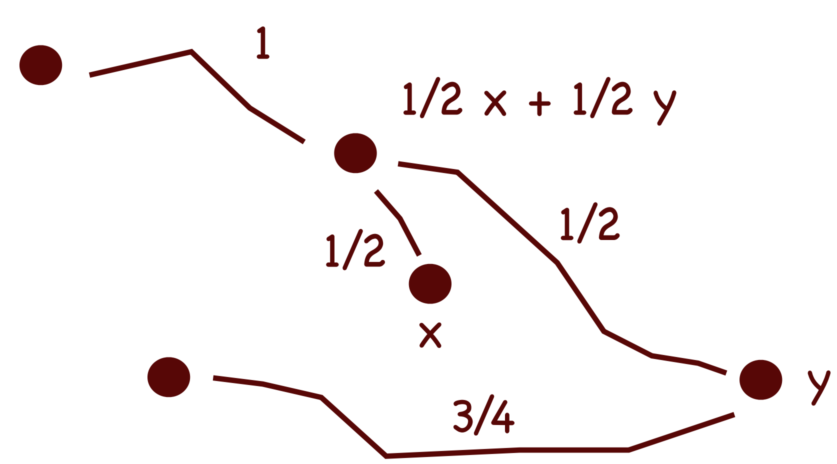
Semantics meets Syntax in Coalgebra

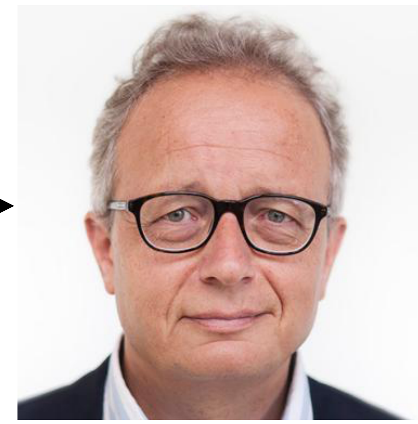
Ana Sokolova
UNIVERSITY of SALZBURG



SYSMICS'19

Joint work with

Ichiro Hasuo
THE UNIVERSITY OF TOKYO



Bart Jacobs
Radboud University

Alexandra Silva
UCL

Harald Woracek
TU WIEN

Filippo Bonchi
IN SUPREMÆ DIGNITATIS 1343
ENS DE LYON

Valeria Vignudelli
ENS DE LYON

I will tell you about:

1. Just the absolute basics of coalgebra

2. (Trace) semantics via determinisation…

3. …enabled by algebraic structure

I will tell you about:

Mathematical framework based on category theory for state-based systems semantics

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for nondeterministic, probabilistic… systems

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systems with algebraic effects

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systems with algebraic effects

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syntax

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

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$$X \xrightarrow{c} FX$$

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object in the base category ***C***

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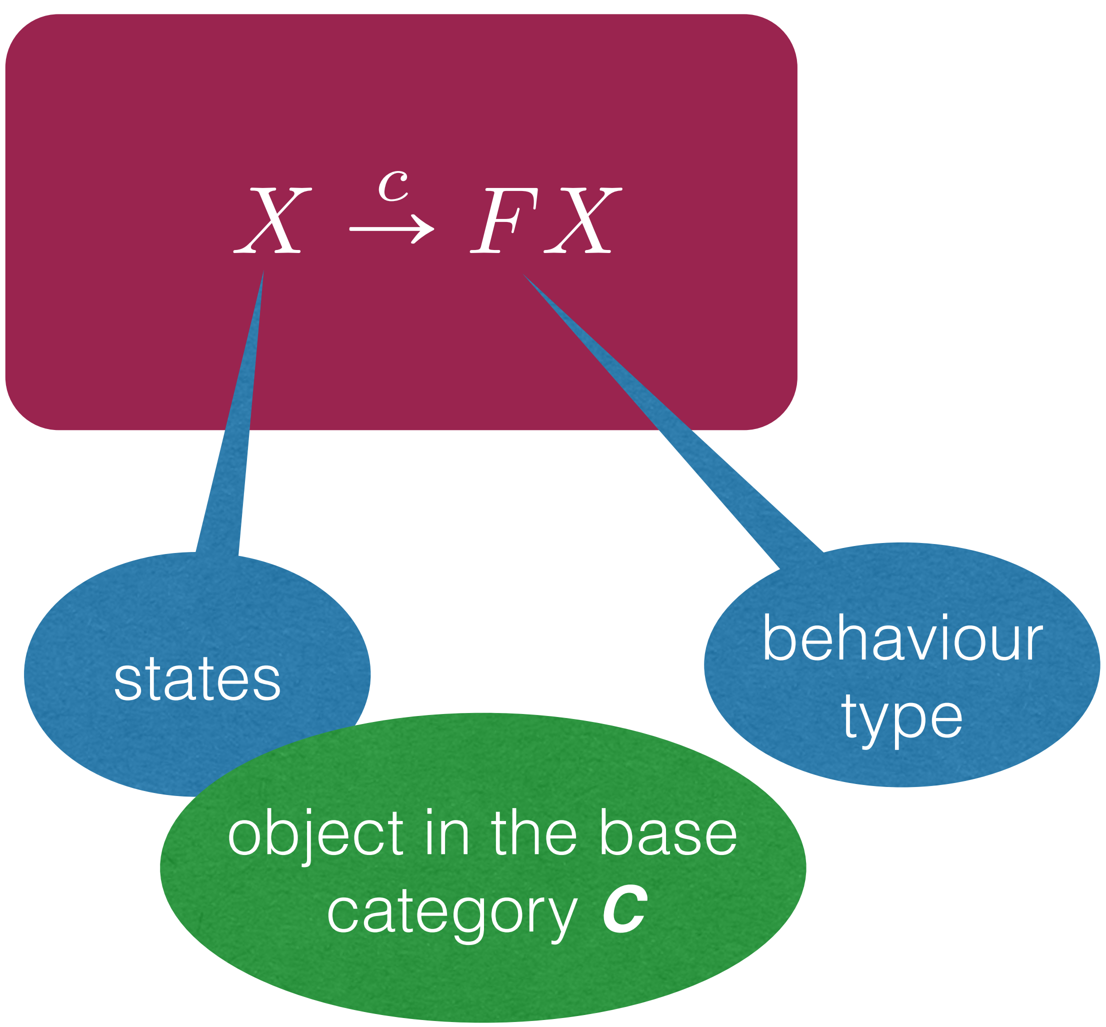

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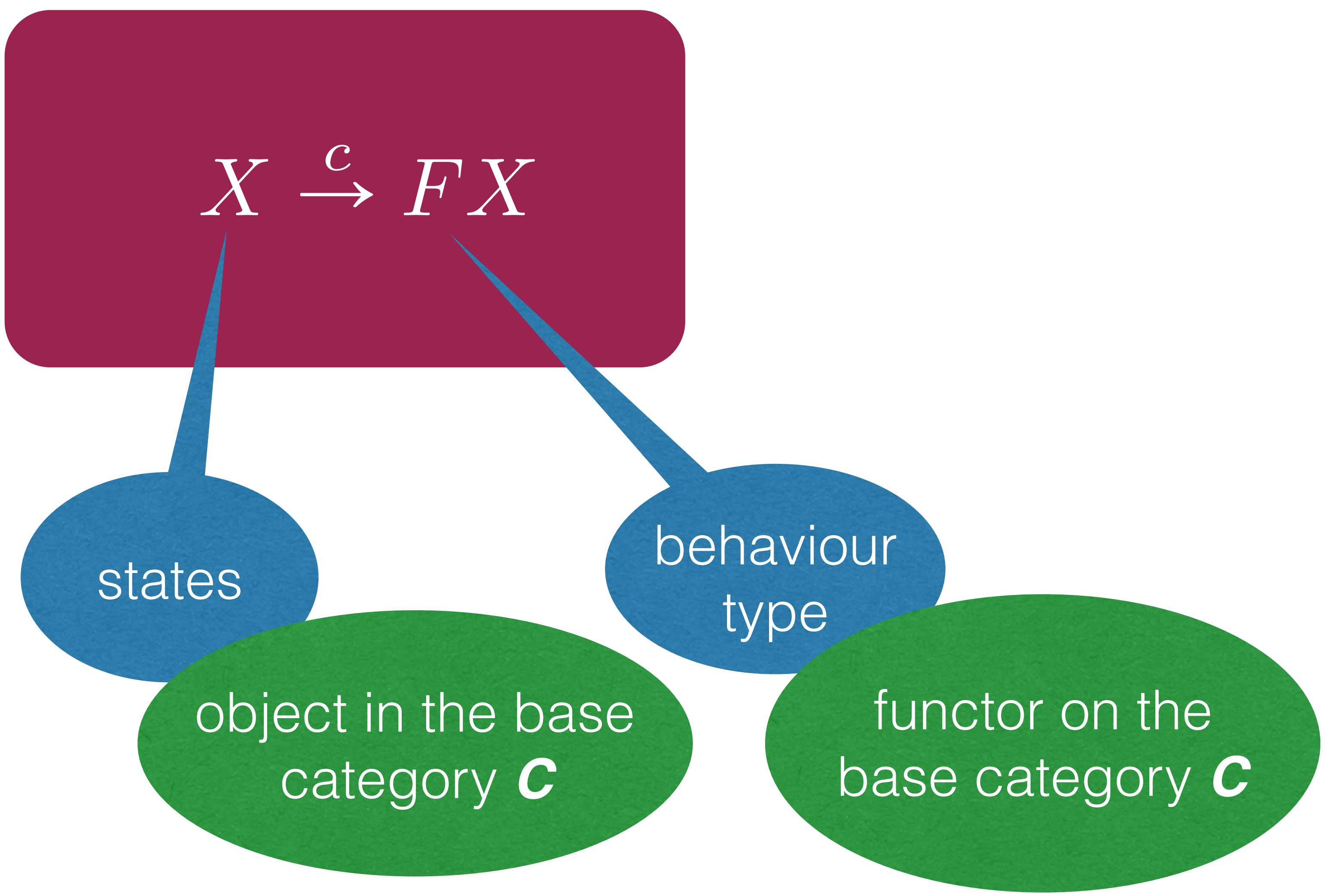

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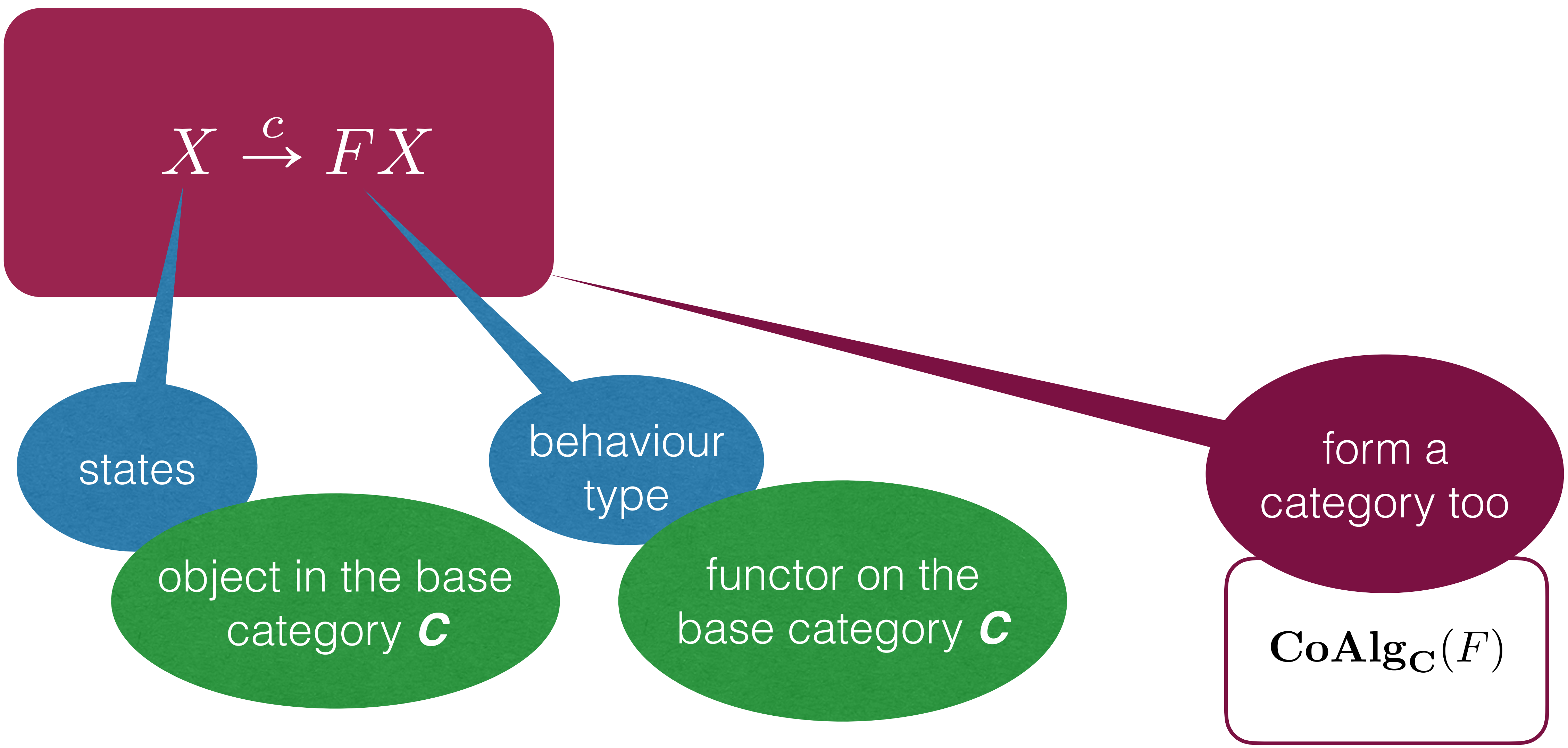

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$$\mathbf{CoAlg}_{\mathbf{C}}(F)$$

generic notion of behavioural equivalence (bisimilarity)

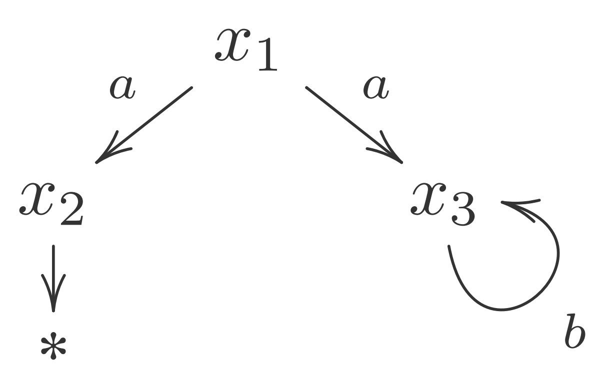
$\approx$

Examples

Examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$



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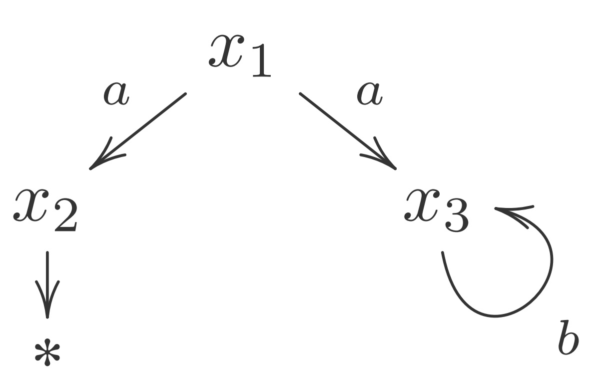
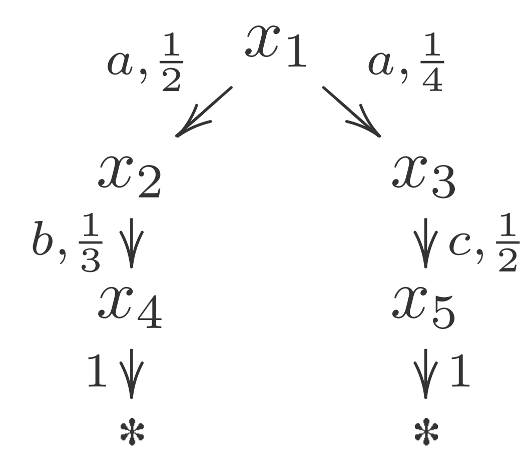


Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



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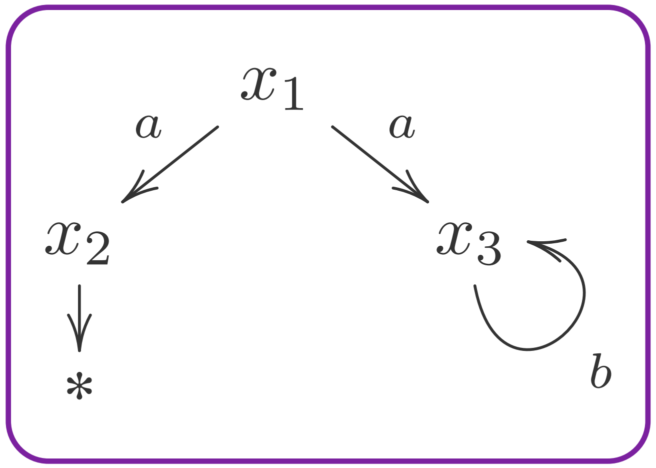

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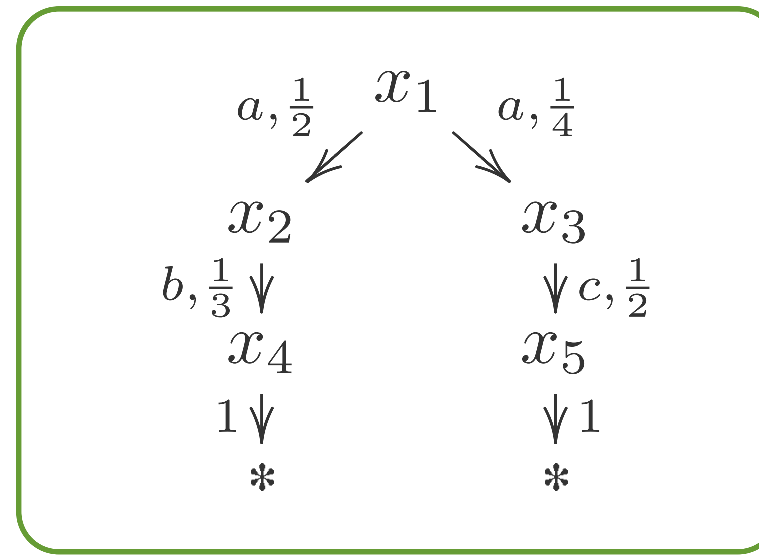

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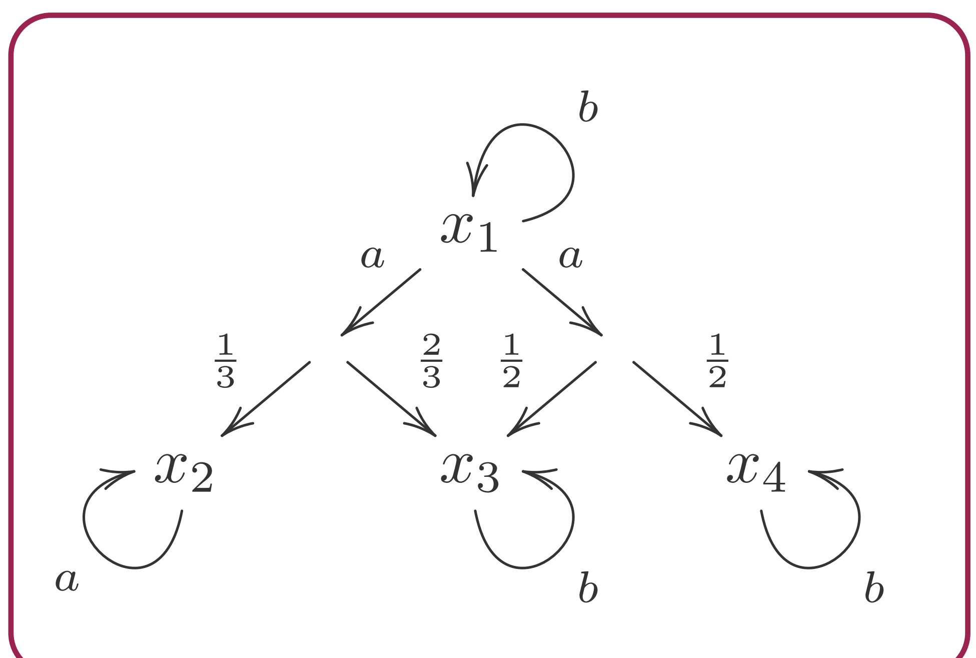

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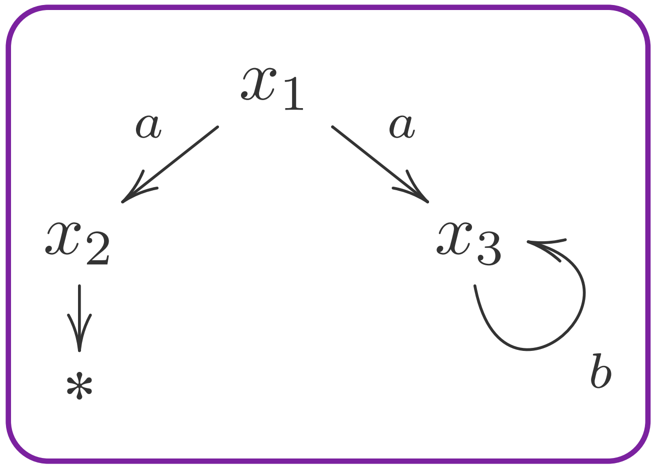

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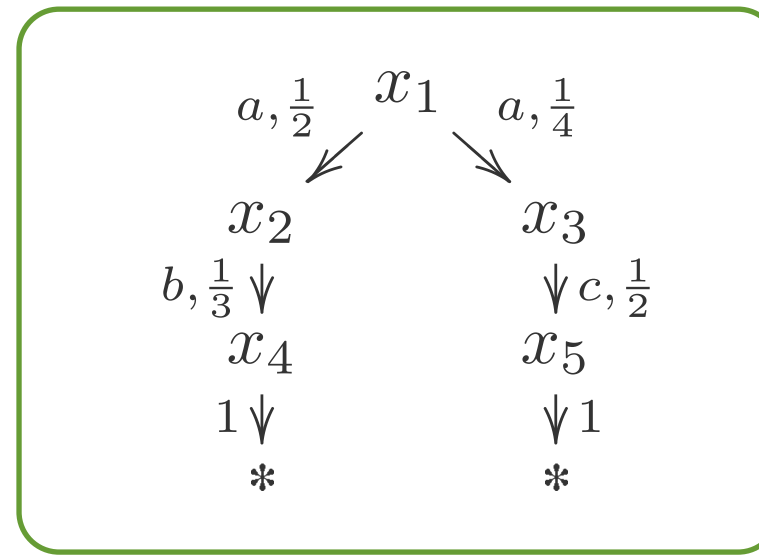

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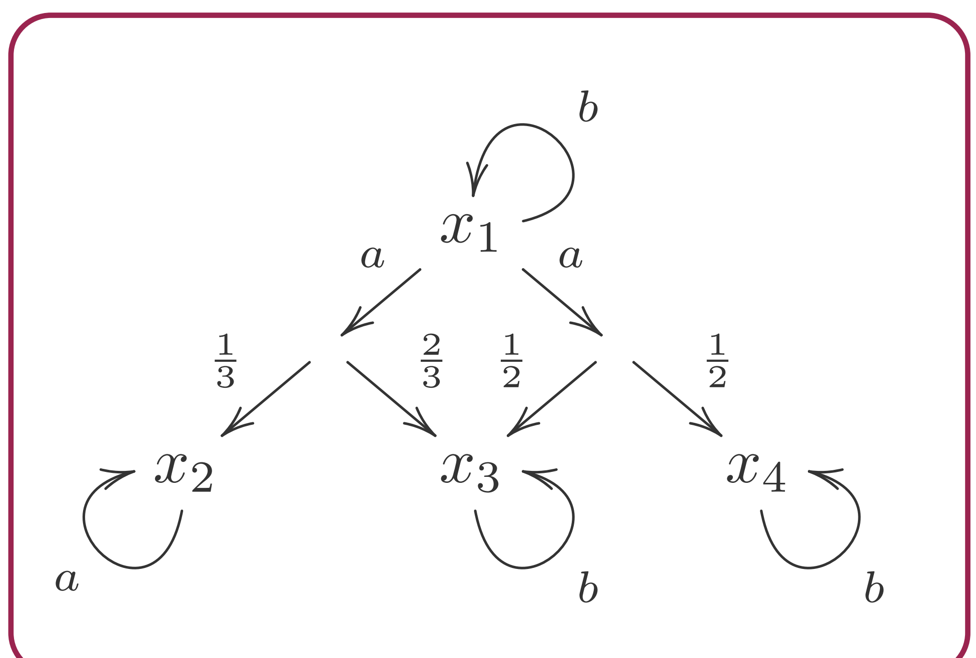

systems with nondeterminism and probability

In general

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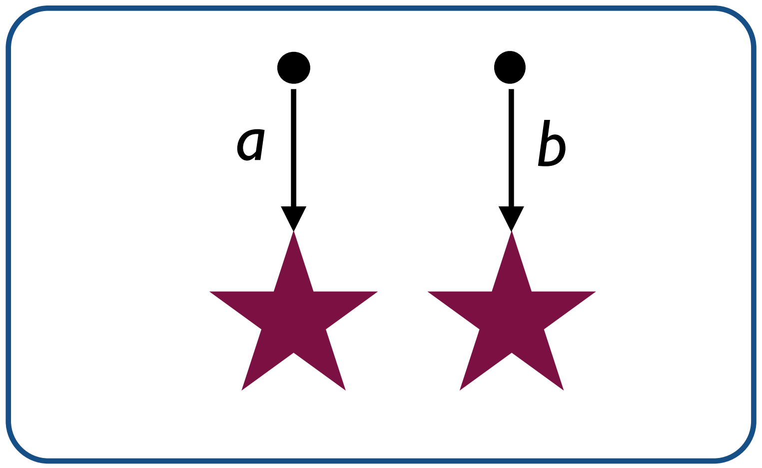
Systems

$X \to (MX)^A$



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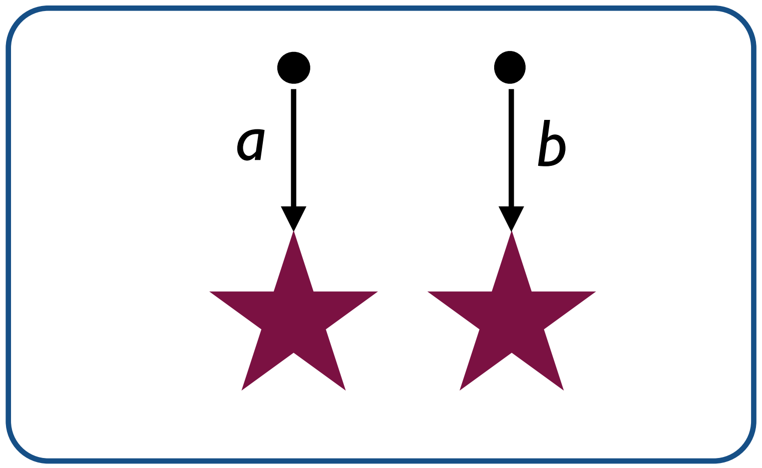
Labelled Transition Systems



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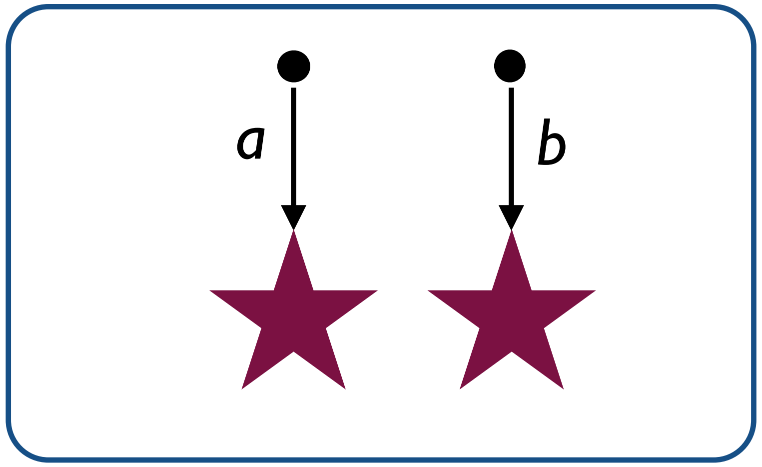

with M-effects

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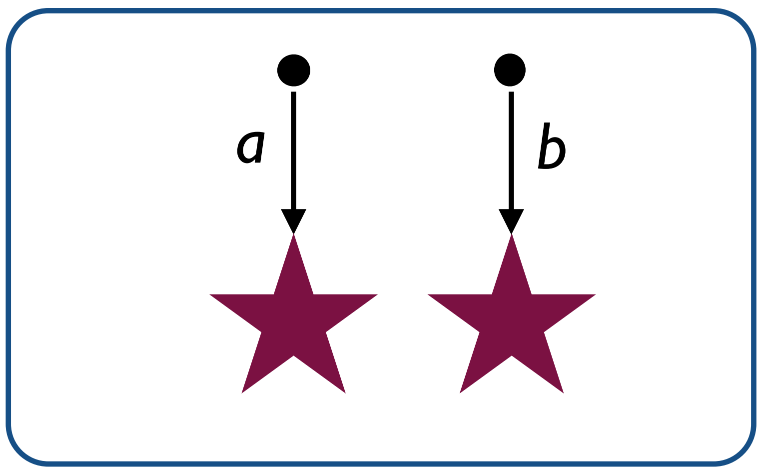

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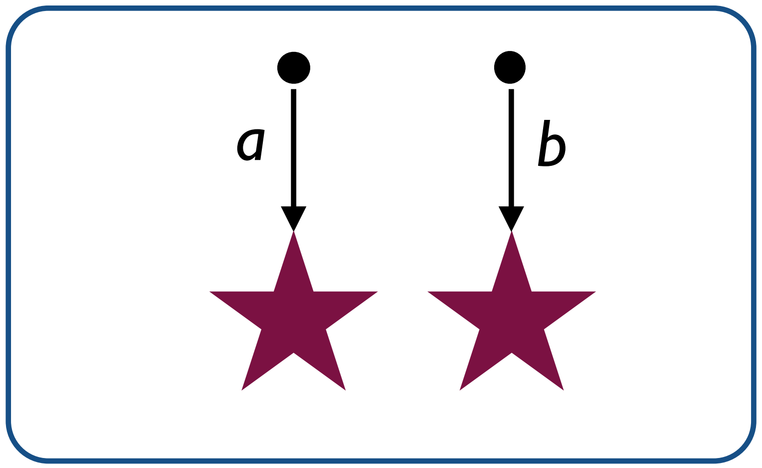
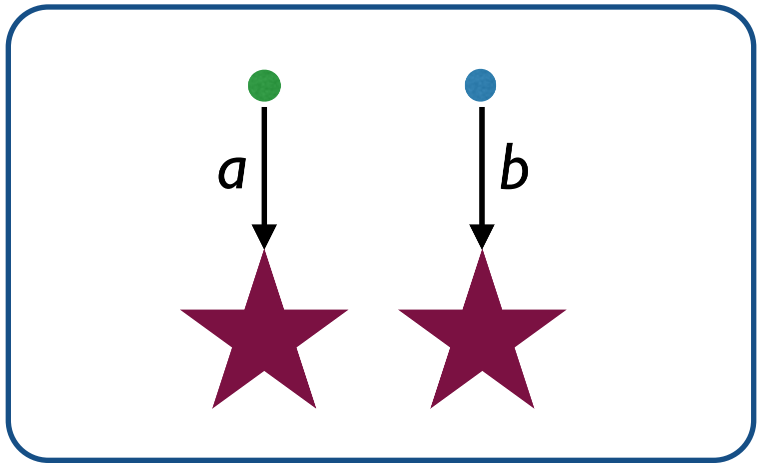


with M-effects

Automata

$X \to O \times (MX)^A$



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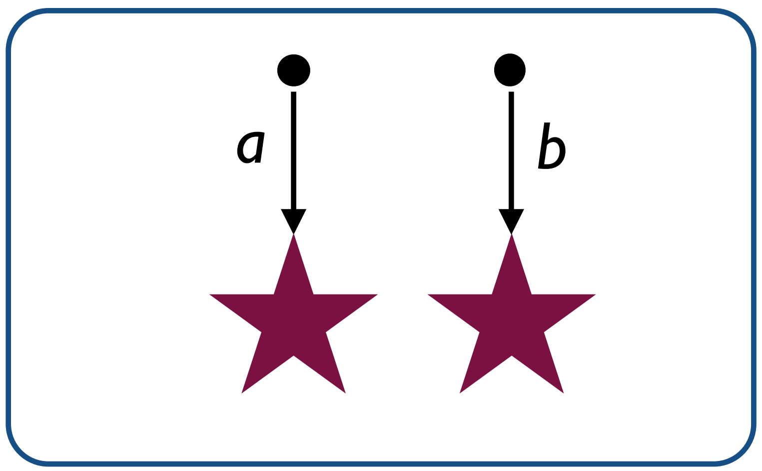

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Automata

with observations in O

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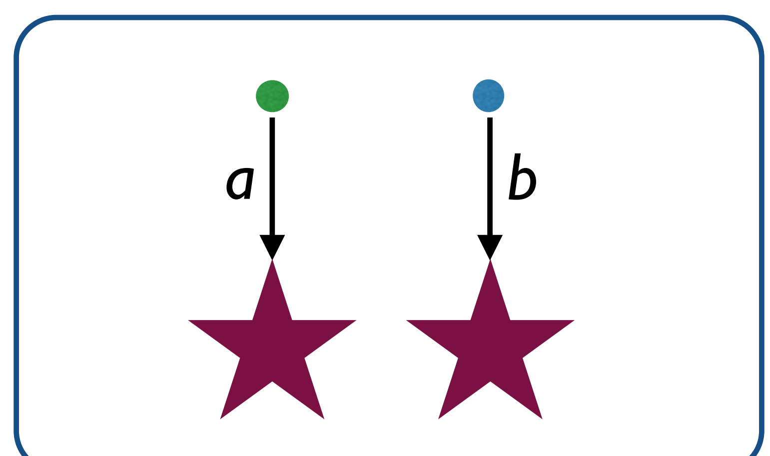

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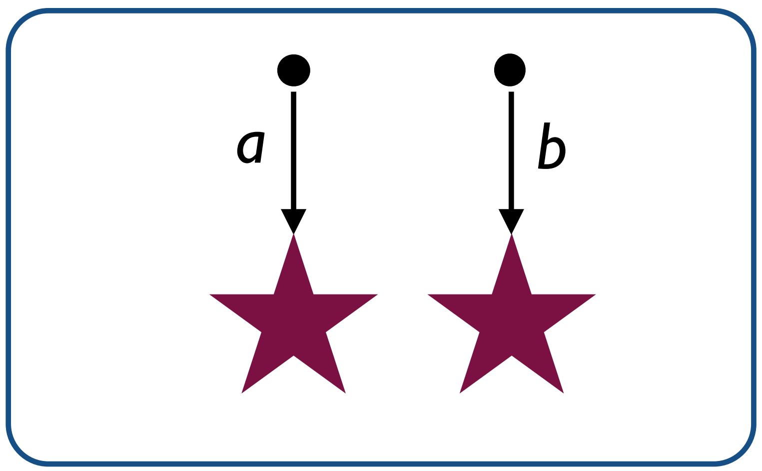

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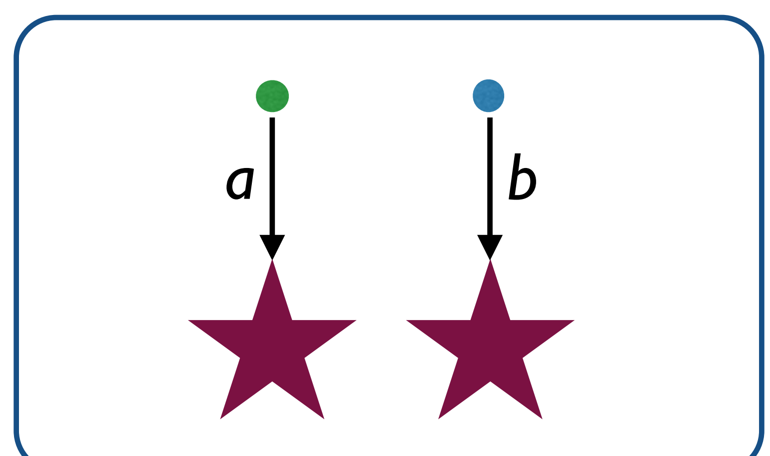

with M-effects

For a monad M

For a monad M

providing algebraic effects

For a monad M

$\mu \colon TT \Rightarrow T$

$\eta \colon Id \Rightarrow T$

providing
algebraic
effects

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NFA

$X \to 2 \times (\mathcal{P}X)^A$

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

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M = $\mathcal{P}$
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Powerset, subsets

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M = $\mathcal{P}\mathcal{D}$???
for nondeterminism and probability

For a monad M

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$X \to ? \times (\mathcal{C}X)^A$

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Convex subsets of distributions

Semantics

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NFA = LTS + termination

$X \to 2 \times (\mathcal{P}X)^A$

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Are the (top states of the) following systems equivalent?

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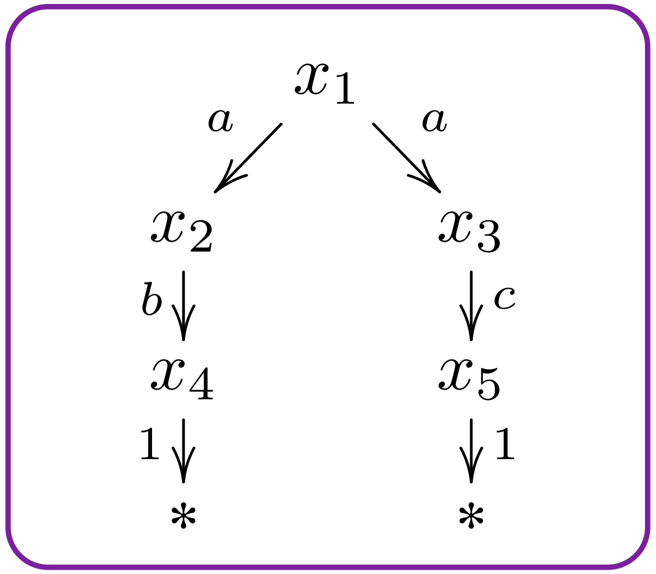

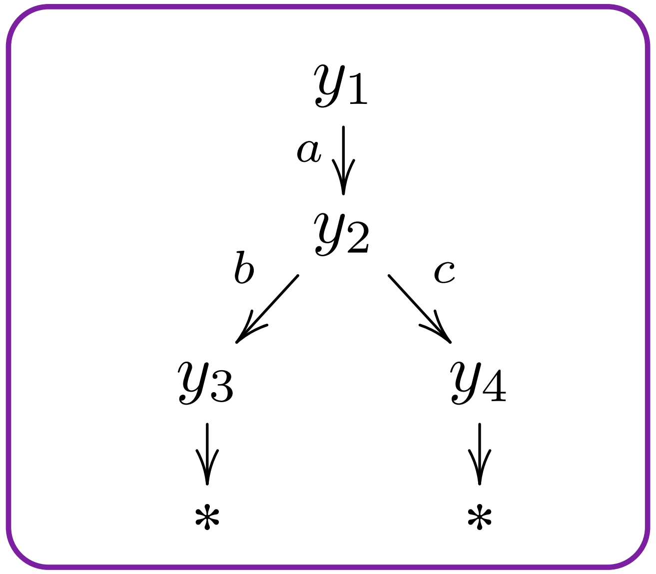

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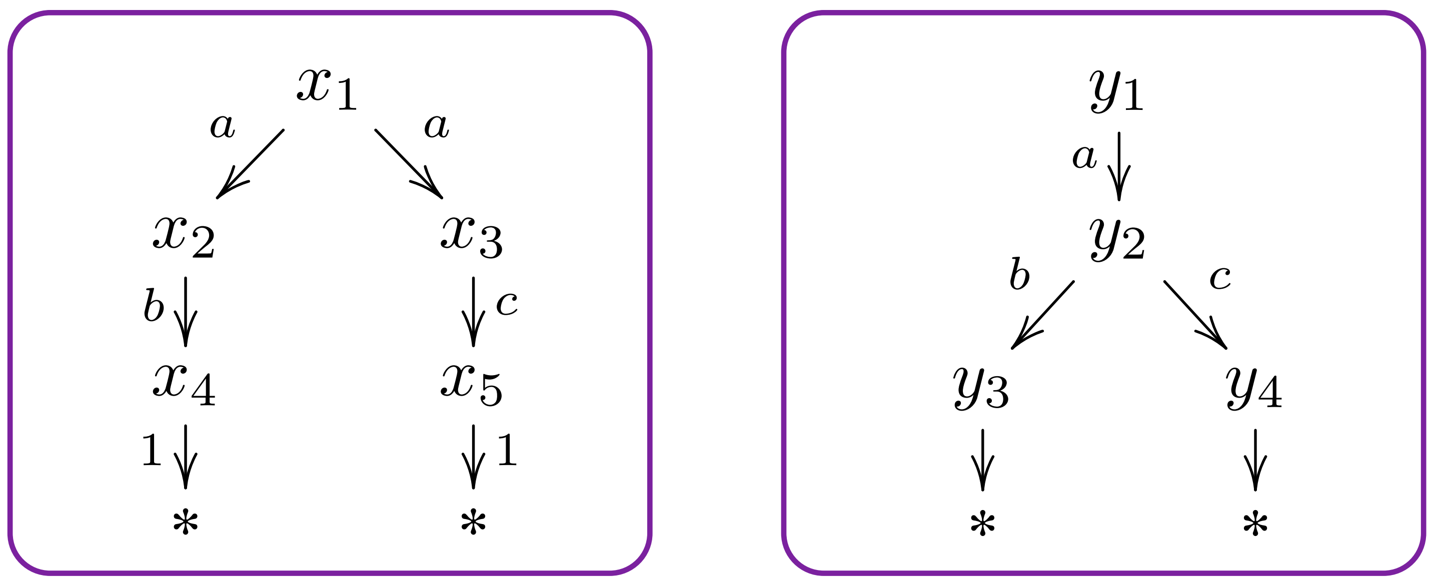

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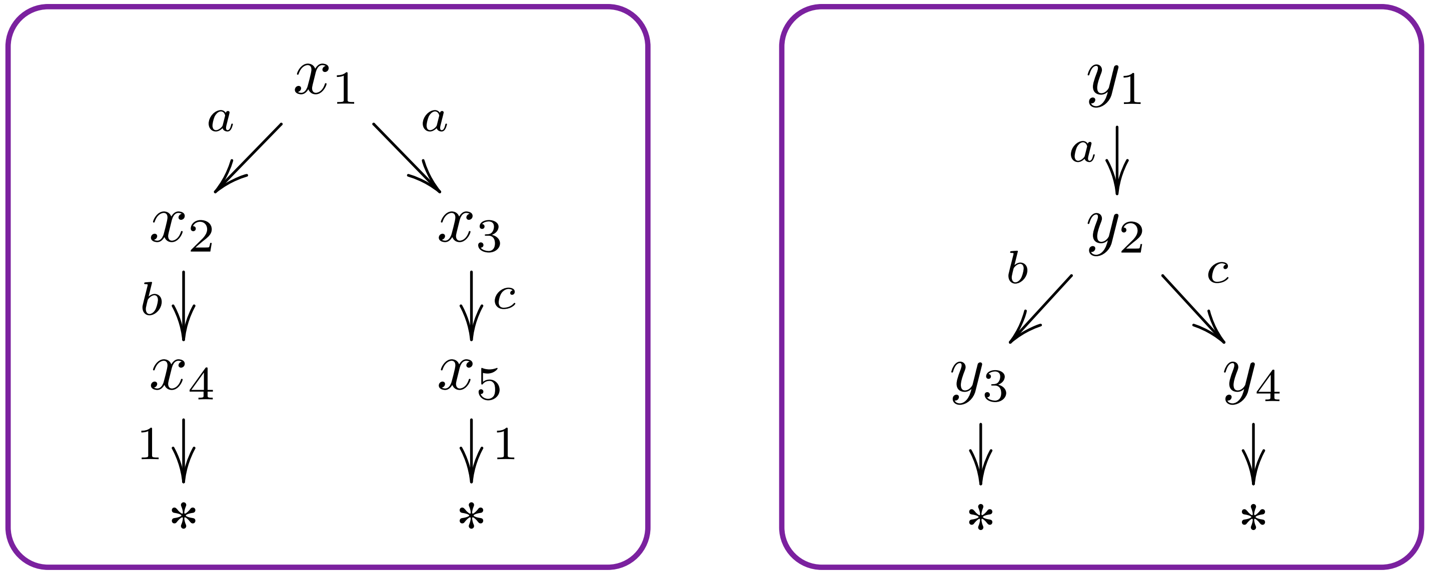

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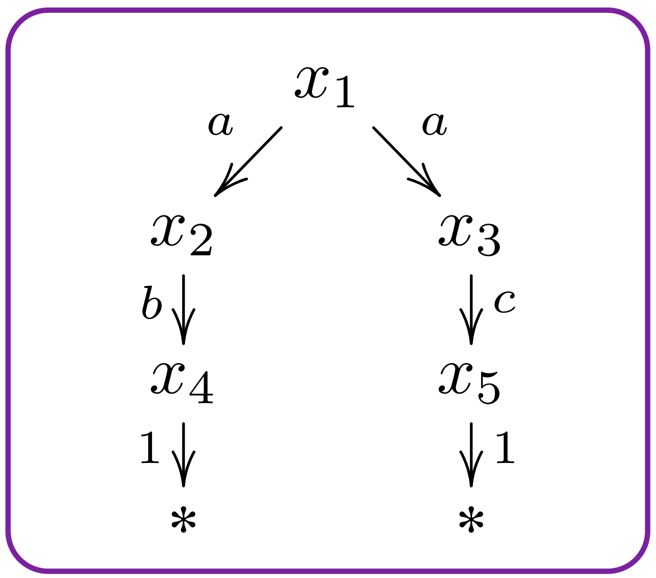

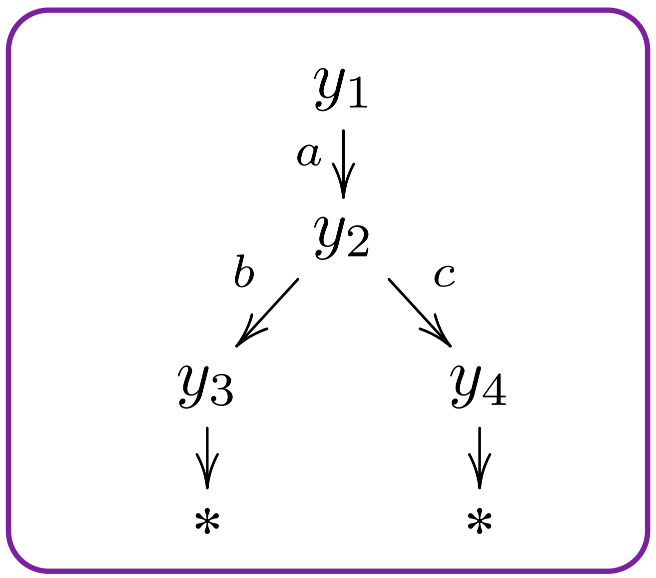

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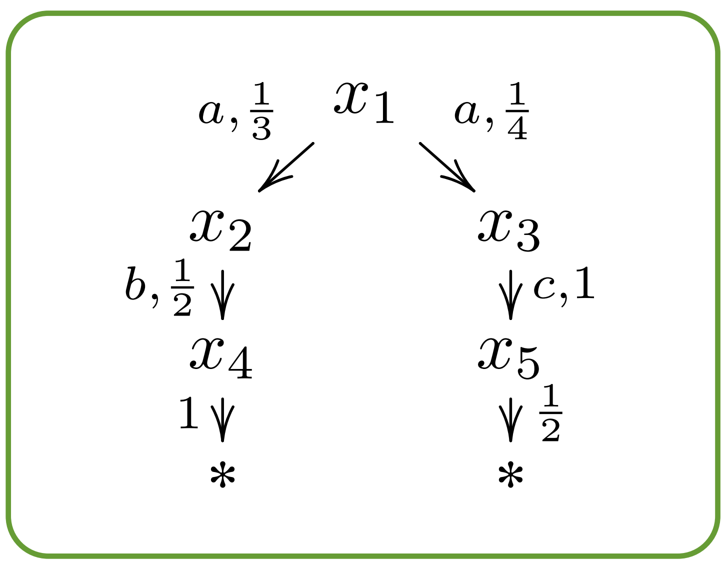

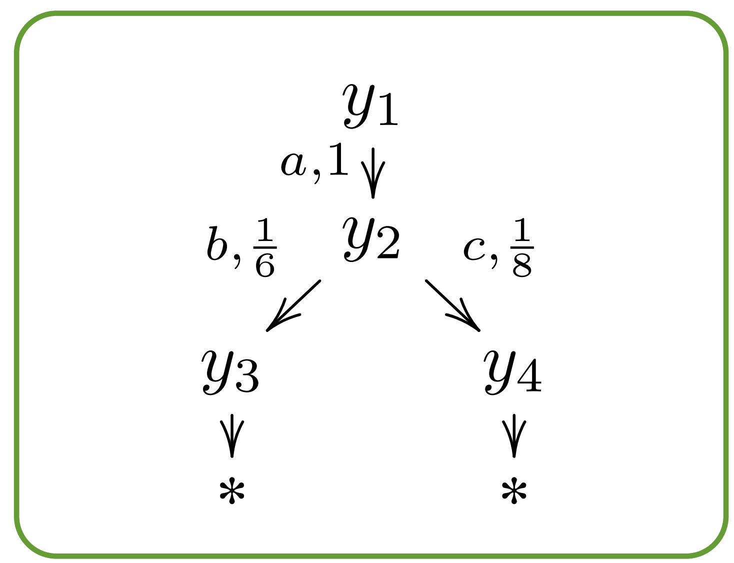

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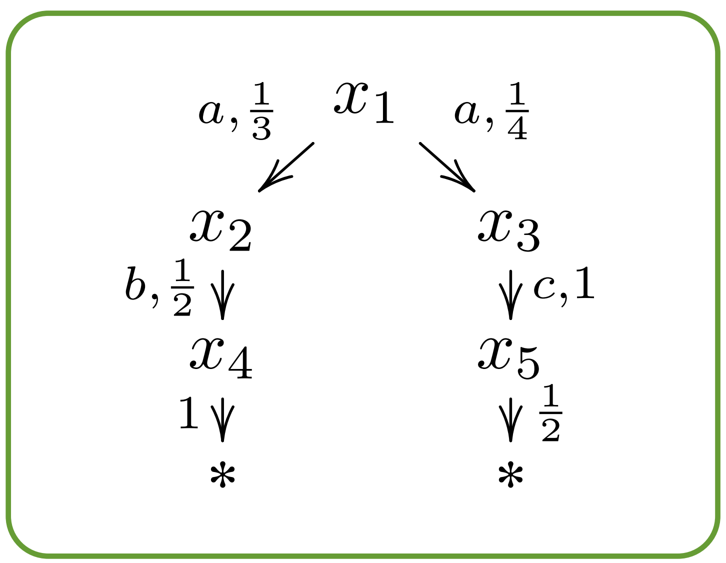

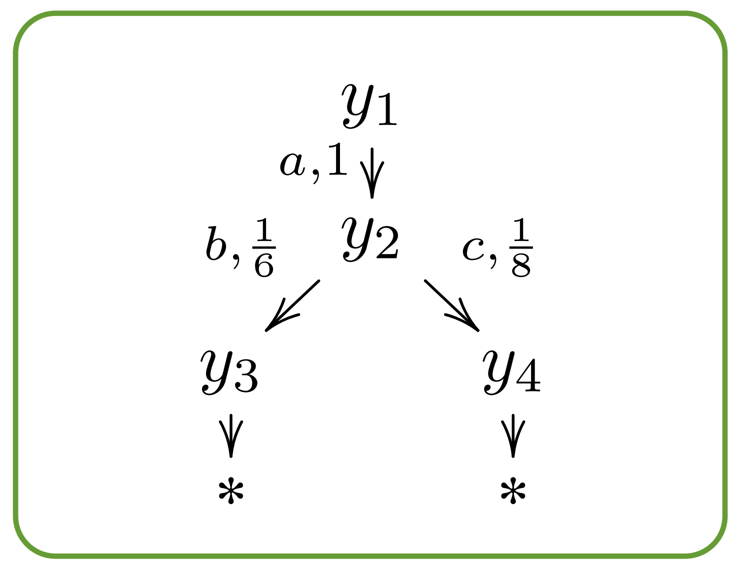

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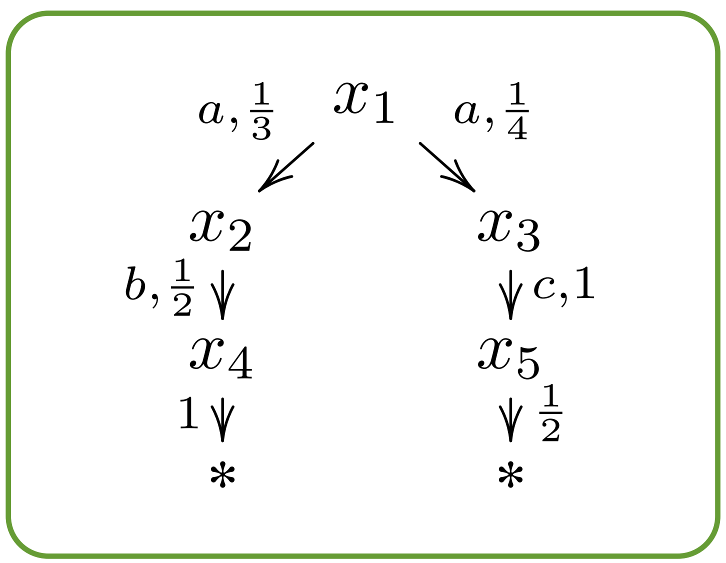

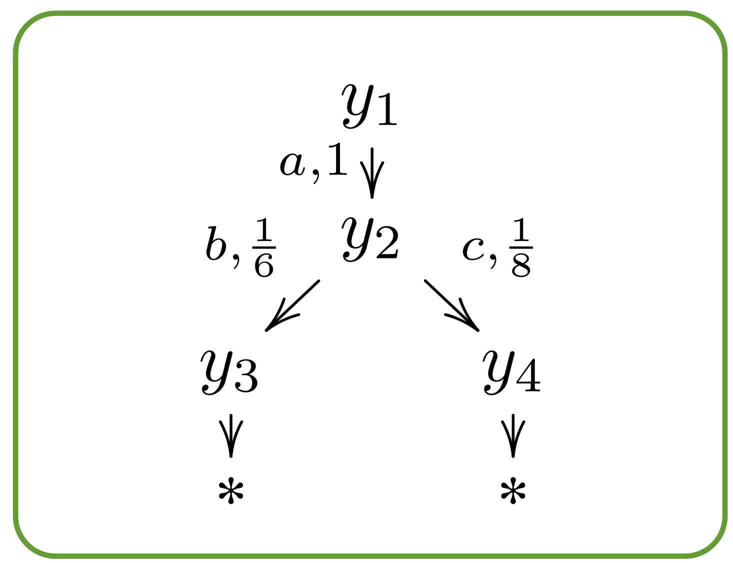

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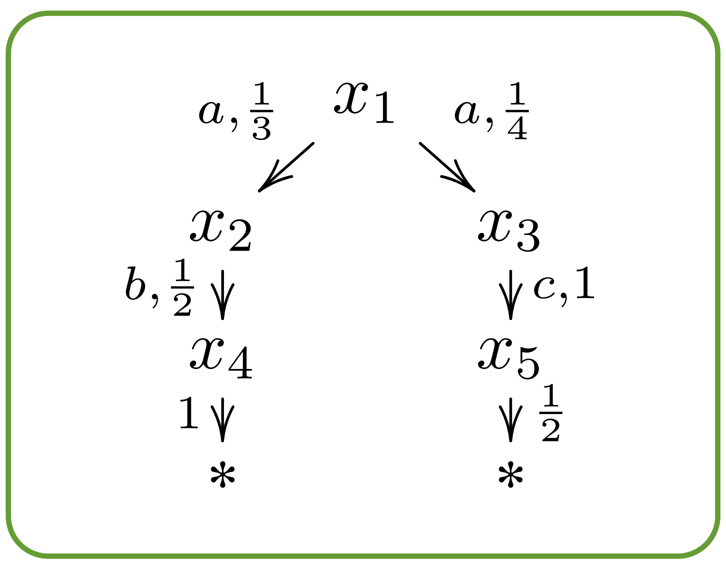

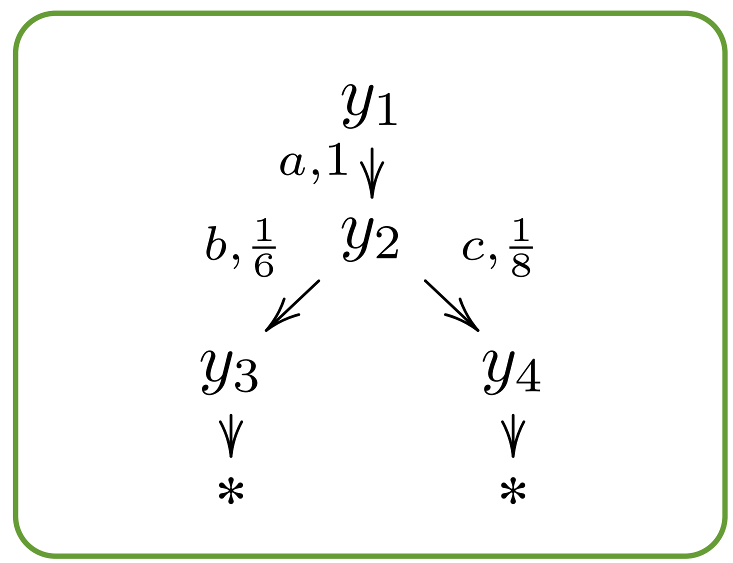

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Trace semantics coalgebraically?

Two ideas: NFA / LTS

(1) unfold branching + transitions on words

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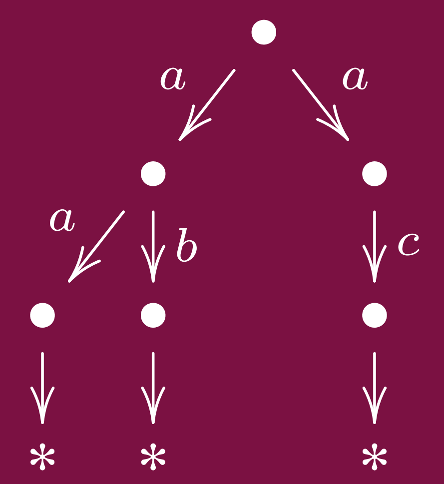

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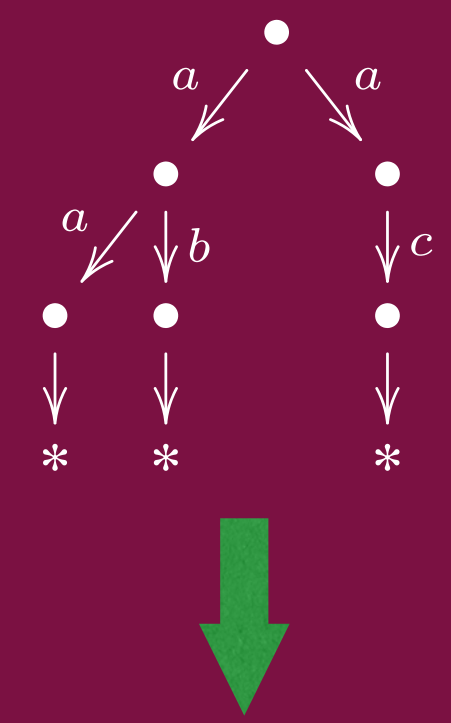

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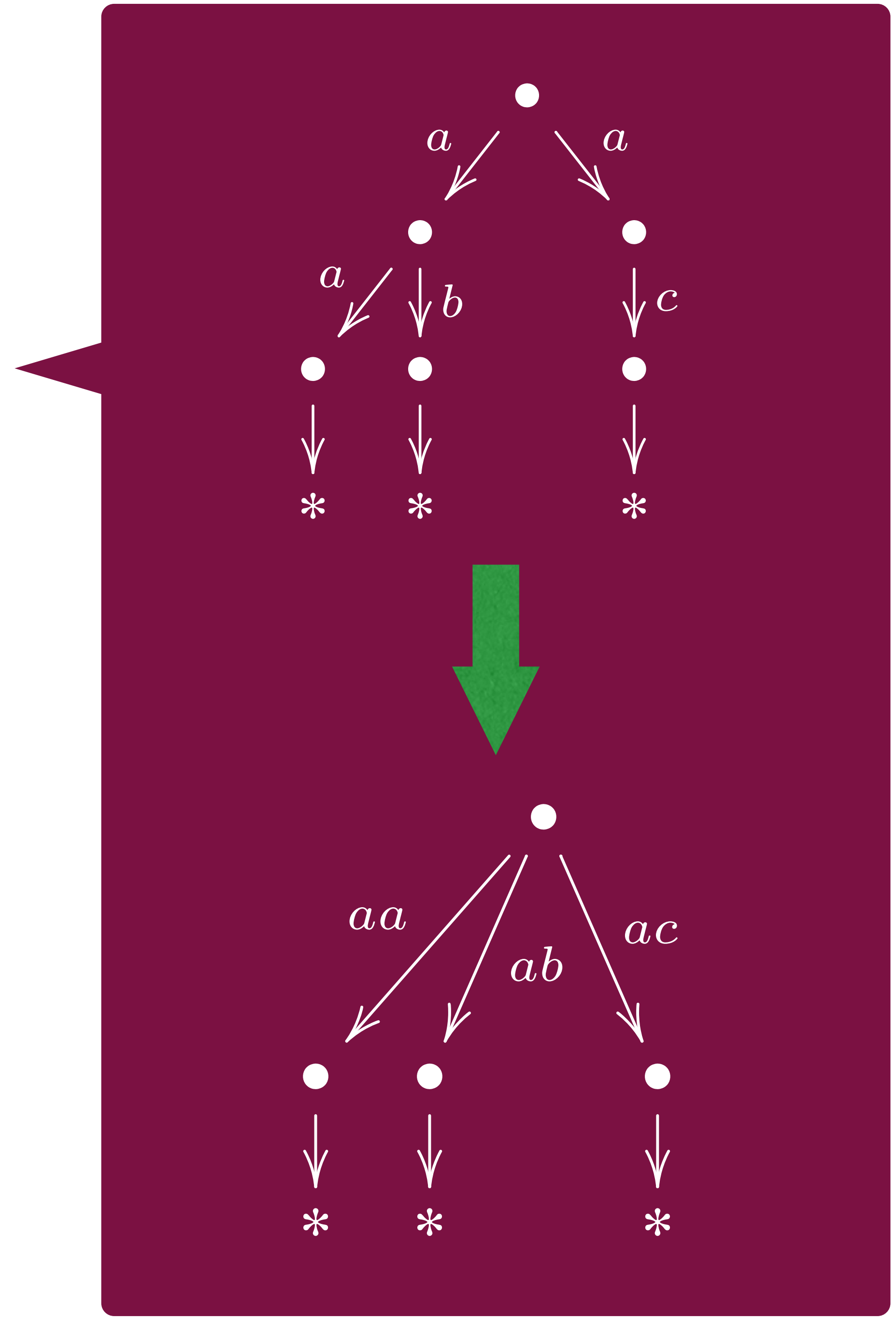

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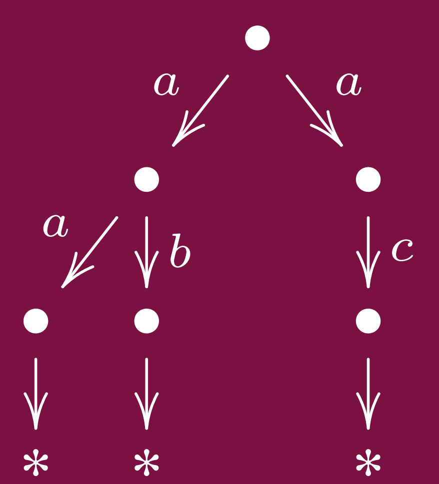
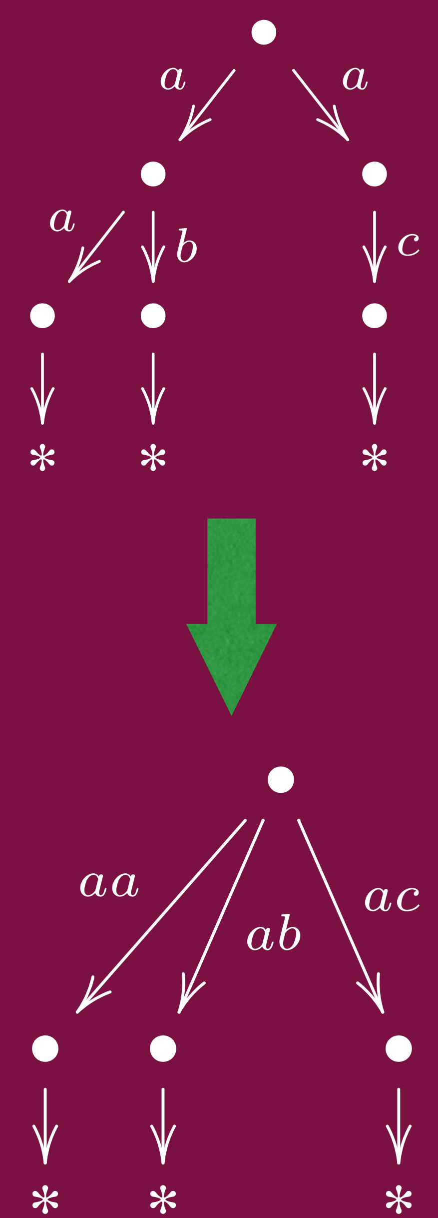
(2) trace = bisimilarity after determinisation





Trace semantics coalgebraically?

NFA / LTS

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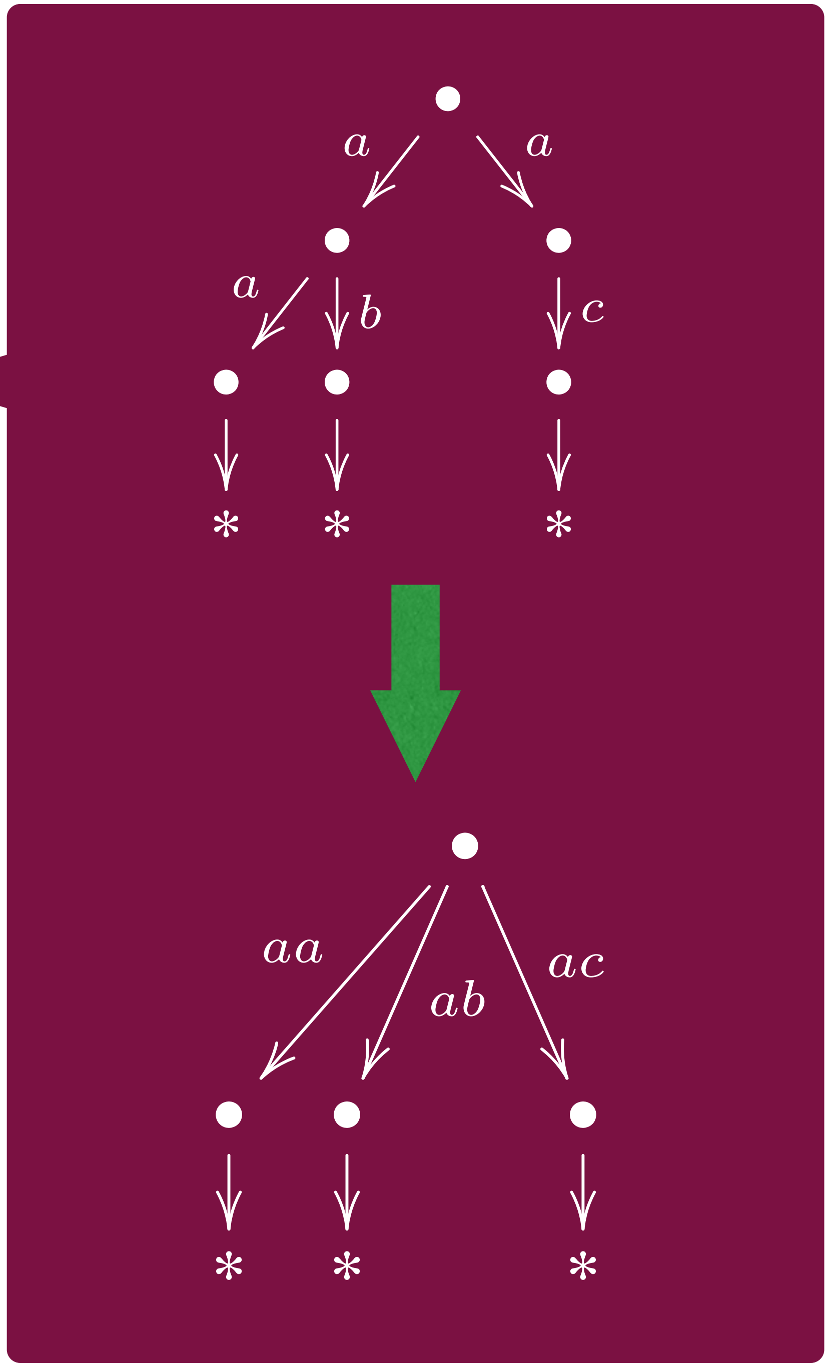

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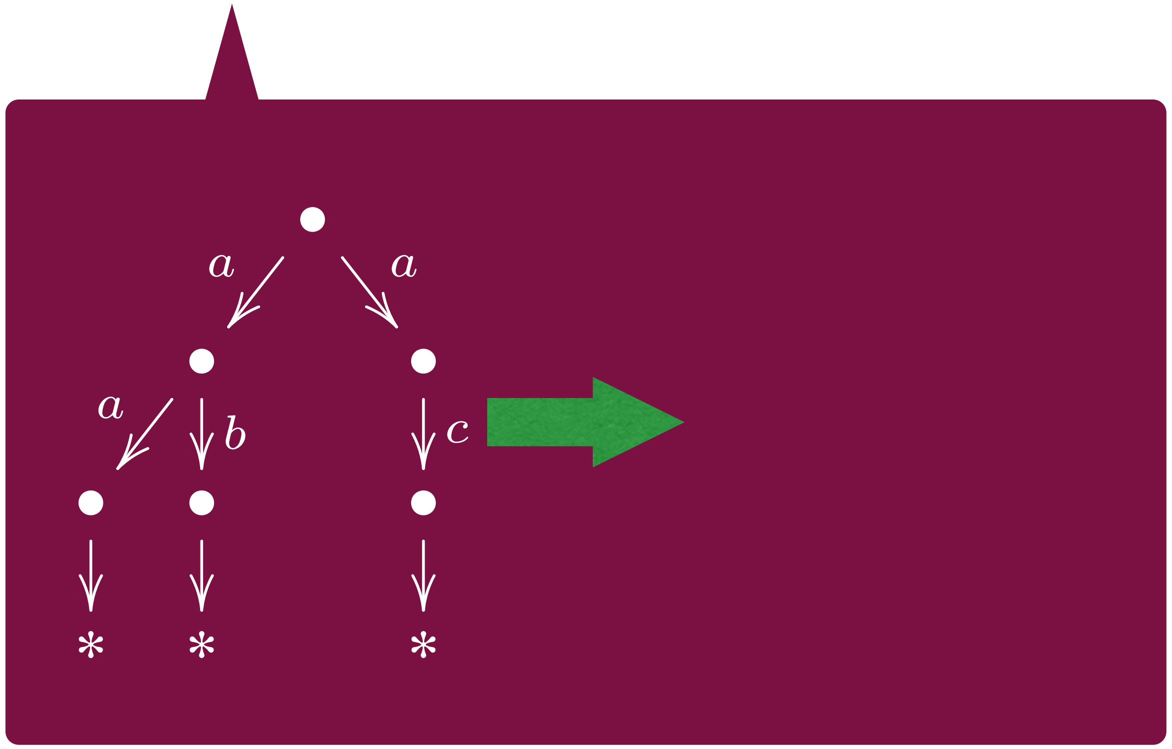

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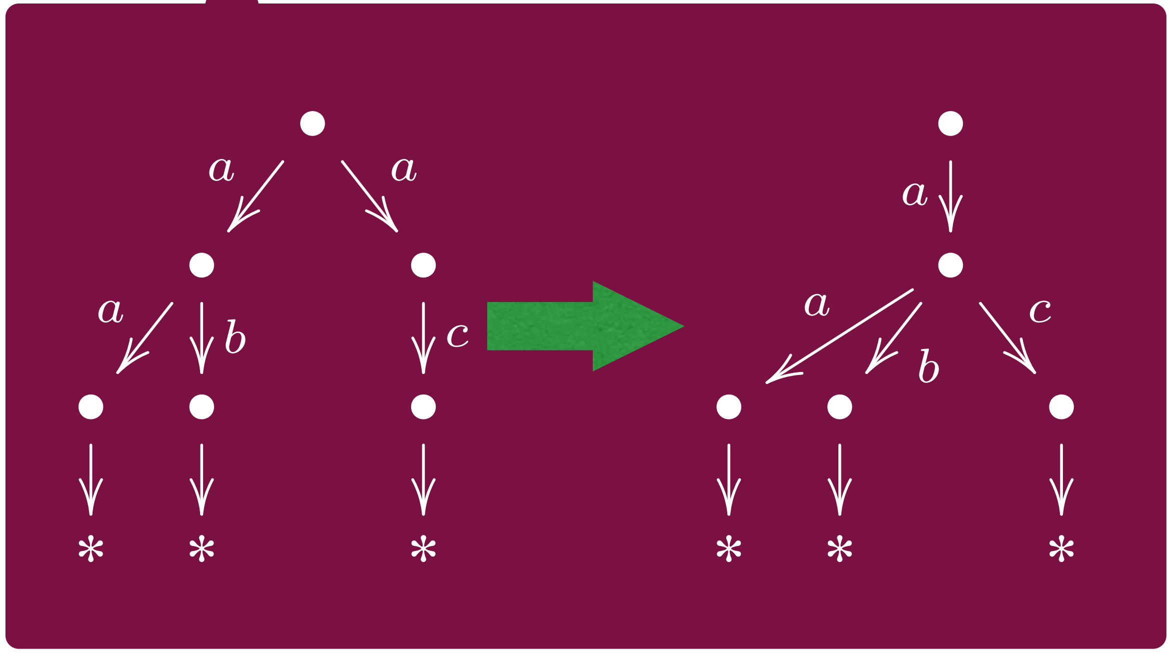

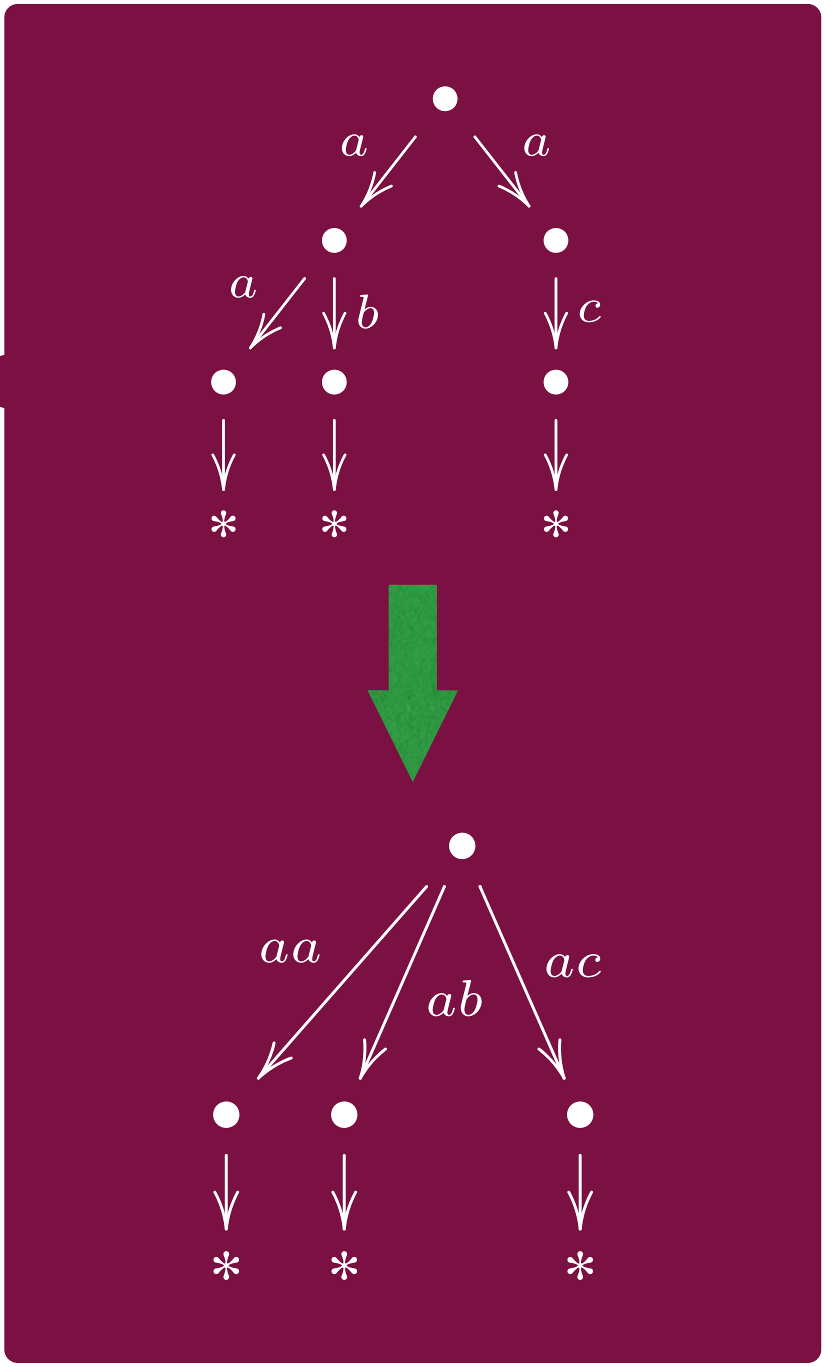

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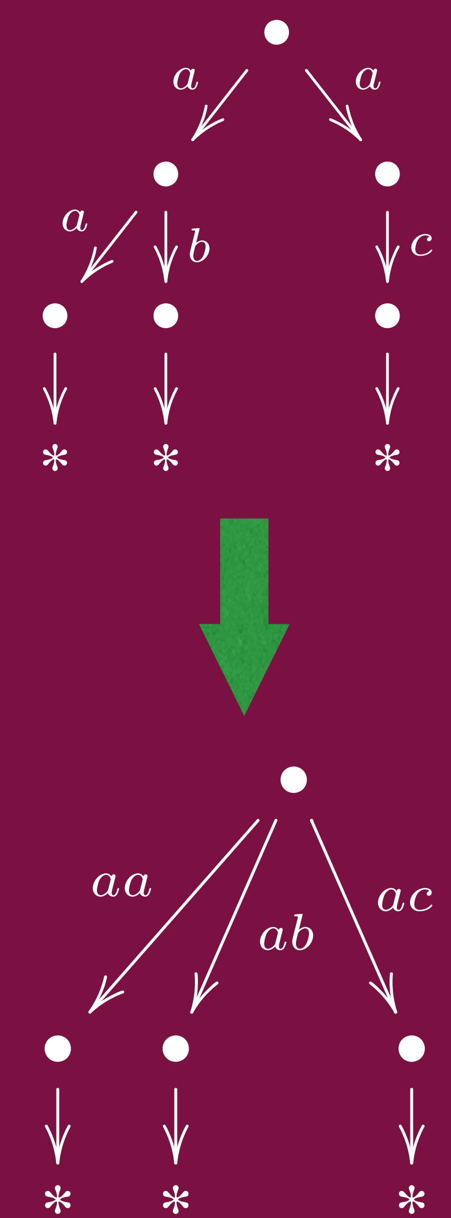

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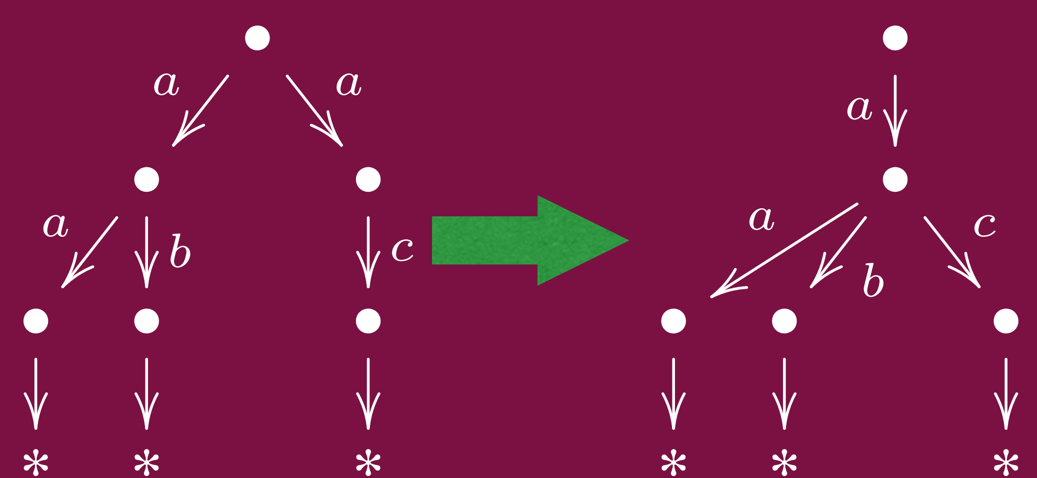

monads !

Trace semantics coalgebraically

Trace semantics coalgebraically

Two approaches:

(1) modelling in a Kleisli category

(2) modelling in an Eilenberg-Moore category

Trace semantics coalgebraically

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algebras of a monad M

Trace semantics coalgebraically

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Hasuo, Jacobs, S. LMCS '07

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we can relate (1) and (2)

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algebras of a monad M

Jacobs, Silva, S. JCSS'15

we can relate (1) and (2)

Traces via determinisation

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

Traces via determinisation

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Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

Determinisation

$MX \to O \times (MX)^A$

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trace = bisimilarity after determinisation

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Algebras for M

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Algebras for M

ideally we have a presentation

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O has to be an M-algebra !

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Algebras for M

ideally we have a presentation

Eilenberg-Moore algebras

Eilenberg-Moore Algebras

abstractly

$\mathcal{EM}(M)$

- objects

$$\begin{array}{c} MA \\ \downarrow a \\ A \end{array}$$

satisfying

$$\begin{array}{ccc} A & \xrightarrow{\eta} & MA \\ & \overset{a}{\diagdown\!\!\diagdown} & \downarrow a \\ & & A \end{array} \qquad \begin{array}{ccc} MMA & \xrightarrow{\mu} & MA \\ Ma \downarrow & & \downarrow a \\ MA & \xrightarrow{a} & A \end{array}$$

- morphisms

$$\begin{array}{c} MA \\ \downarrow a \\ A \end{array} \xrightarrow{h} \begin{array}{c} MB \\ \downarrow b \\ B \end{array}$$

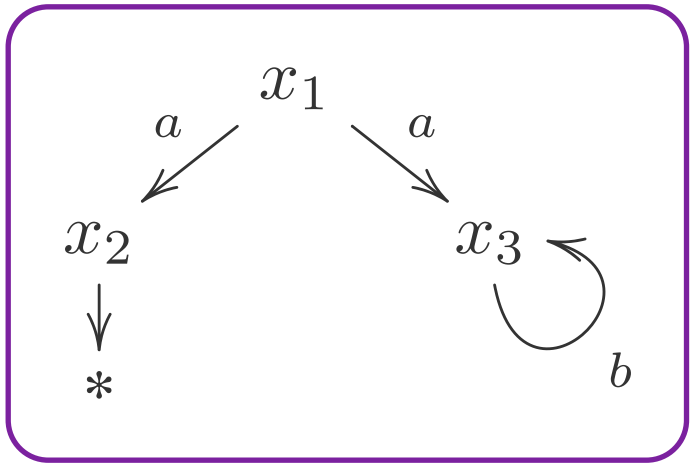
$$\begin{array}{ccc} MA & \xrightarrow{Mh} & MB \\ a \downarrow & & \downarrow b \\ A & \xrightarrow{h} & B \end{array}$$

Traces via determinisation

Traces via determinisation

NFA

$X \to 2 \times (\mathcal{P}X)^A$



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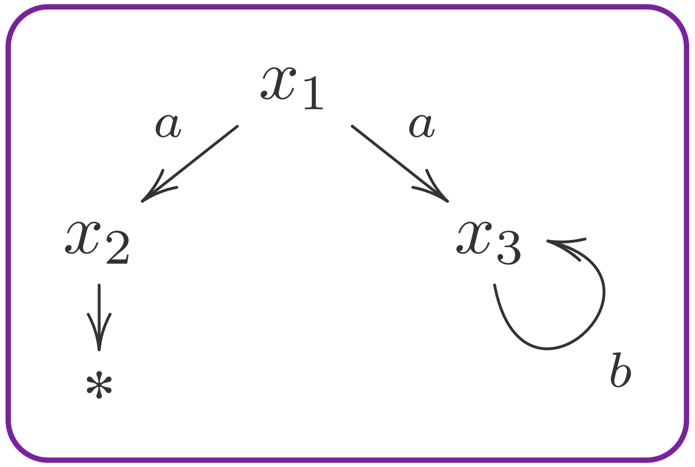

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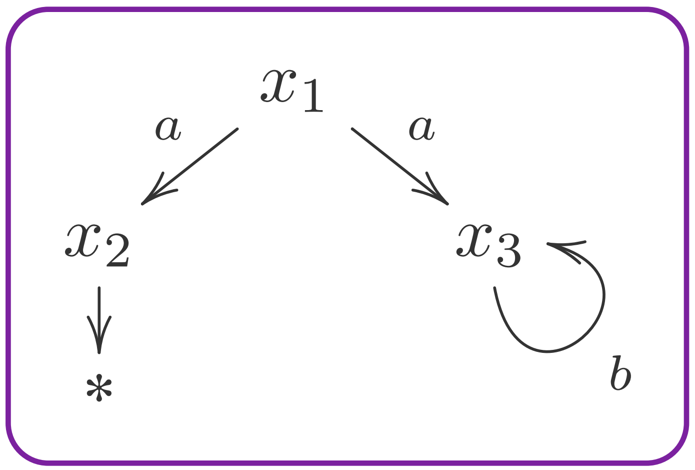
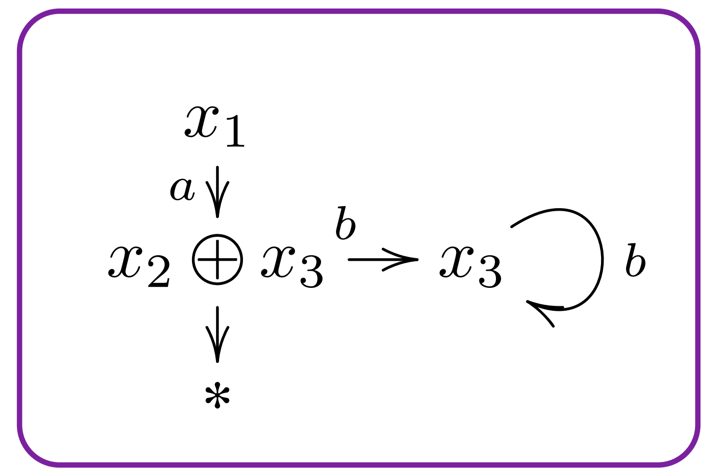


DFA

$\mathcal{P}X \to 2 \times (\mathcal{P}X)^A$



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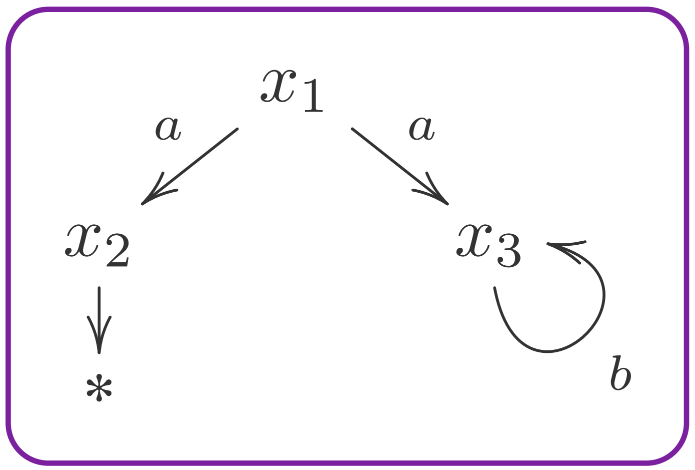

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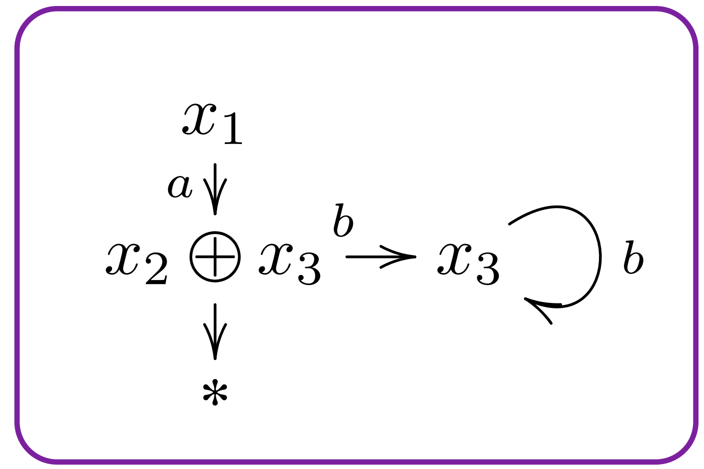

trace = bisimilarity after determinisation

Traces via determinisation

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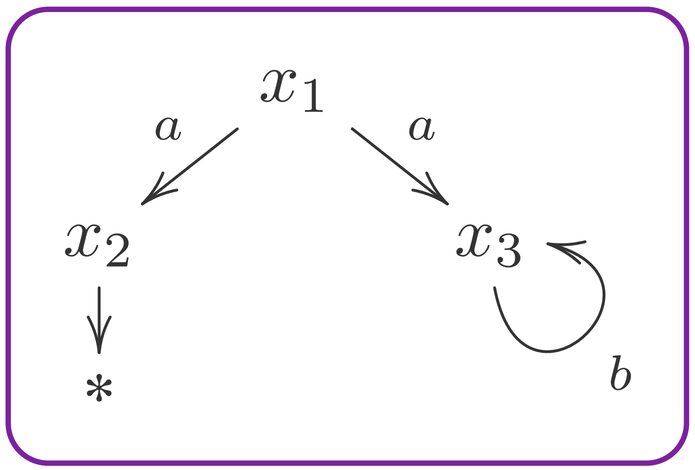

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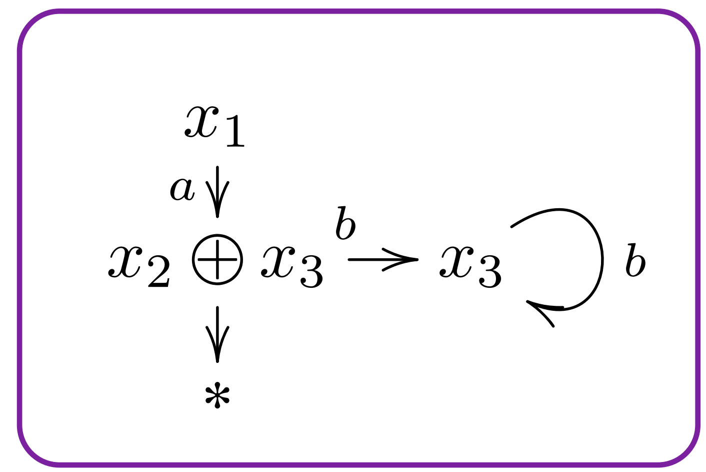

trace = bisimilarity after determinisation

Algebras for $\mathcal{P}$

Traces via determinisation

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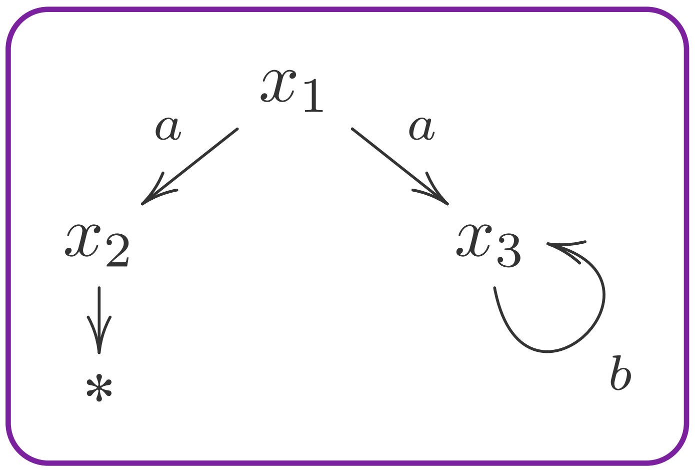

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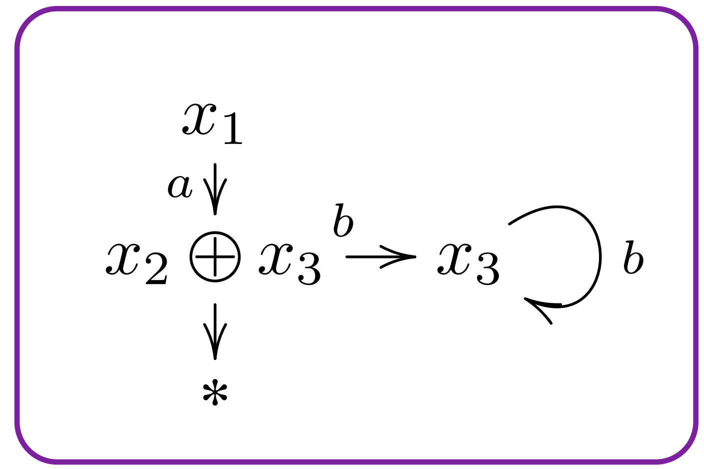

trace = bisimilarity after determinisation

Algebras for $\mathcal{P}$

join semilattices with bottom

Traces via determinisation

NFA

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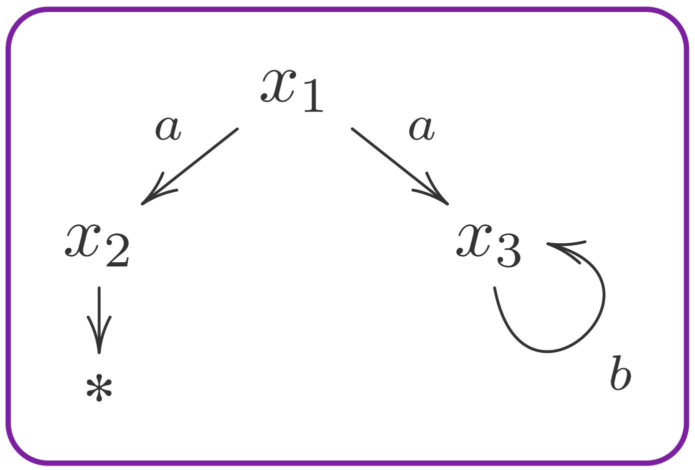

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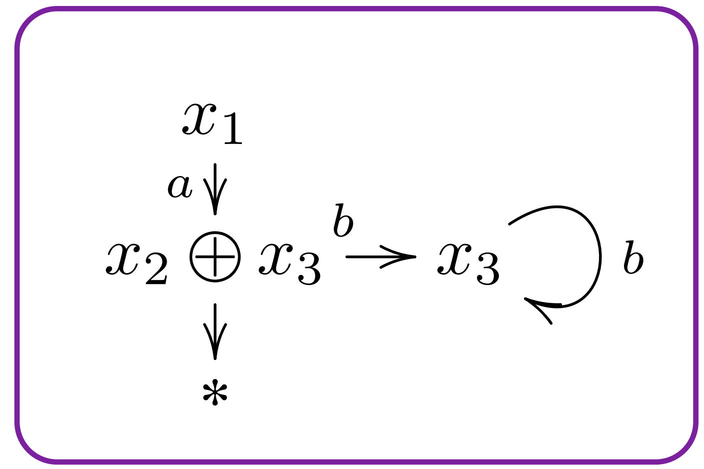


trace = bisimilarity after determinisation

Algebras for $\mathcal{P}$

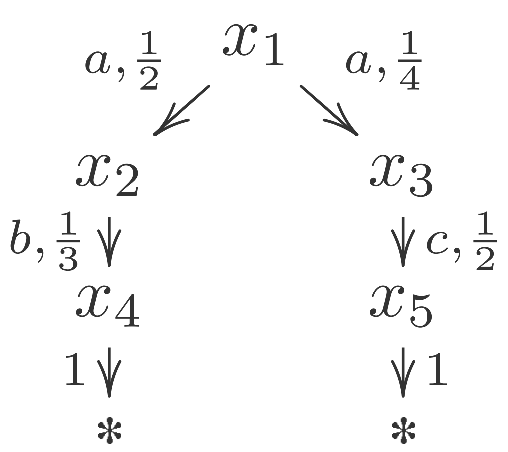
join semilattices with bottom

finite powerset !

Traces via determinisation

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



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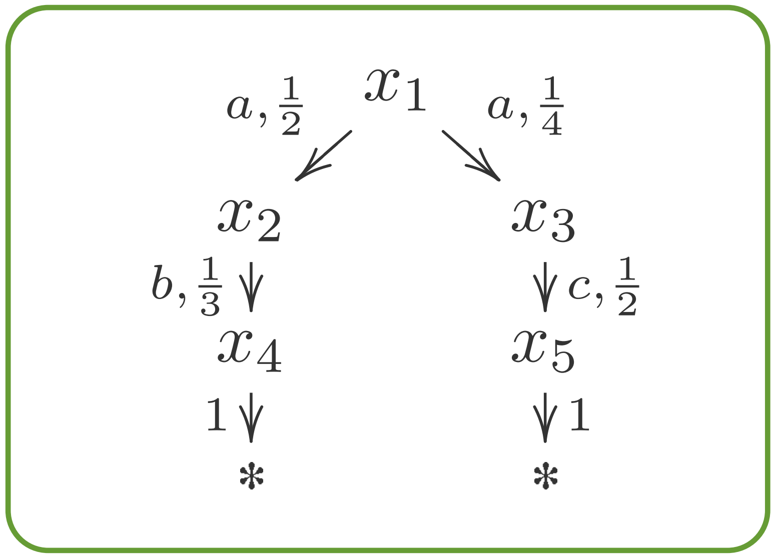

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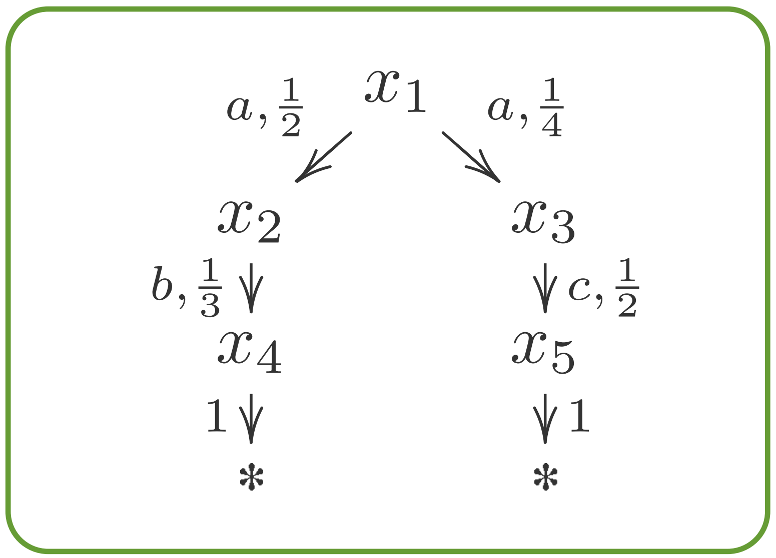
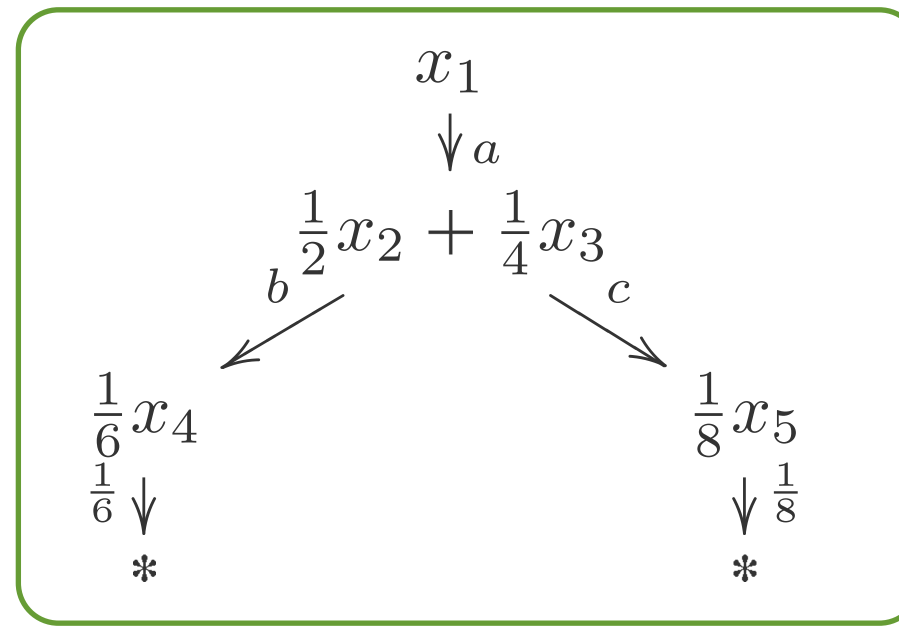


DFA

$\mathcal{D}_{\leq 1} X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



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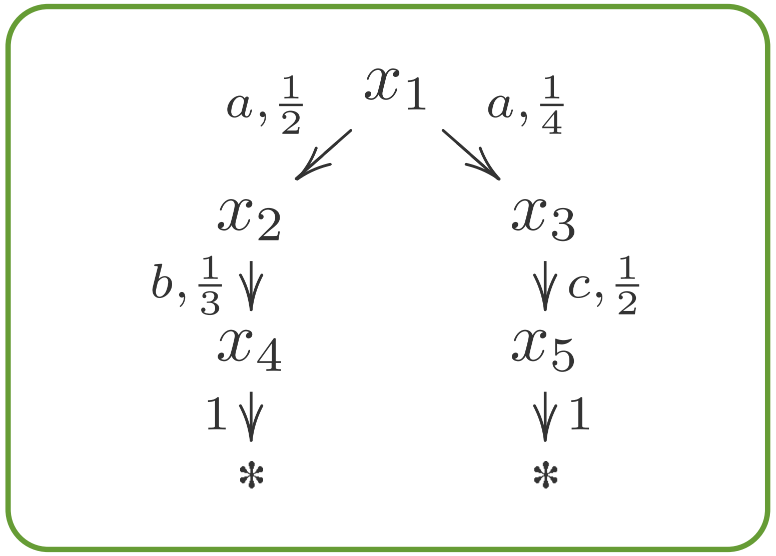

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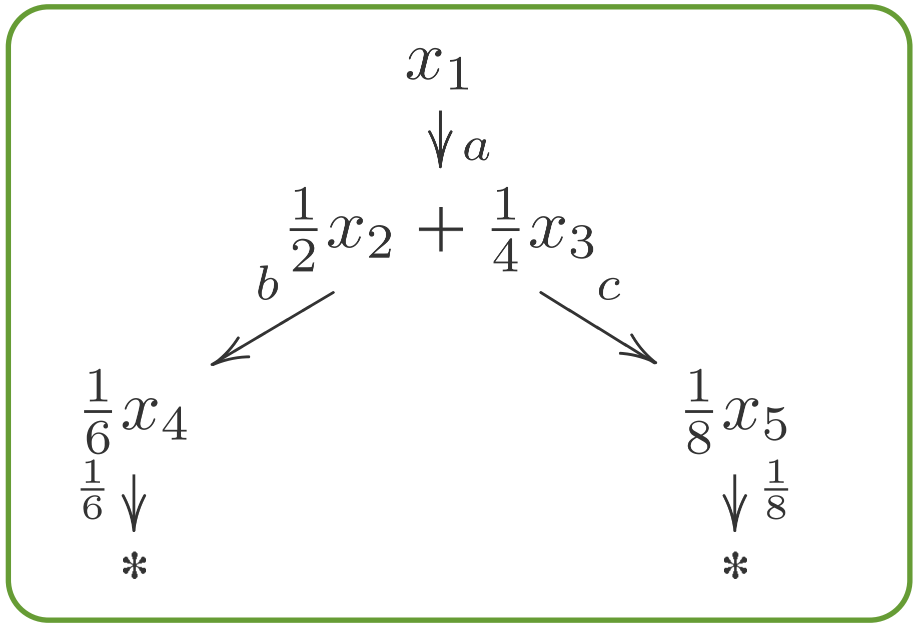

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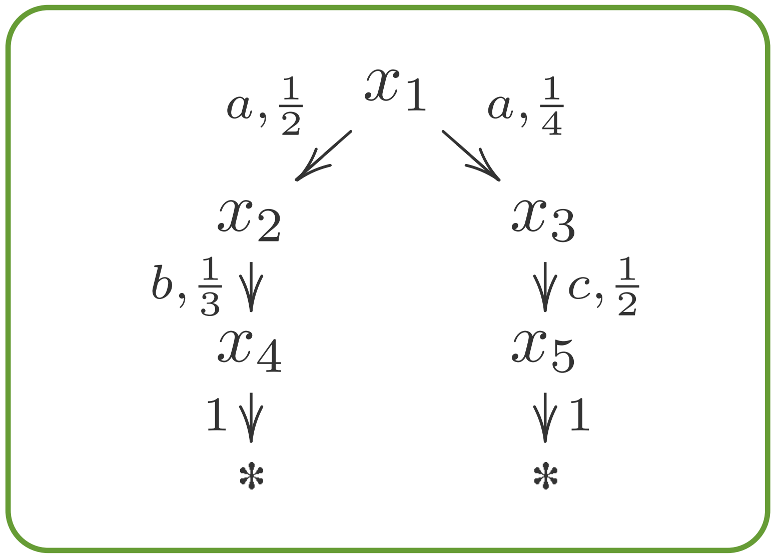

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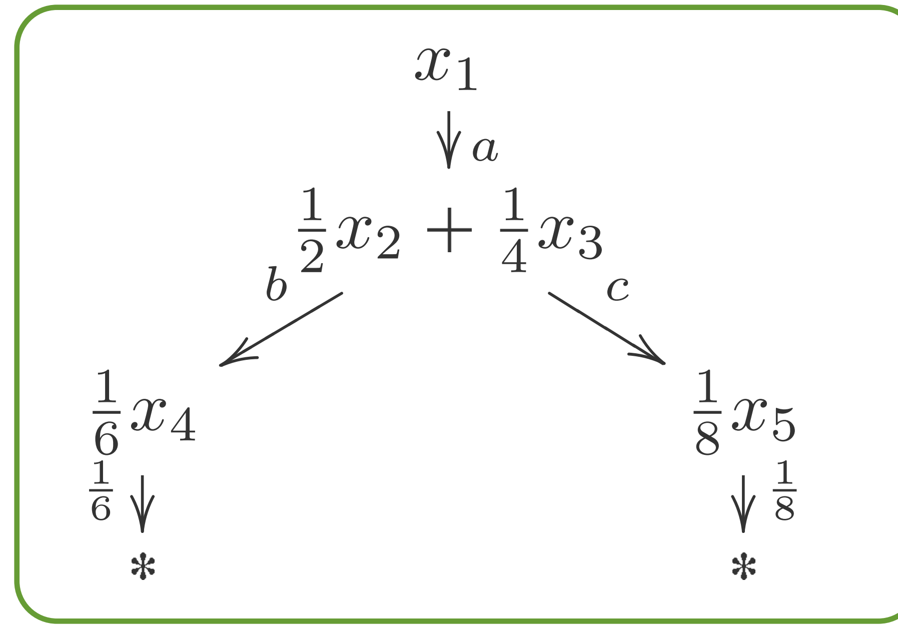

Algebras for $\mathcal{D}_{(\leq 1)}$

trace = bisimilarity after determinisation

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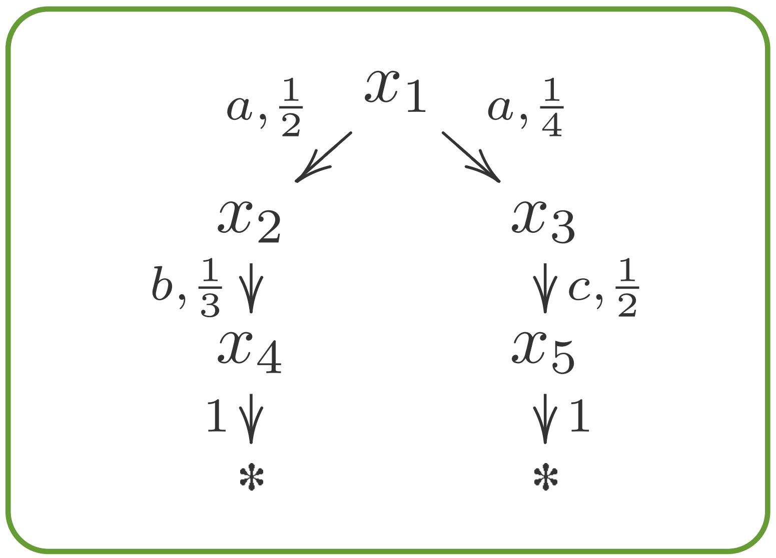

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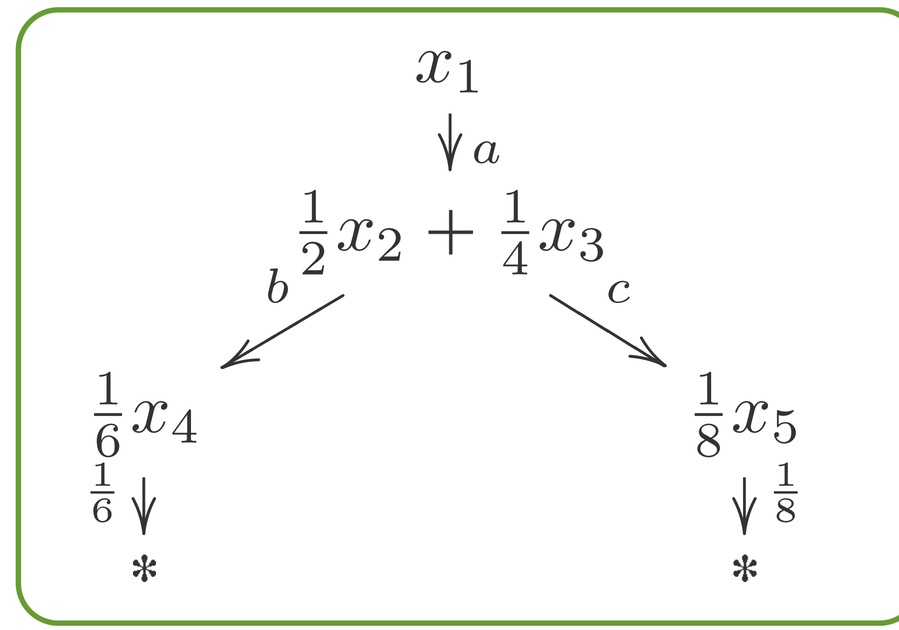

trace = bisimilarity after determinisation

Algebras for $\mathcal{D}_{(\leq 1)}$

(positive) convex algebras

Traces via determinisation

Rabin PA

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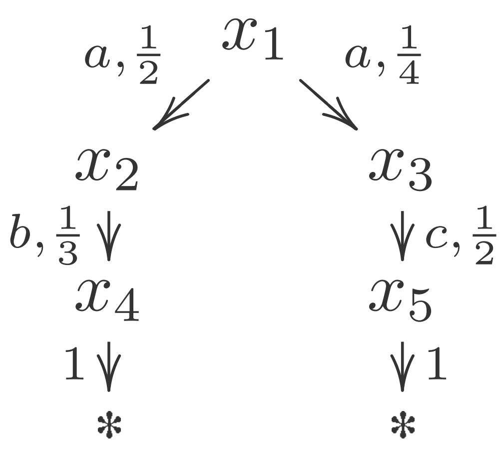

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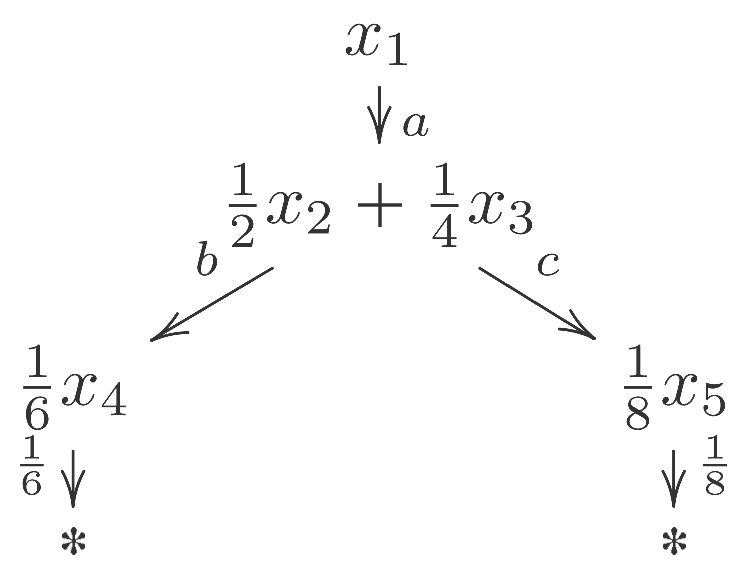


Algebras for $\mathcal{D}_{(\leq 1)}$

(positive) convex algebras

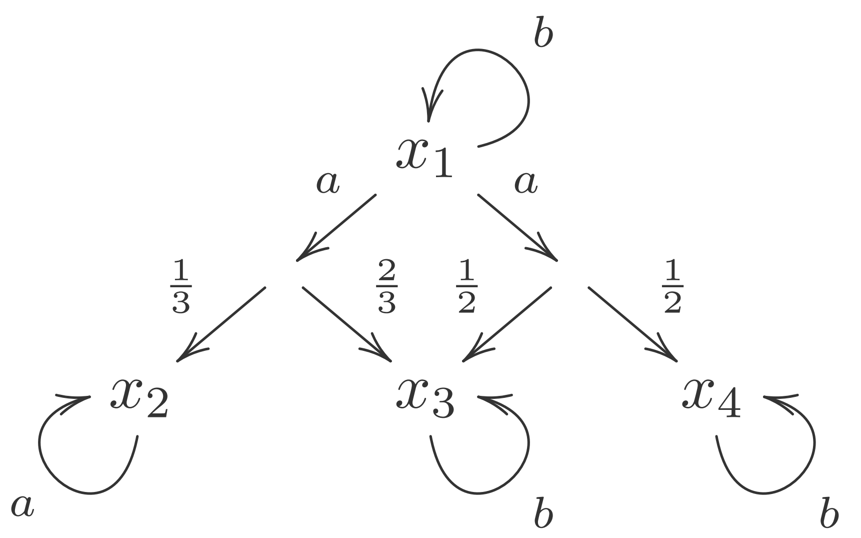
finitely supported (sub)distributions!

trace = bisimilarity after determinisation

Traces via determinisation

Simple PA

$X \to ? \times (\mathcal{C}X)^A$



Traces via determinisation

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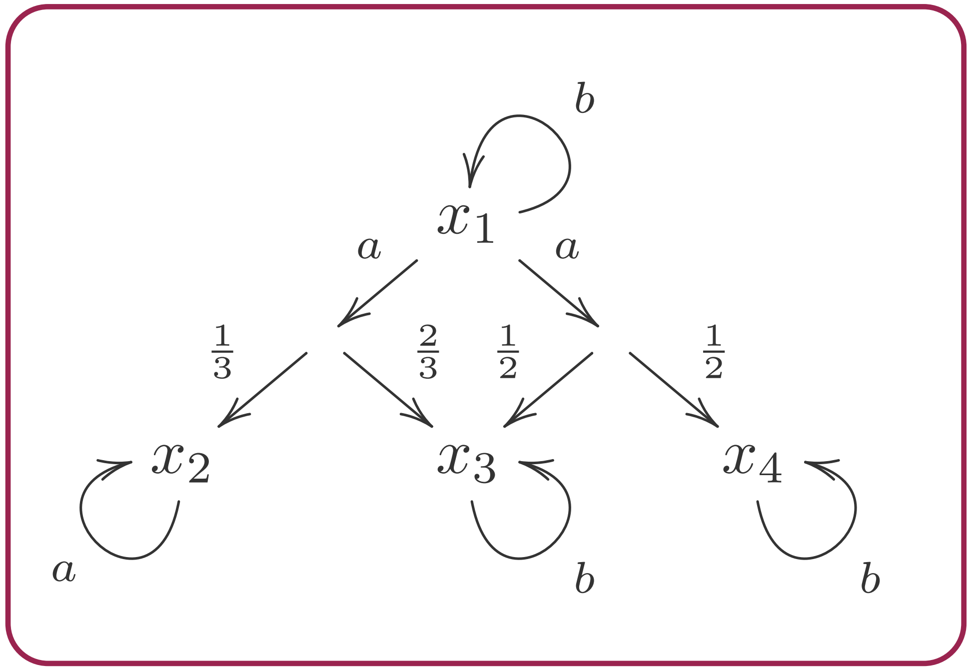

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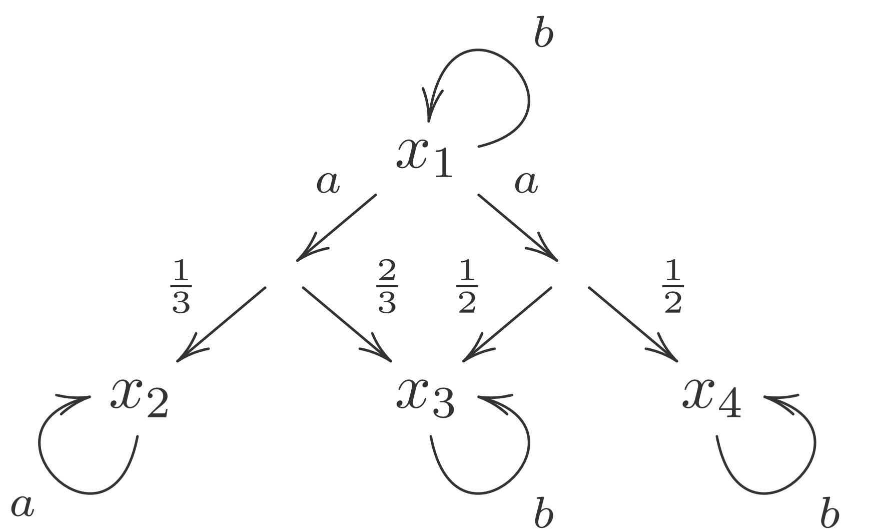
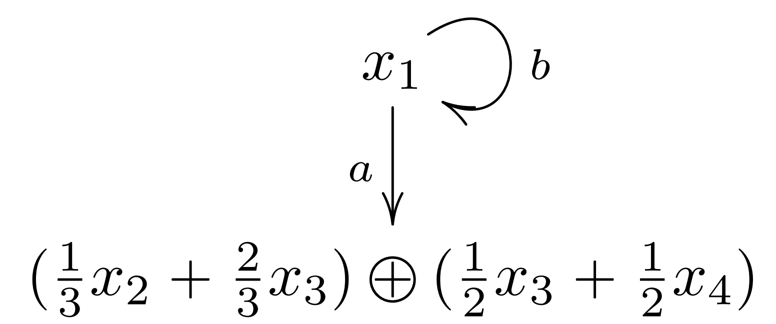


DFA

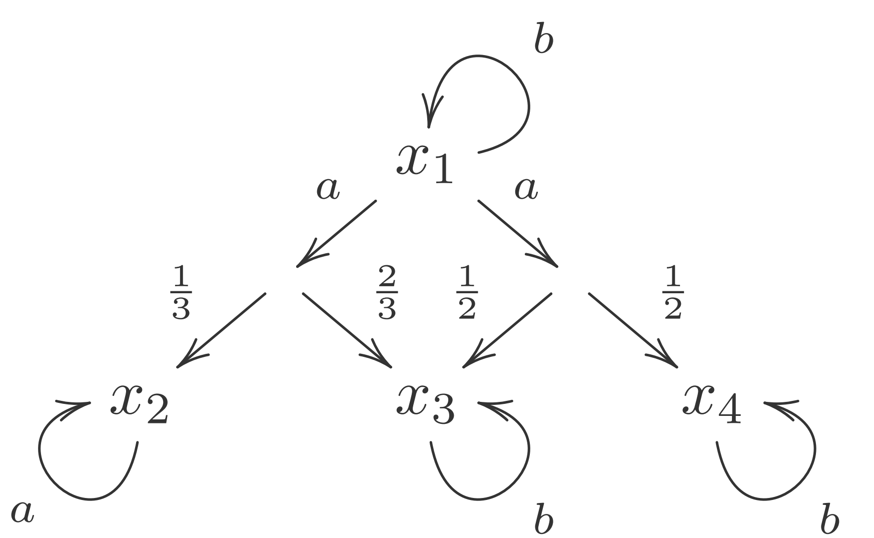
$\mathcal{C}X \to ? \times (\mathcal{C}X)^A$



Traces via determinisation

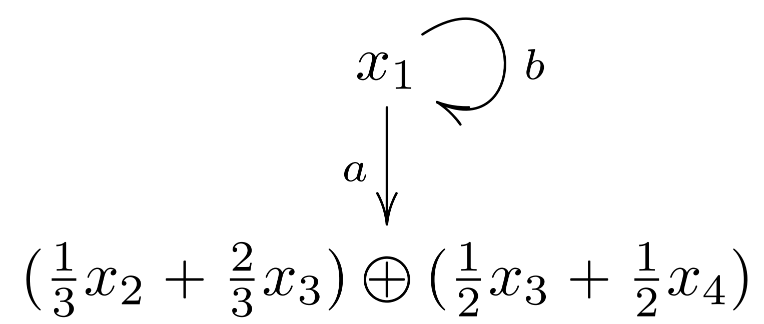
Simple PA

$X \rightarrow ? \times (\mathcal{C}X)^A$



DFA

$\mathcal{C}X \rightarrow ? \times (\mathcal{C}X)^A$

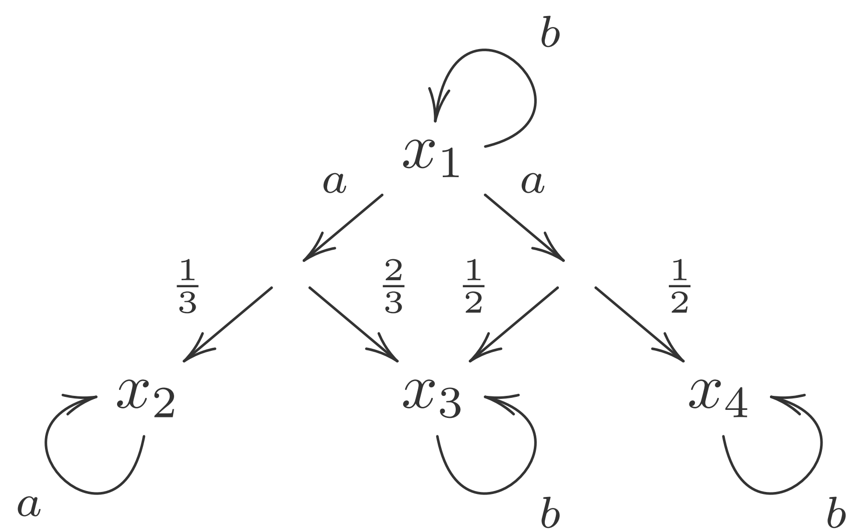


trace = bisimilarity after determinisation

Traces via determinisation

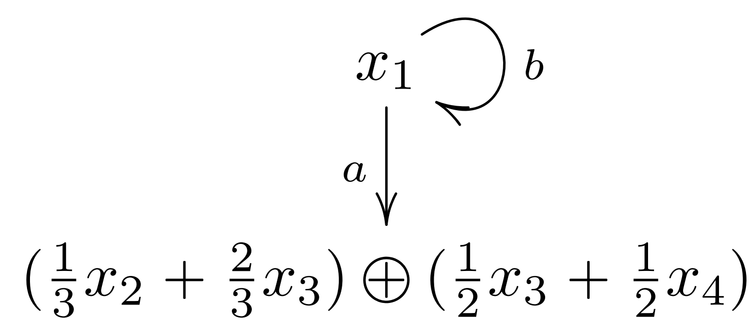
Simple PA

$X \to ? \text{ x } (\mathcal{C}X)^A$



DFA

$\mathcal{C}X \to ? \text{ x } (\mathcal{C}X)^A$



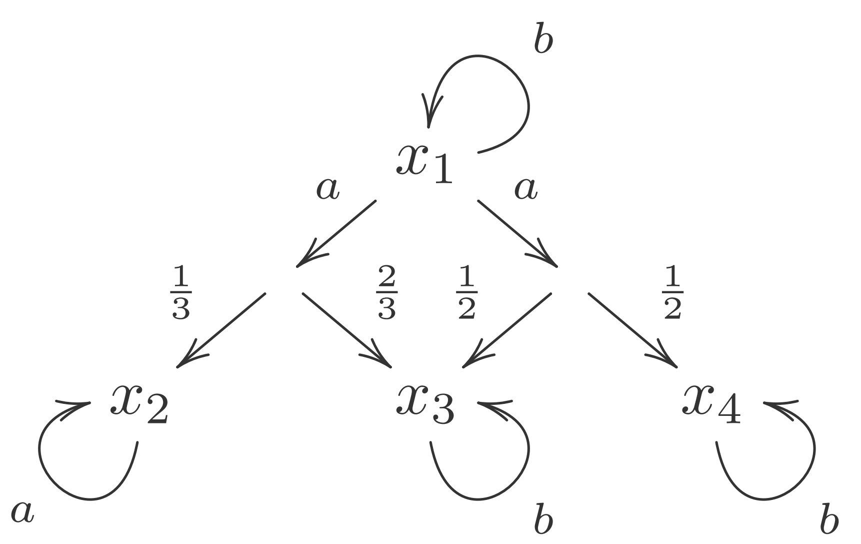
trace = bisimilarity after determinisation

Algebras for *C*

Traces via determinisation

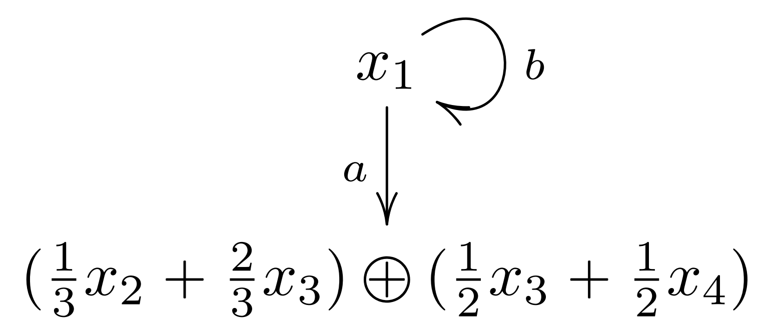
Simple PA

X → ? x ($\mathcal{C}$X)^A



DFA

$\mathcal{C}$X → ? x ($\mathcal{C}$X)^A



trace = bisimilarity after determinisation

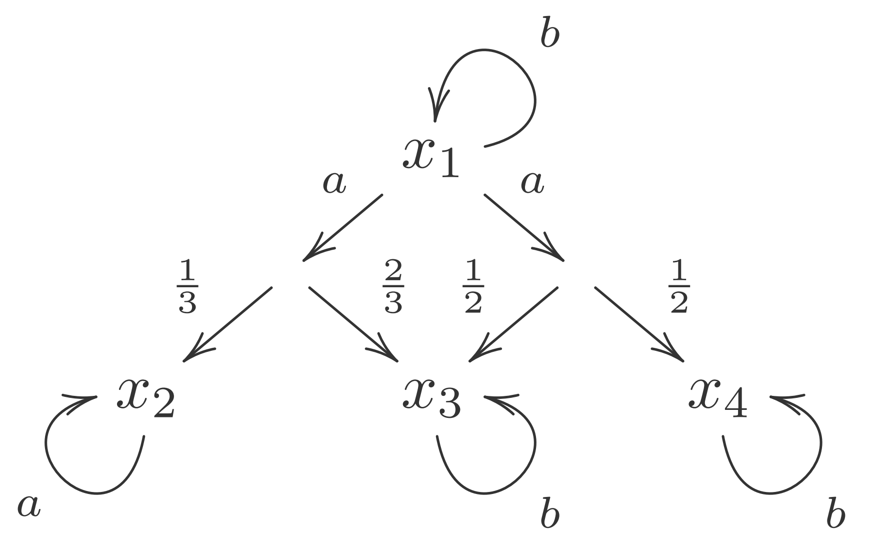
Algebras for C

convex semilattices

Traces via determinisation

Simple PA

$X \to ? \times (\mathcal{C}X)^A$



DFA

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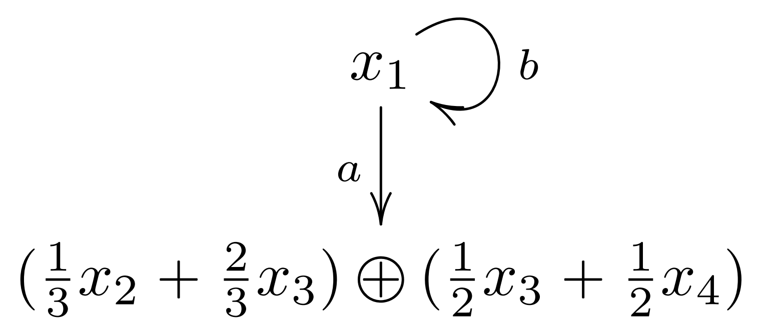

trace = bisimilarity after determinisation

Algebras for C

convex semilattices

finitely generated convex sets of distr…

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

finitely generated convex sets of distr…

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

$$\mathbb{A} = (A, \oplus, +_p)$$

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

Bonchi, S., Vignudelli '19

finitely generated convex sets of distr…

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

Bonchi, S., Vignudelli ‘19

finitely generated convex sets of distr…

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

$$(x \oplus y) \oplus z \overset{(A)}{=} x \oplus (y \oplus z)$$
$$x \oplus y \overset{(C)}{=} y \oplus x$$
$$x \oplus x \overset{(I)}{=} x$$

$$(x +_q y) +_p z \overset{(A_p)}{=} x +_{pq} (y +_{\frac{p(1-q)}{1-pq}} z)$$
$$x +_p y \overset{(C_p)}{=} y +_{1-p} x$$
$$x +_p x \overset{(I_p)}{=} x$$

$$(x \oplus y) +_p z \overset{(D)}{=} (x +_p z) \oplus (y +_p z)$$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

semilattice

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

$$(x \oplus y) \oplus z \overset{(A)}{=} x \oplus (y \oplus z)$$
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Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

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Bonchi, S., Vignudelli '19

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$p \in (0,1)$

$$\begin{aligned} (x \oplus y) \oplus z &\overset{(A)}{=} x \oplus (y \oplus z) \\ x \oplus y &\overset{(C)}{=} y \oplus x \\ x \oplus x &\overset{(I)}{=} x \end{aligned}$$

semilattice

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Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

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distributivity

Many general properties
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proof technique

Three things to take home:

1. Semantics via determinisation
is easy for systems / automata with M-effects

2. Having a presentation for M gives us syntax

3. Having the syntax makes determinisation natural !

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Thank You !