Quantitatively Relaxed Data Structures

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The goal

- Trading correctness for performance
- In a controlled way with quantitative bounds

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measure the error from correct behavior

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Stack - incorrect behavior

push(a)push(b)push(c)pop(a)pop(b)

- Trading correctness for performance
- In a controlled way with quantitative bounds

correct in a relaxed stack
... 2-relaxed? 3-relaxed?

measure the error from correct behavior

Stack example

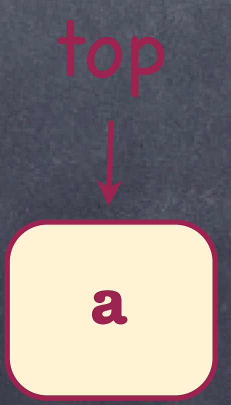
```
push(a)push(b)push(c)pop(a)pop(b)
```

state evolution

Stack example

`push(a)push(b)push(c)pop(a)pop(b)`

state evolution



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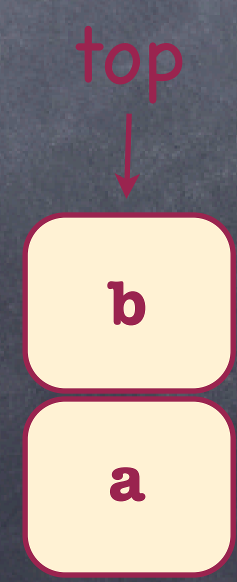

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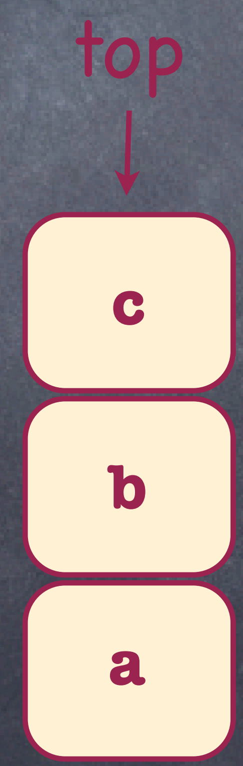

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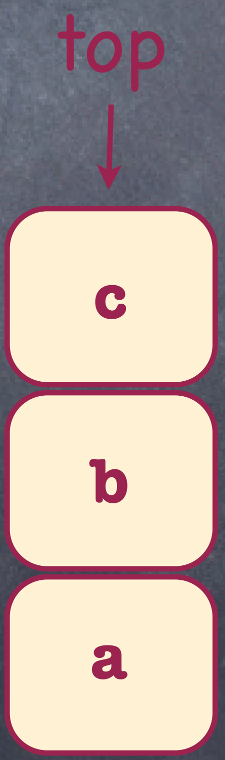

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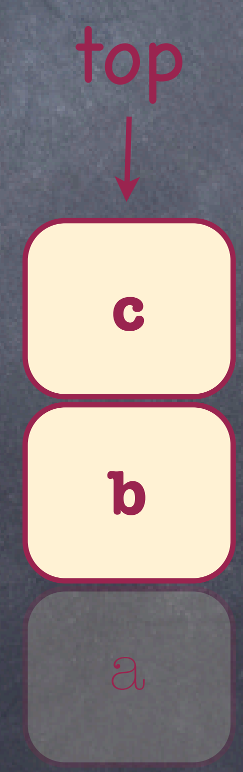

???

How much does this error cost?

Stack example

push(a)push(b)push(c)pop(a)pop(b)

state evolution

top

c

b

a

Cost 2

Stack example

push(a)push(b)push(c)pop(a)pop(b)

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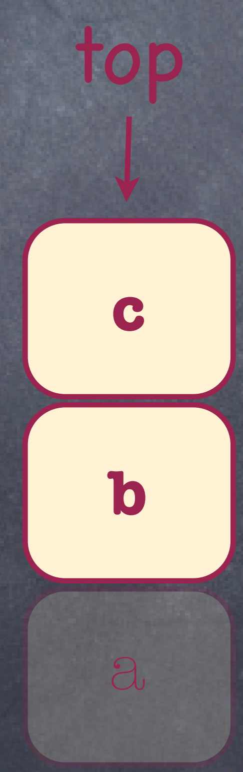

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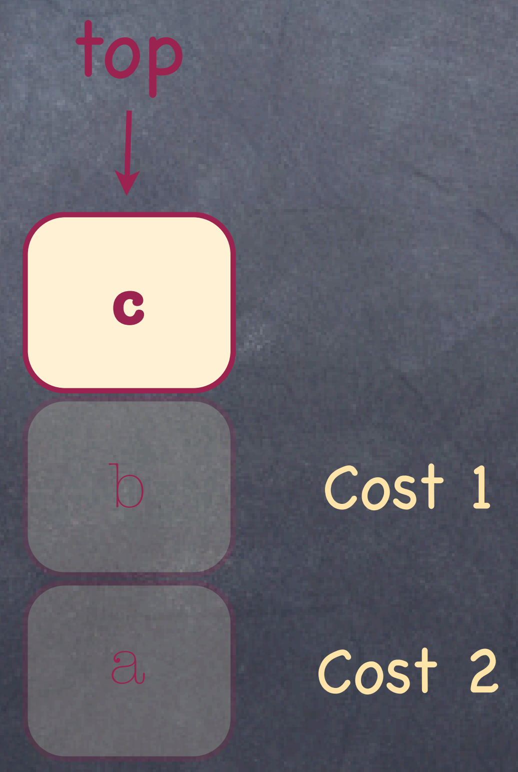

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`push(a)push(b)push(c)pop(a)pop(b)`

state evolution

top

c

b — Cost 1

a — Cost 2

Total cost?

Stack example

push(a)push(b)push(c)pop(a)pop(b)

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c

b

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Cost 1

Cost 2

max = 2

sum = 3

Total cost?

Why relax?

- It is theoretically interesting
- Provides potential for better performing concurrent implementations

...

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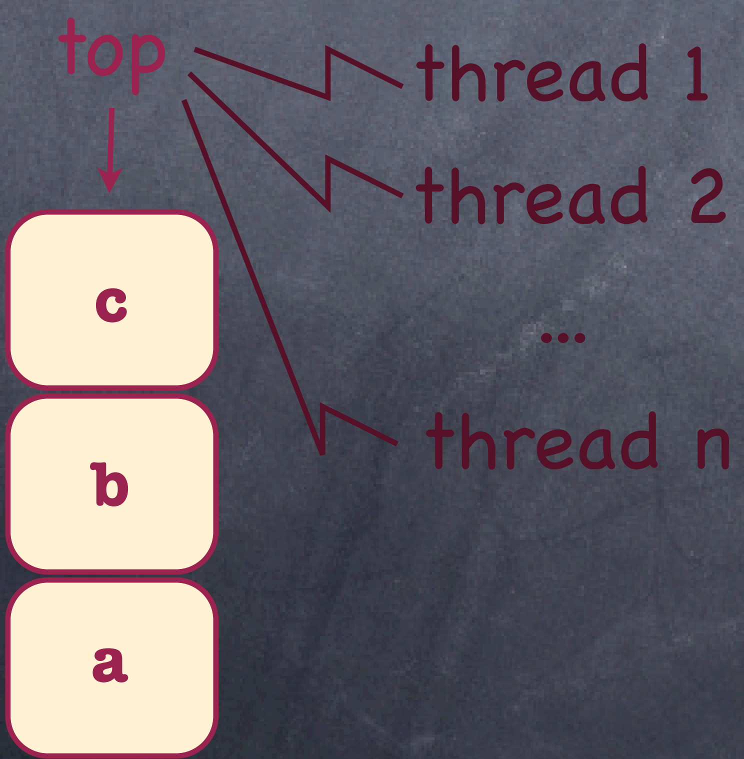
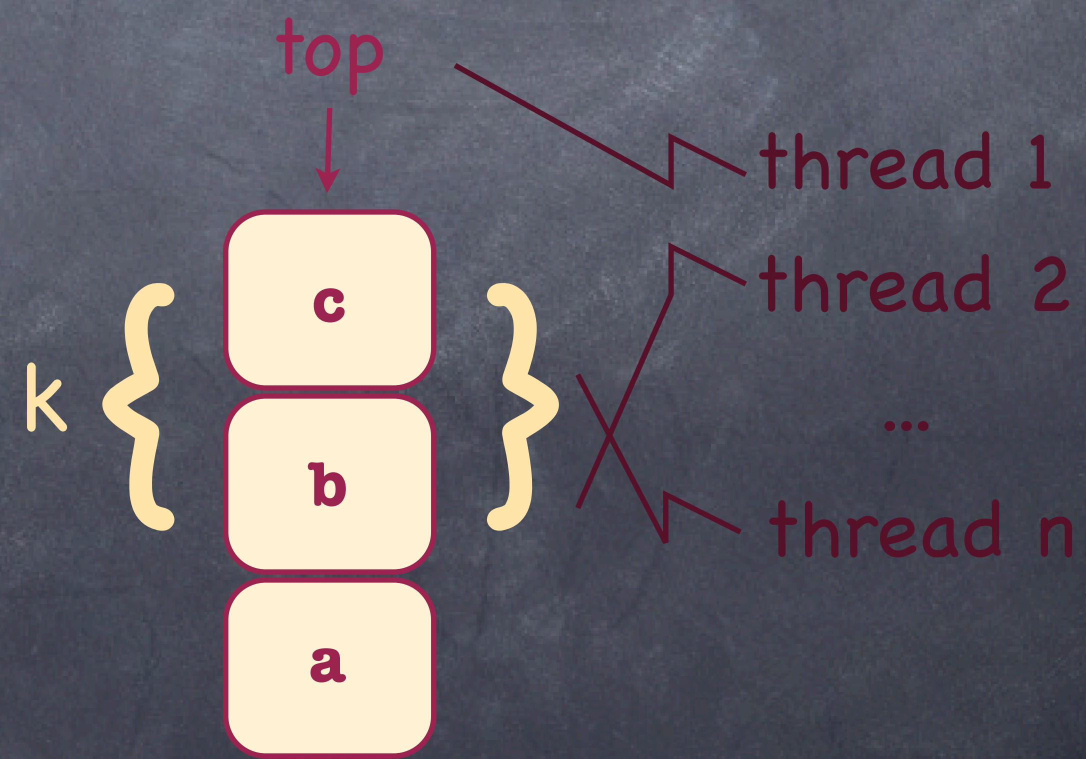
Stack



k-Relaxed stack



What we have

- Framework
  - for semantic relaxations
- Generic example
  - for ordered data structures
- Concrete relaxation examples
  - stacks, queues, priority queues,..
- Efficient concurrent implementations
  - of relaxation instances

Enough introduction

☺

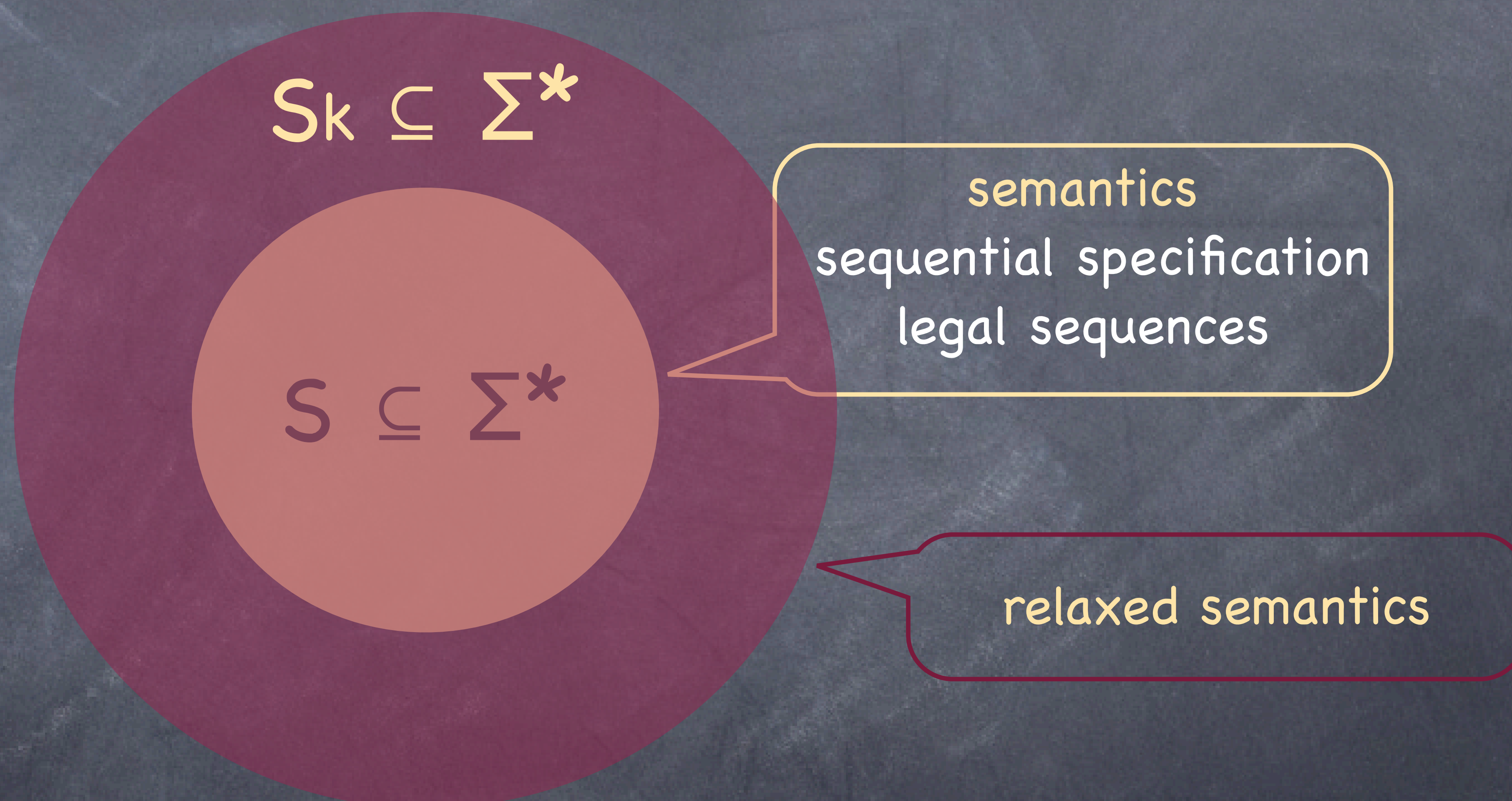
The big picture

$S \subseteq \Sigma^*$

semantics
sequential specification
legal sequences

Σ - methods with arguments

The big picture



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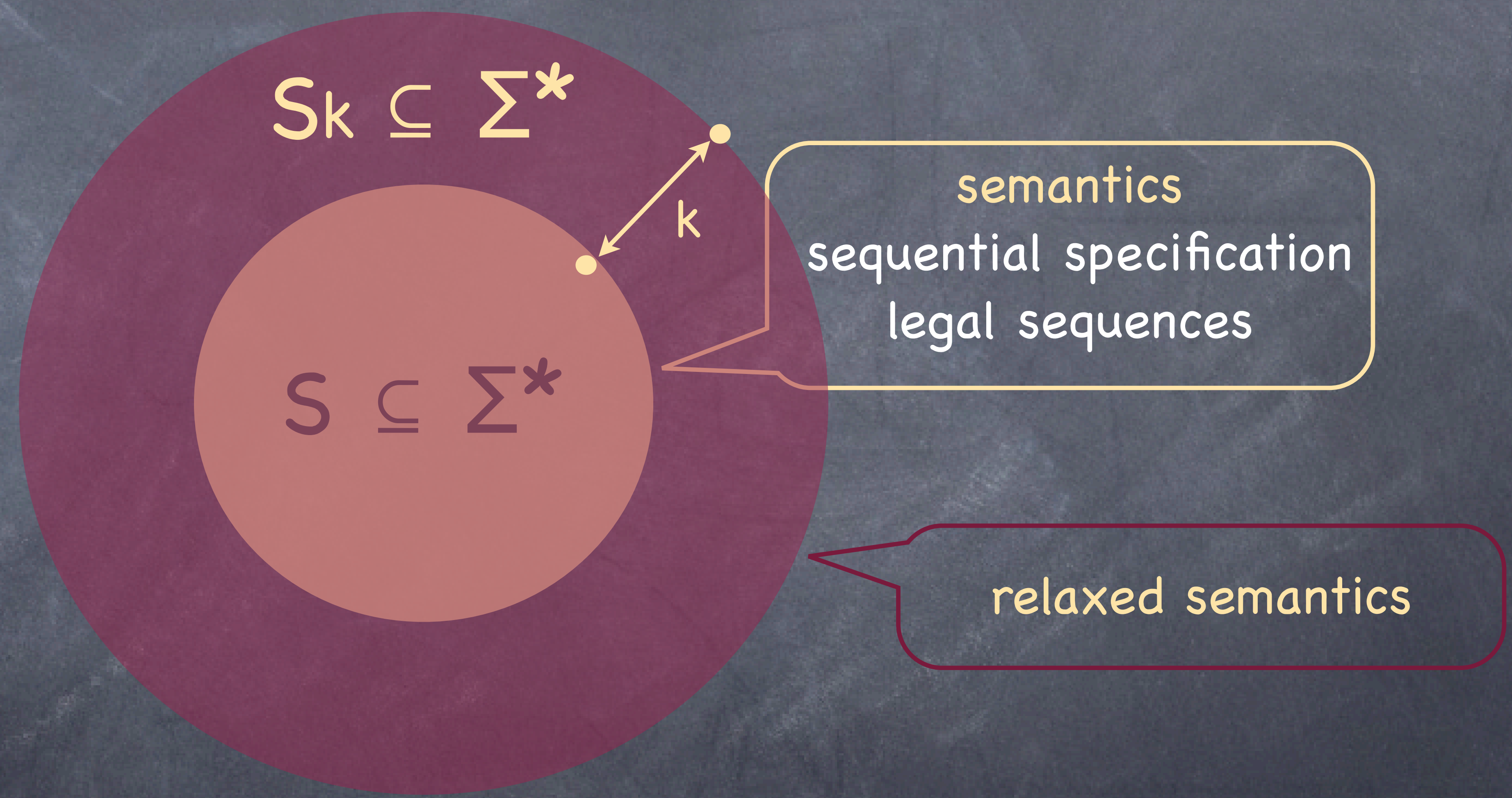

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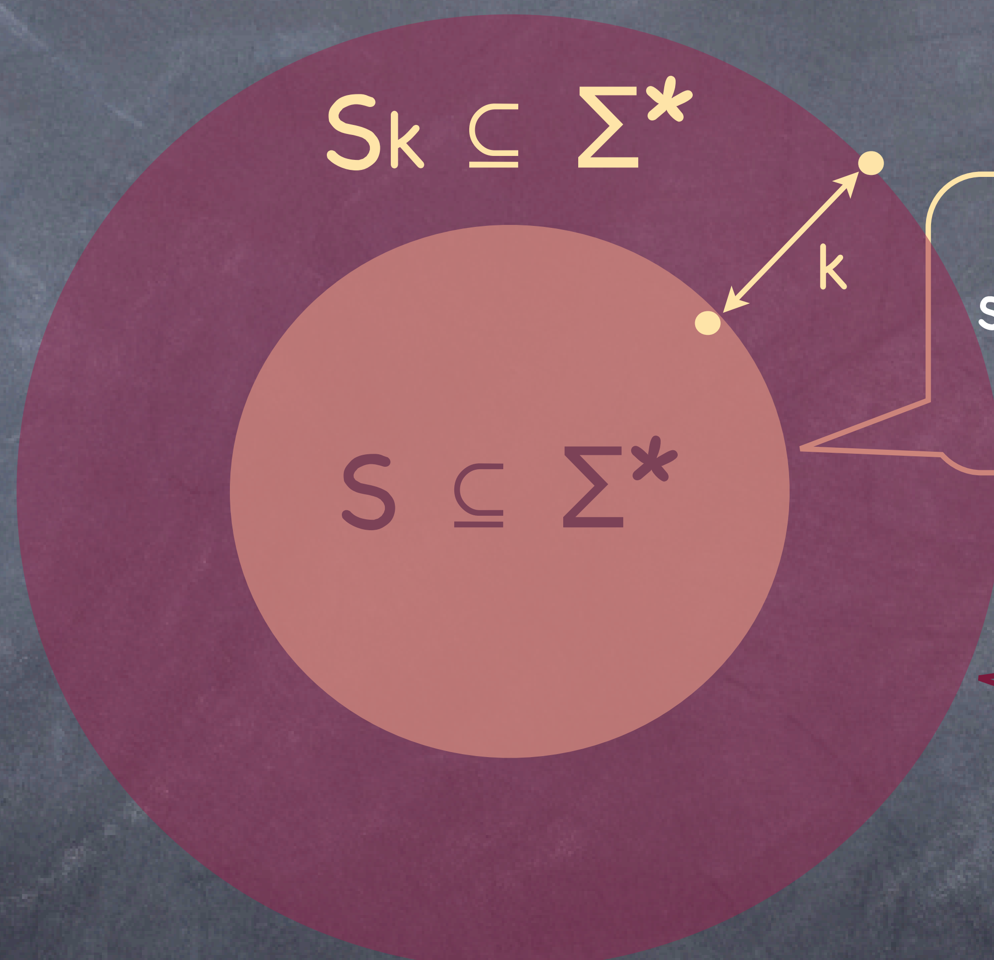

semantics
sequential specification
legal sequences

relaxed semantics

leads to relaxed linearizability

Σ - methods with arguments

Theoretical challenge

There are natural concrete relaxations...

Stack

Each **pop** pops one of the k-youngest elements

Each **push** pushes .....

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k-out-of-order relaxation

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makes sense also for queues, priority queues, ...

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Each **pop** pops one of the k-youngest elements

Each **push** pushes .....

k-out-of-order relaxation

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How is it reflected by a distance between sequences?

one distance for all?

Syntactic distances do not help

$\mathtt{push(a)[push(i)pop(i)]^n push(b)[push(j)pop(j)]^m pop(a)}$

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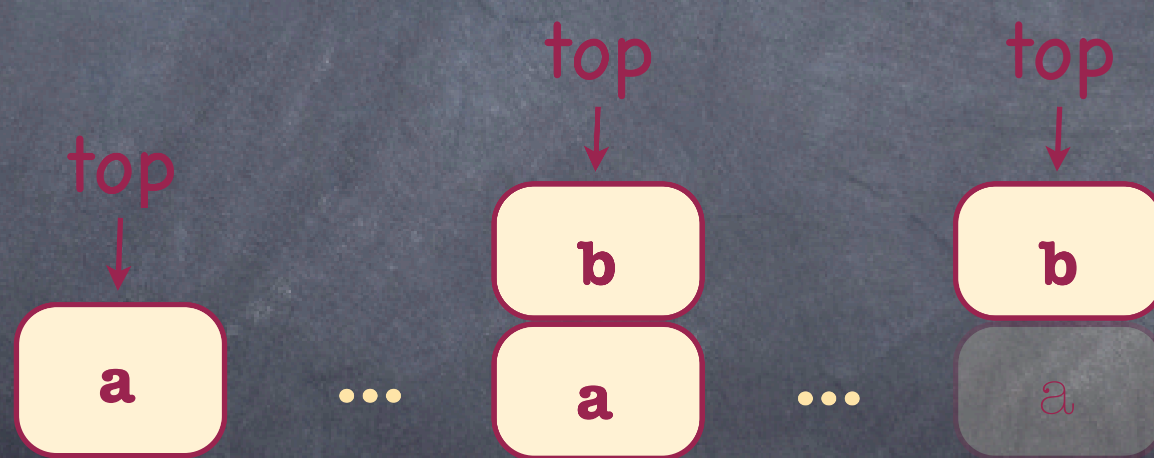
is a 1-out-of-order stack sequence



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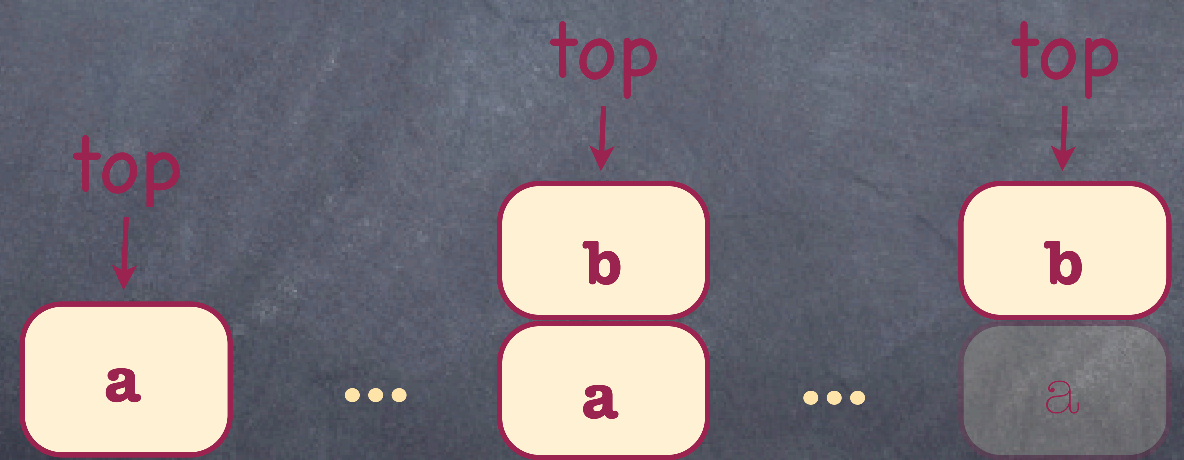

its permutation distance is unbounded

Semantic distances need a notion of state

- States are equivalence classes of sequences in S
- Two sequences in S are equivalent
if they have an indistinguishable future

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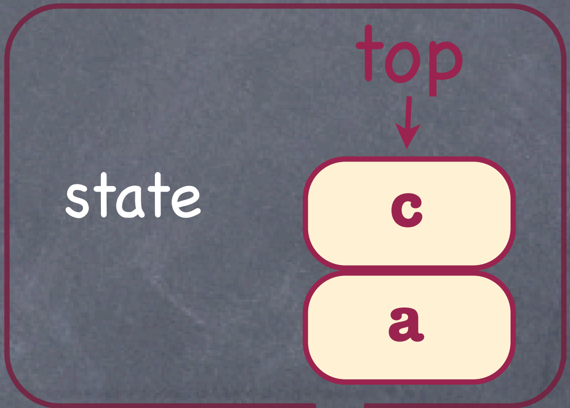

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state

top

c

a

- Two sequences in S are equivalent if they have an indistinguishable future

$$\mathbf{x \equiv y \quad \Leftrightarrow \quad \forall u \in \Sigma^*.\ (xu \in S \ \Leftrightarrow\ yu \in S)}$$

Semantics goes operational

- $S \subseteq \Sigma^*$ is the sequential specification

states

labels

initial state

- $LTS(S) = (S/_\equiv, \Sigma, \rightarrow, [\varepsilon]_\equiv)$ with

transition relation

$$[s]_\equiv \xrightarrow{m} [sm]_\equiv \quad \Leftrightarrow \quad sm \in S$$

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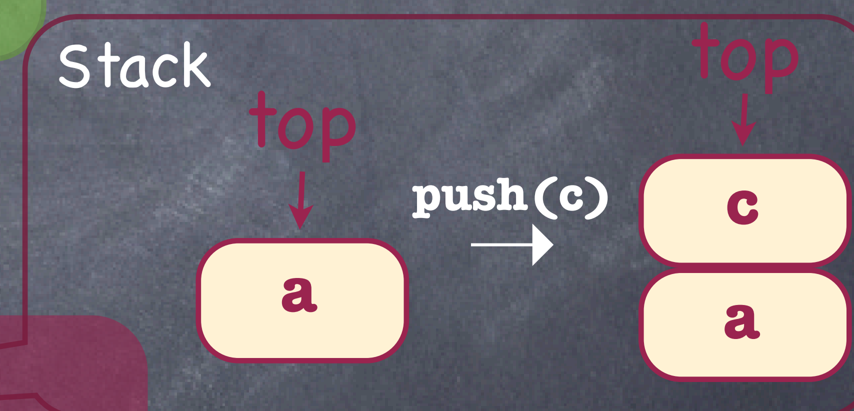

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The framework

- Completion of LTS(S)
- Transition costs
- Path cost function

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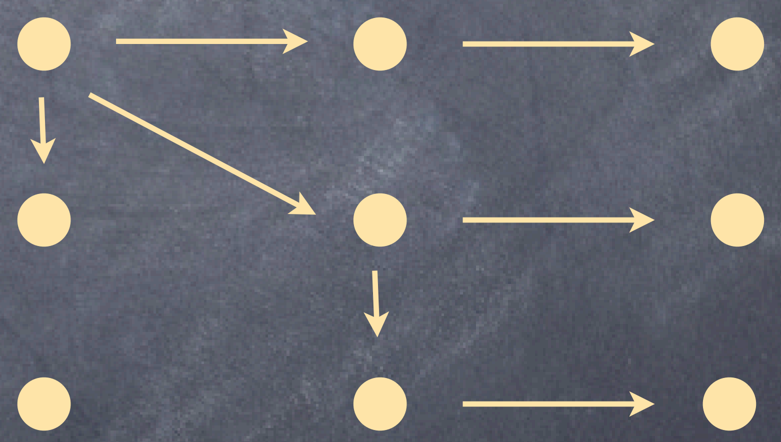
Σ - singleton



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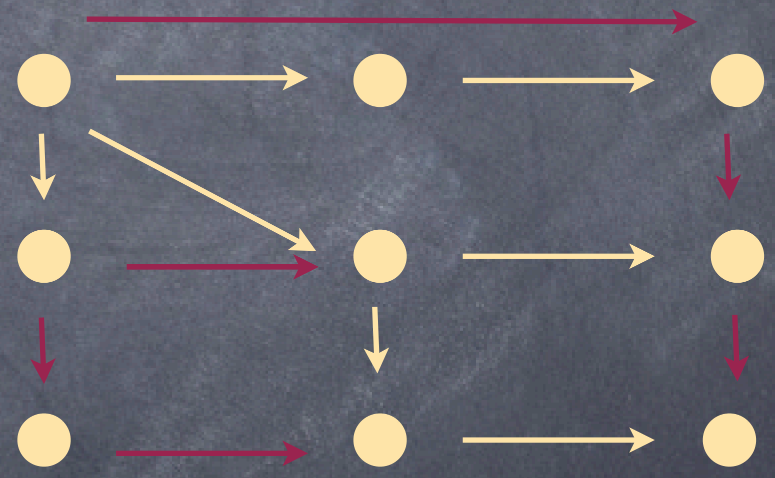

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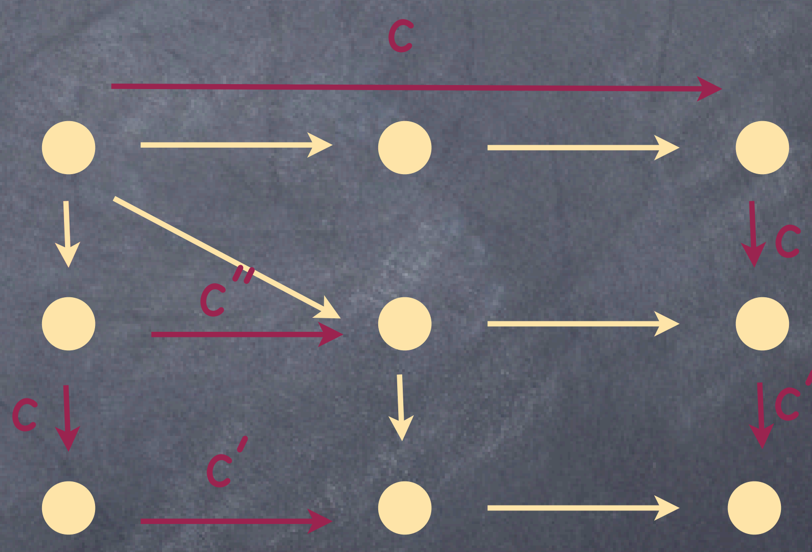

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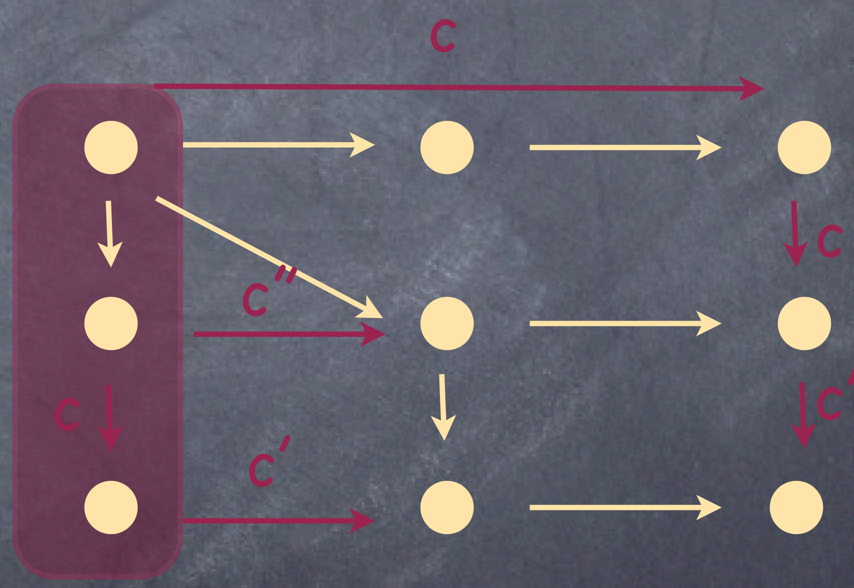

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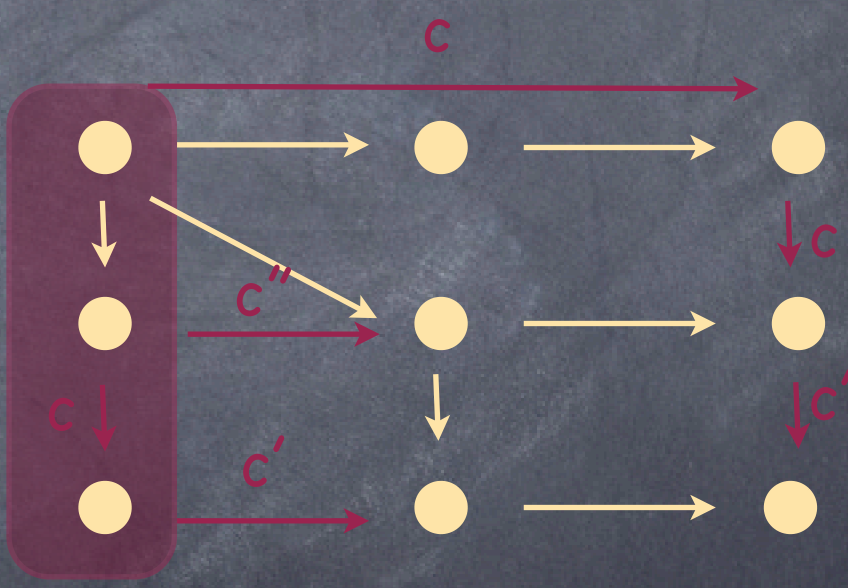

distance - minimal cost on all paths labelled by the sequence

For the user

- Pick your favorite data structure S
- Add desired incorrect transitions and assign them transition costs
- Choose a path cost function

distance and relaxation follow

For the user

The framework clears the head,
direct concrete relaxations are also possible

- Pick your favorite data structure S
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Cost 1

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Total cost

Stack example

- Canonical representative of a state
- Add incorrect transitions with costs

- Possible path cost functions max, sum,...

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Sequence of **push**'s with no matching **pop**

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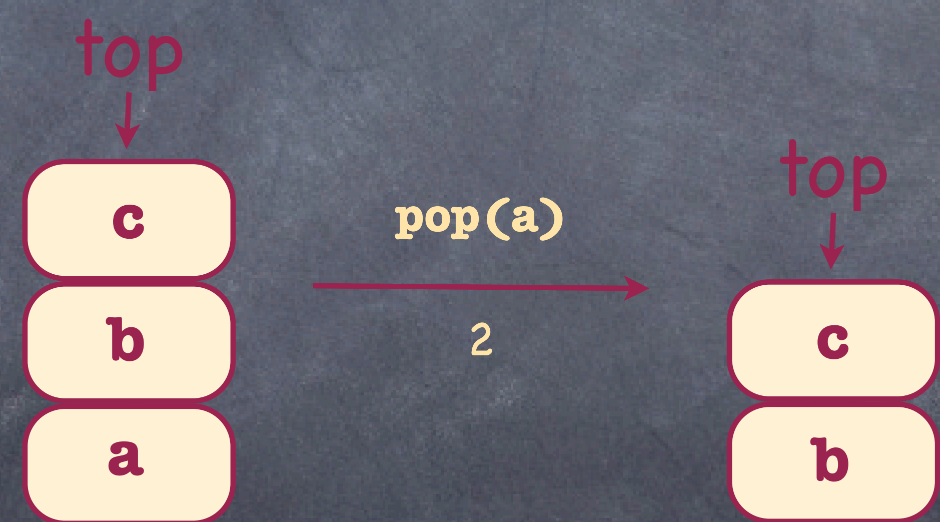

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Let's generalize

Generic out-of-order

segment_cost($q \xrightarrow{m} q'$) $= |\mathbf{v}|$ — transition cost

where $\mathbf{v}$ is a sequence of minimal length s.t.

(1) removing $\mathbf{v}$ enables a transition

(2) inserting $\mathbf{v}$ enables a transition

goes with different path costs

Generic out-of-order

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(1) $[\mathbf{uvw}]_\equiv = q$, $\mathbf{uvw}$ is minimal, $\mathbf{uw}$ is minimal

(1.1) $[\mathbf{uw}]_\equiv \xrightarrow{m} [\mathbf{u'w}]_\equiv$, $[\mathbf{u'vw}]_\equiv = q'$

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Out-of-order stack

- Canonical representative of a state
- Add incorrect transitions with segment-costs
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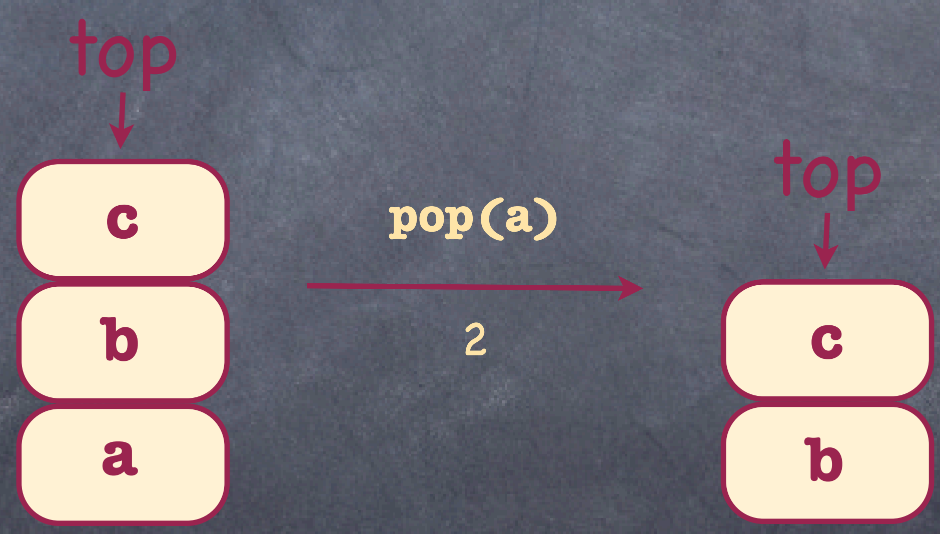

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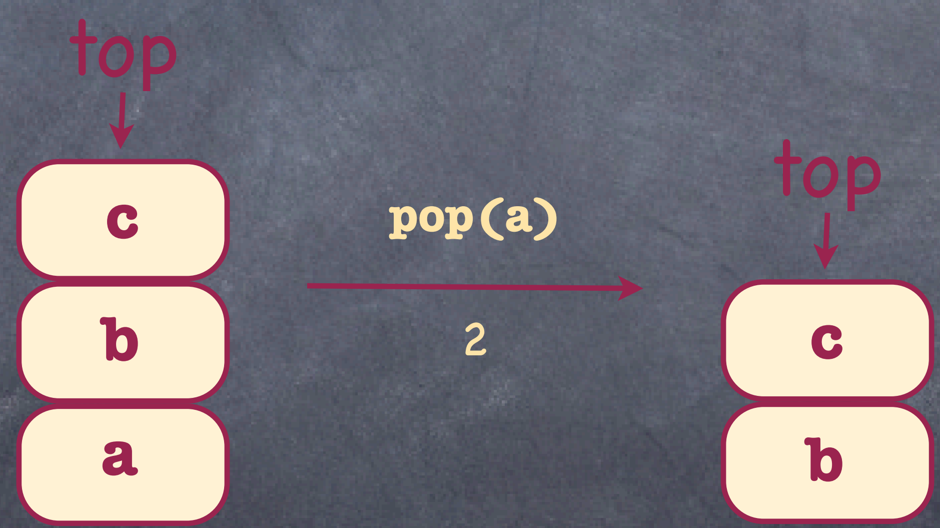

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also ``shrinking window''
restricted out-of-order

Out-of-order queue

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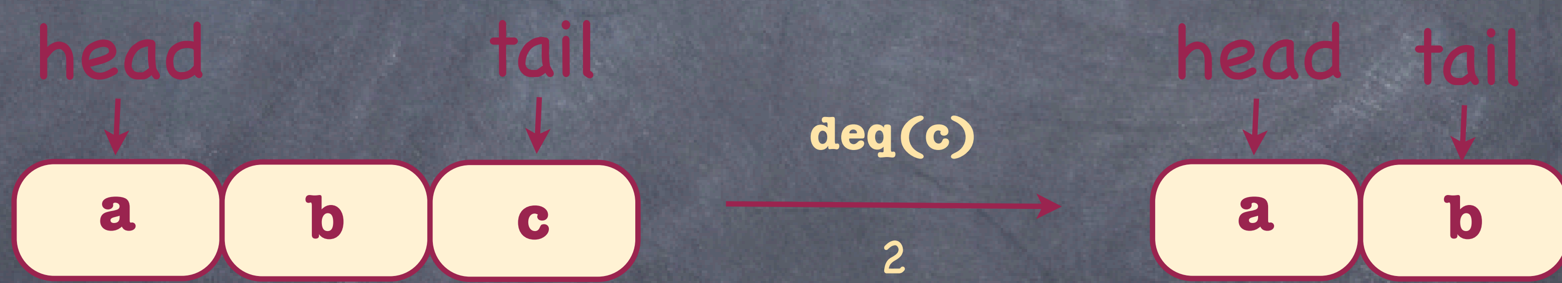

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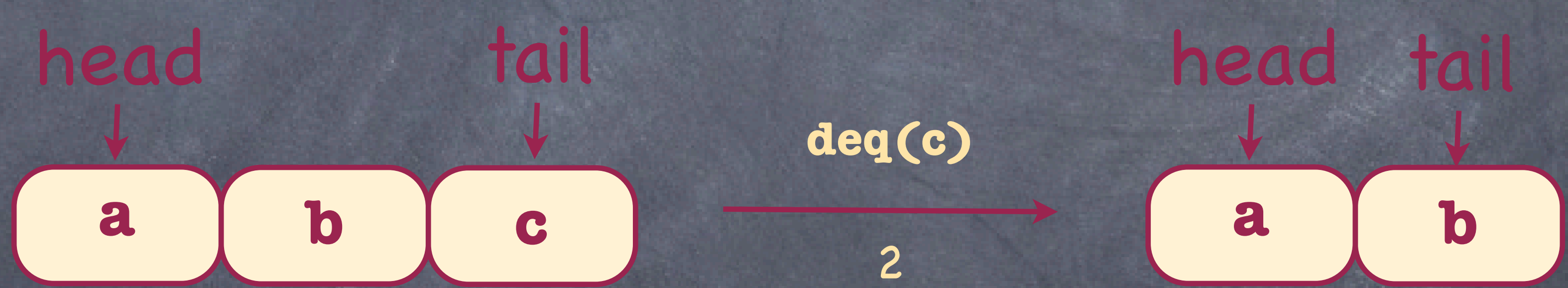

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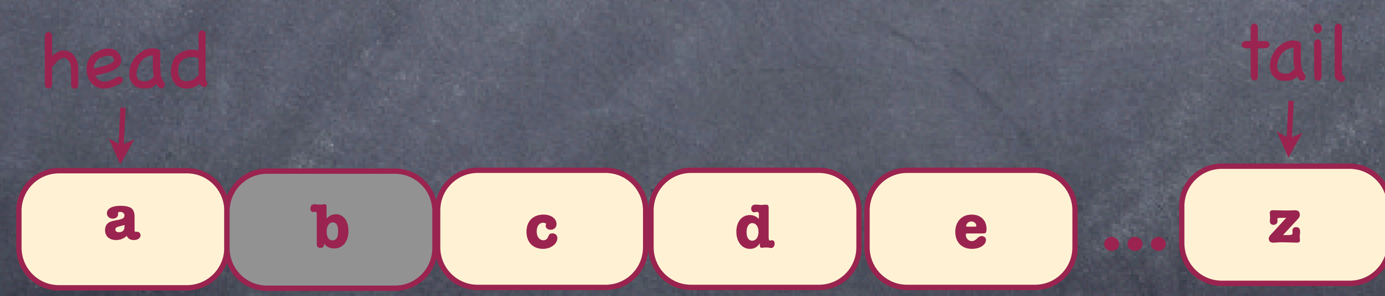
Out-of-order variants

Queue



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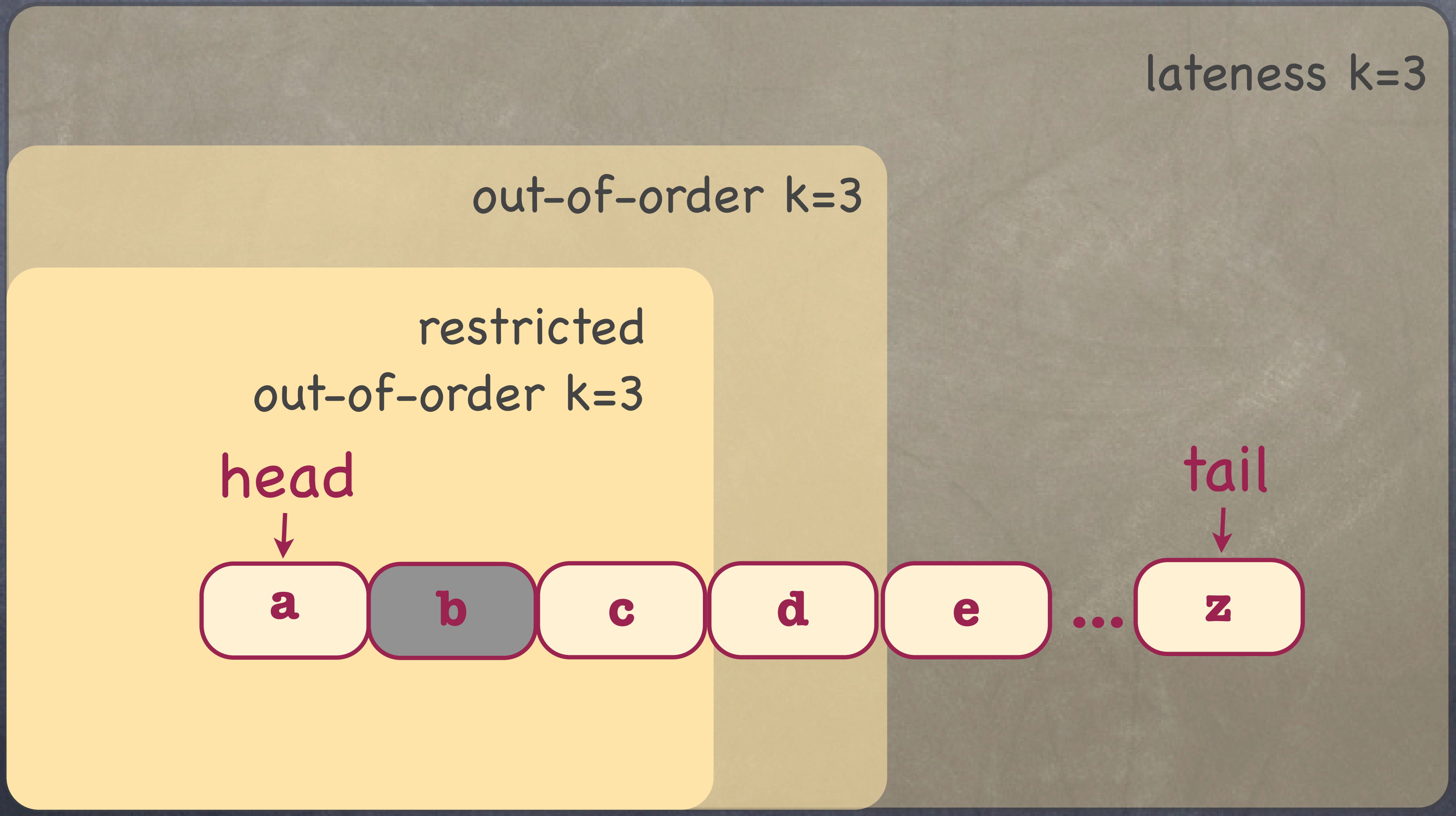

How about implementations? Performance?

Short-term history

- SCAL queues [KPRS'11]
- Quasi linearizability theory and implementations [AKY'10]
- Some straightforward implementations [HKPSS'12]
- Efficient lock-free segment queue [KLP'12]

(almost) all implement restricted out-of-order

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distributed, one k-queue

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Lessons learned

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Being relaxed not necessarily means better performance

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Well-performing implementations of relaxed specifications do exist!

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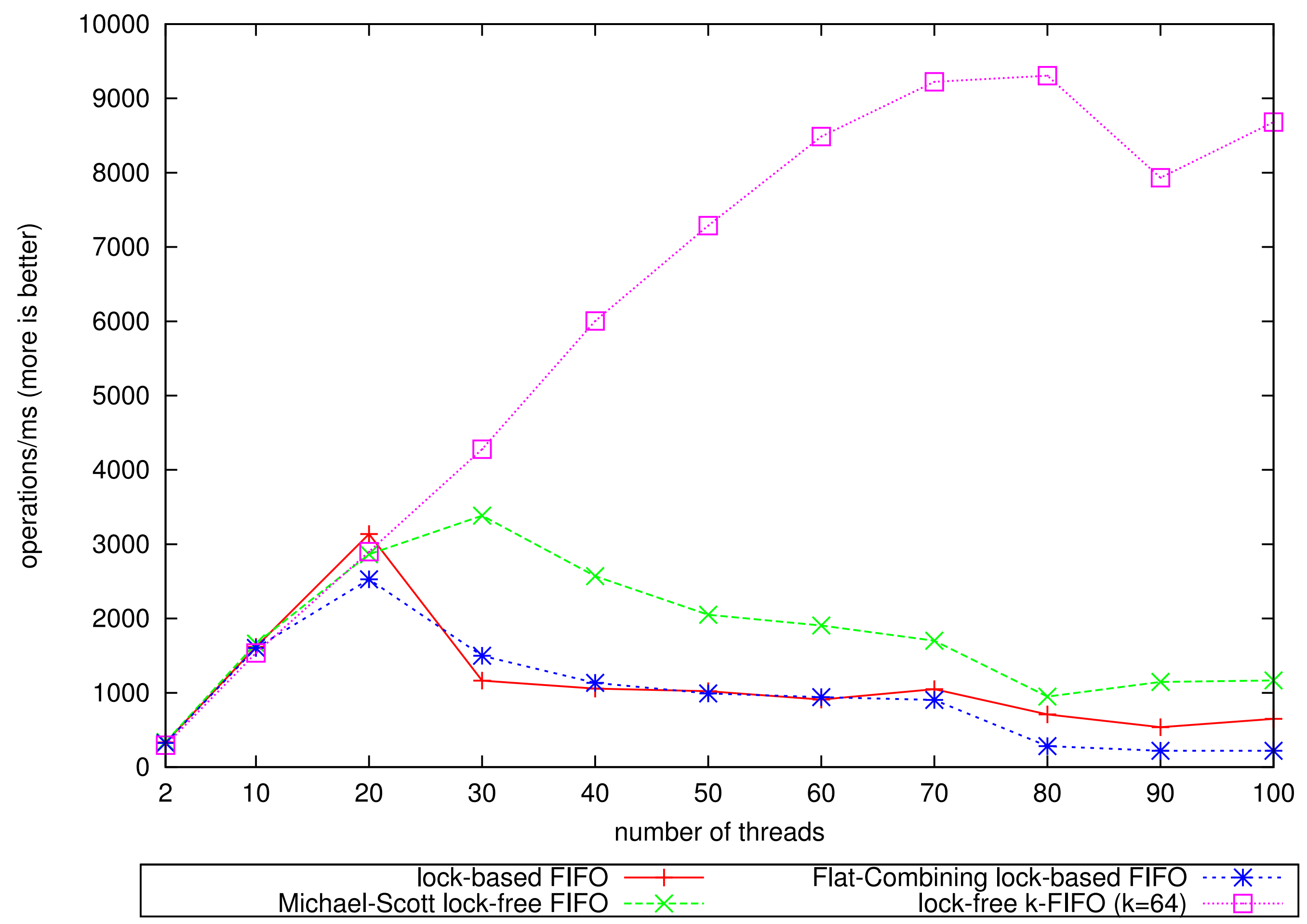
Well-performing implementations of relaxed specifications do exist!

Let's see them!

Queue

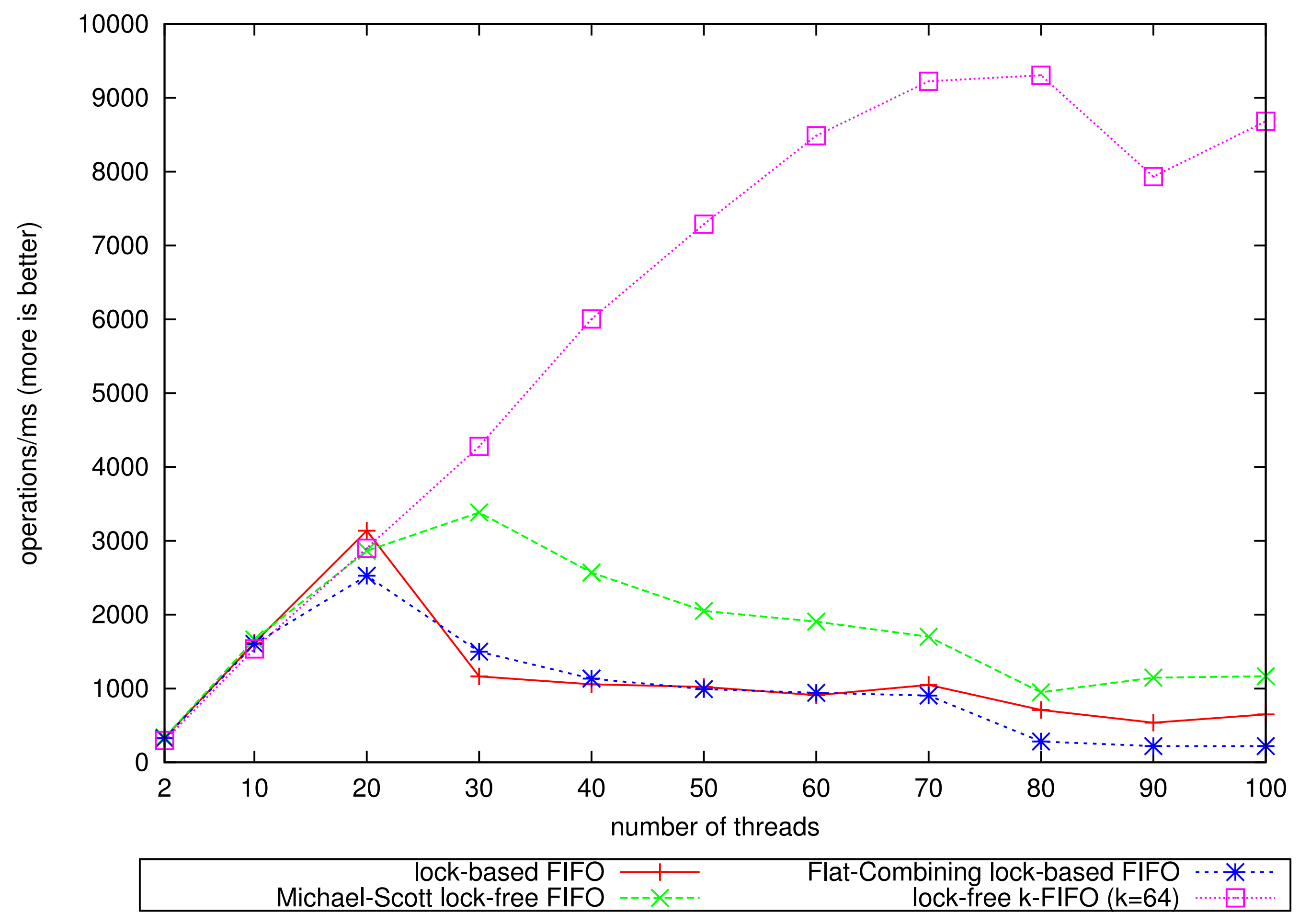
Scalability comparison



Queue

Scalability comparison

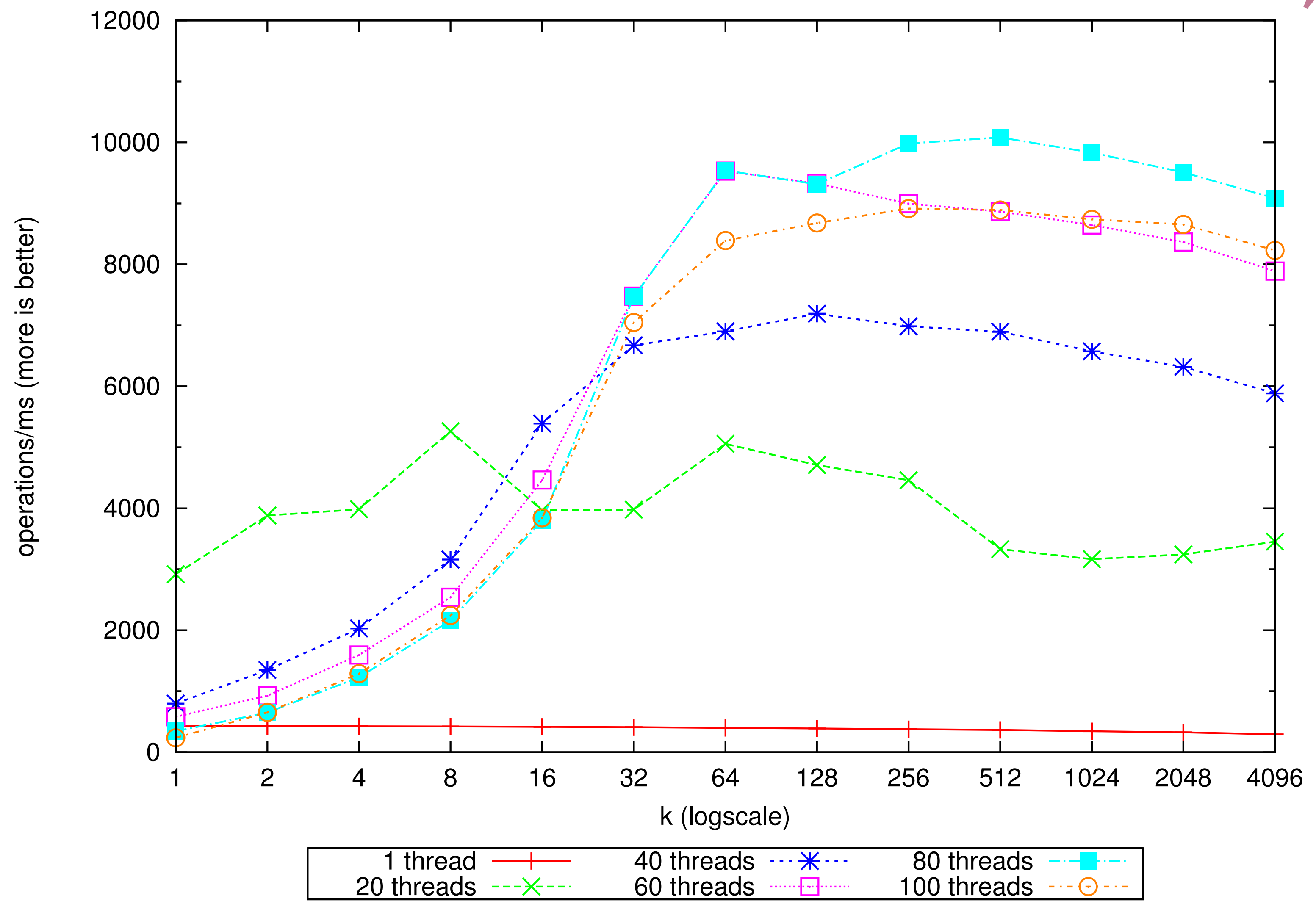
80-core machine



Queue

The more relaxed, the better

lock-free segment queue



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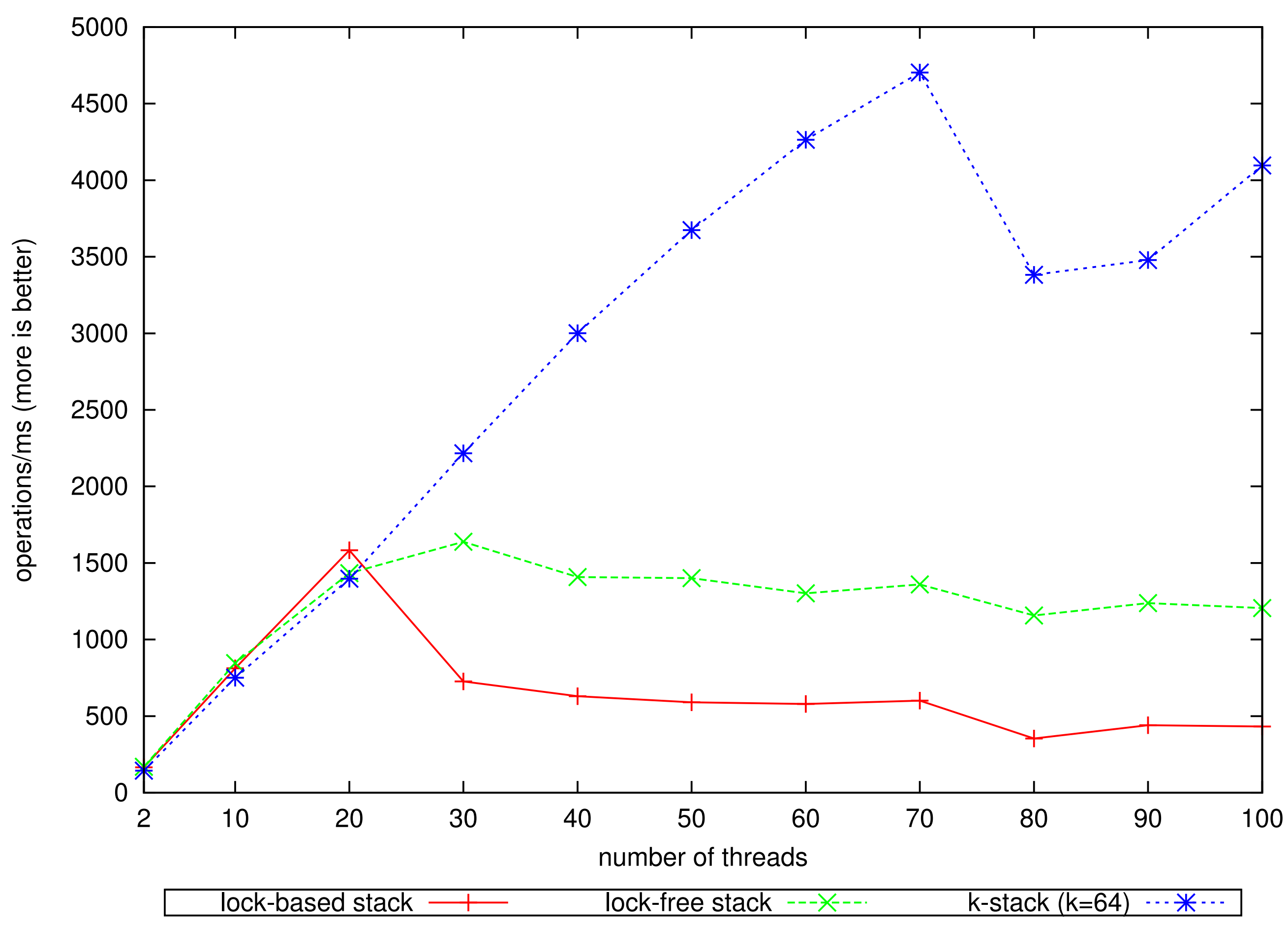
Scalability comparison



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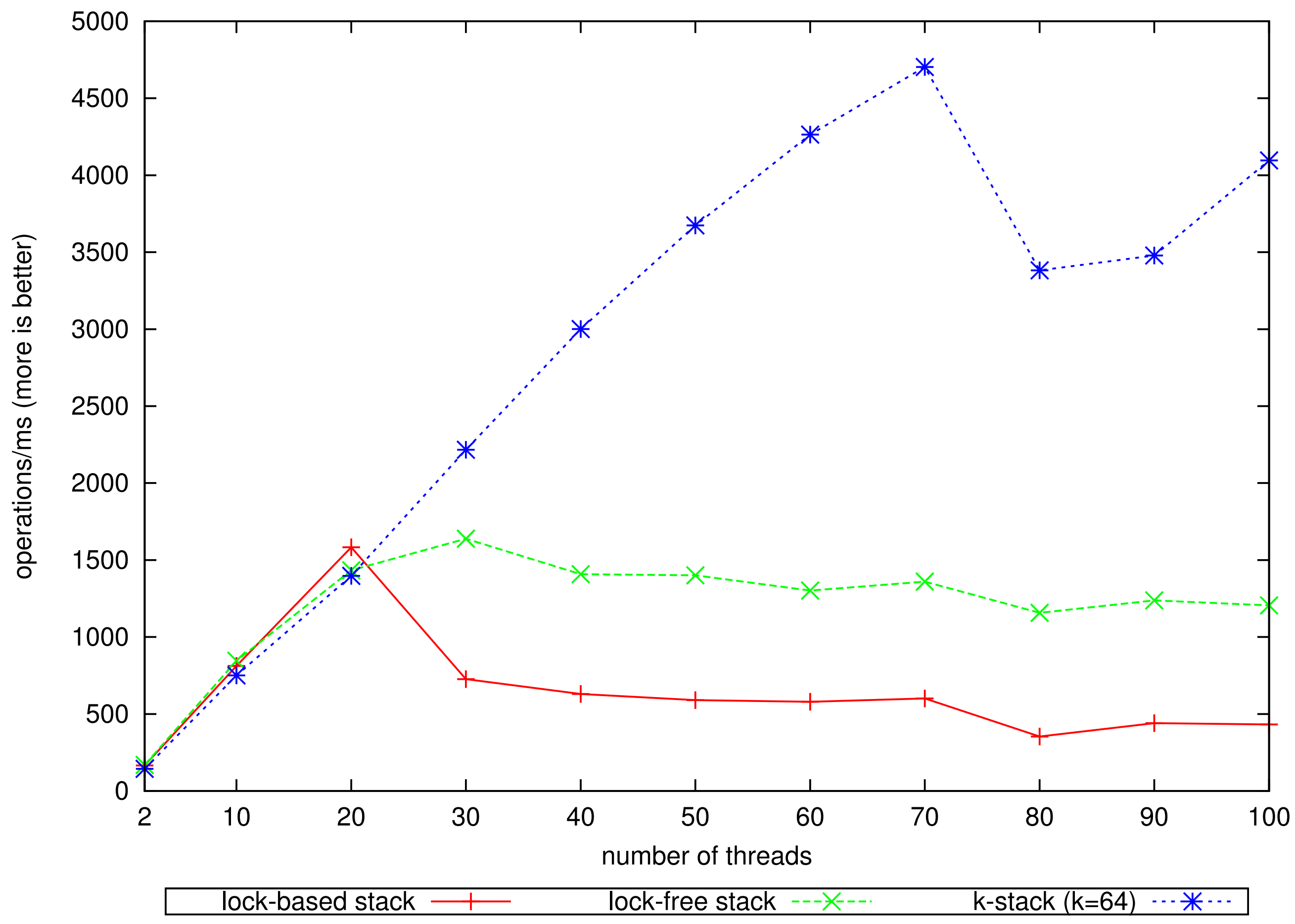

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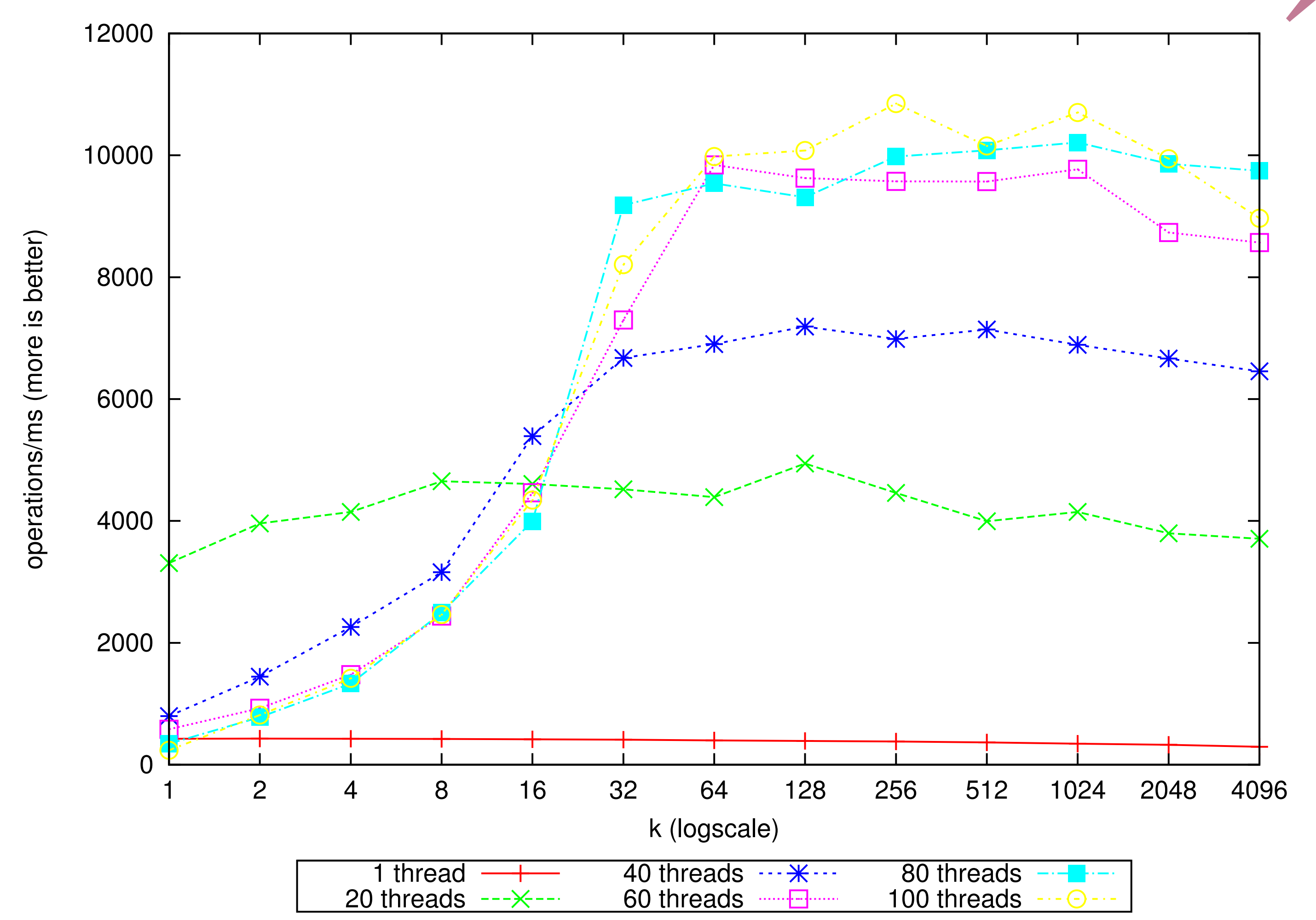

Final remarks

Contributions

Framework for quantitative relaxations
generic relaxation, concrete examples,
efficient implementations exist

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all kinds of

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Difficult open problem

From practice to theory it works...
How to get from theory to practice?

Final remarks

all kinds of

Contributions

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Difficult open problem

THANK YOU

From practice to theory it work
How to get from theory to practice?