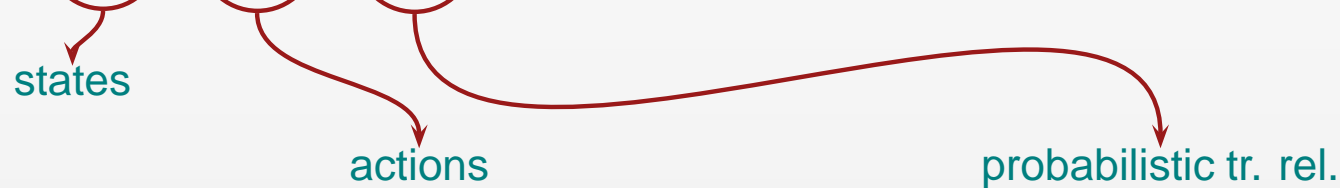
probabilistic tr. rel.

$$s_1 \xrightarrow{a[p]} s_2 \text{ for } P(s_1, a, s_2) = p$$

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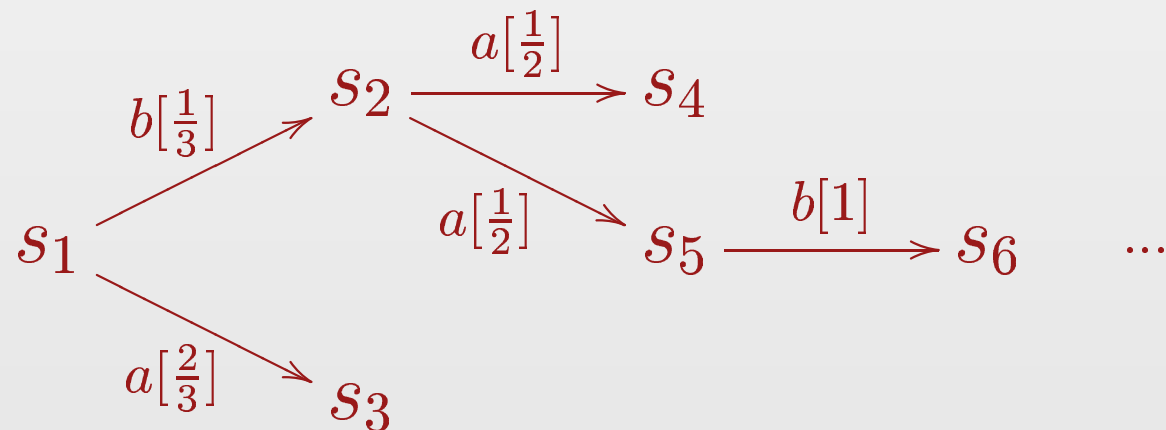
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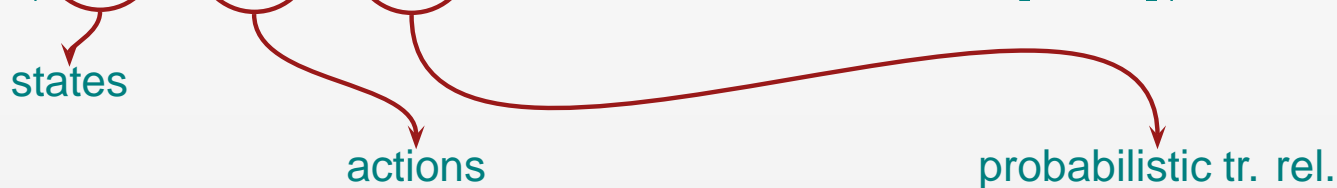
Example:



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Note: g. s. $\langle S, A, P \rangle = \mathcal{G}_A$ coalgebra $\langle S, A, \alpha \rangle$ for the bifunctor $\mathcal{G} = \mathcal{D}(\mathcal{I} \times \mathcal{I}) + 1$ with

$$\alpha(s)(a, s') = P(s, a, s')$$

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$\mathcal{G} = \mathcal{D}(\mathcal{I} \times \mathcal{I}) + 1$: \mathcal{D} - distribution functor

$$\mathcal{D}\mathcal{S} = \{\mu : \mathcal{S} \rightarrow [0, 1], \mu[\mathcal{S}] = 1\}, \quad \mu[X] = \sum_{s \in X} \mu(s)$$

$$\mathcal{D}f : \mathcal{D}\mathcal{S} \rightarrow \mathcal{D}\mathcal{T}, \quad \mathcal{D}f(\mu)(t) = \mu[f^{-1}(\{t\})]$$

Strong and weak - generative

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R - equivalence **strong bisimulation**
transfer condition:

$$P(s, a, C) = P(t, a, C)$$

for $C \in S/R$ and $P(s, a, C) = \sum_{s' \in C} P(s, a, s')$

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R - equivalence **weak bisimulation**
transfer condition:

$$\text{Prob}(s, \tau^* \hat{a} \tau^*, C) = \text{Prob}(t, \tau^* \hat{a} \tau^*, C)$$

for $C \in S/R$ and $\hat{a} = a$ if $a \in A \setminus \{\tau\}$ and $\hat{\tau} = \varepsilon$

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$\sim_{\text{GEN}}, \approx_{\text{GEN}}$ - strong, weak bisimilarity

Home for $*$ -extensions

bifunctor \mathcal{G}^*

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for

$$f = \langle f_1, f_2 \rangle: A \times S \rightarrow B \times T$$

$$\nu: \mathcal{P}(A) \times \mathcal{P}(S) \rightarrow [0, 1]$$

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Properties:

- \mathcal{G}_A^* w.p. total pullbacks
- it does **not** w.p. pullbacks

Paths - generative systems

generative system $\langle S, A, \alpha \rangle$ i.e. $\langle S, A, P \rangle$

sets $\text{Paths}(s)$, $\text{FPaths}(s)$, $\text{CPaths}(s)$

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Γ - set of cones,

$$\Gamma \subseteq \mathcal{P}(\text{CPaths}(s))$$

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$$\text{Prob}(\pi \uparrow) = P(s, a_1, s_1) \cdot \dots \cdot P(s_{k-1}, a_k, s_k)$$

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then: Prob is a pre-measure, and
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then: Φ is indeed a $*$ -translation

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given Φ and $\tau \subseteq A$

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for $B_i = h_\tau^{-1}(w_i)$, $w_i \in A_\tau$ and $C_j \in S/R$

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proof: difficult for \Rightarrow

Conclusion

- general notion of weak bisimulation
 - * two phase approach
 - * - extension and hiding internal actions
 - * weak bisimulation is strong
- from coalgebraic bisimulation to transfer conditions
- correspondence results for
 - * LTS
 - * generative probabilistic systems