Quantitatively Relaxed Concurrent Data Structures

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University of Leicester 14.12.2012

Semantics of concurrent data structures

- Sequential specification - set of legal sequences
- Correctness condition - linearizability

Semantics of concurrent data structures

Stack - legal sequence

push(a)push(b)pop(b)

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begin-push(a)begin-push(b) end-push(a) end-push(b)begin-pop(b)end-pop(b)

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linearizable wrt seq.spec.

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Semantics of concurrent data structures

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we relax this

- Sequential specification - set of legal sequences

linearizable wrt seq.spec.

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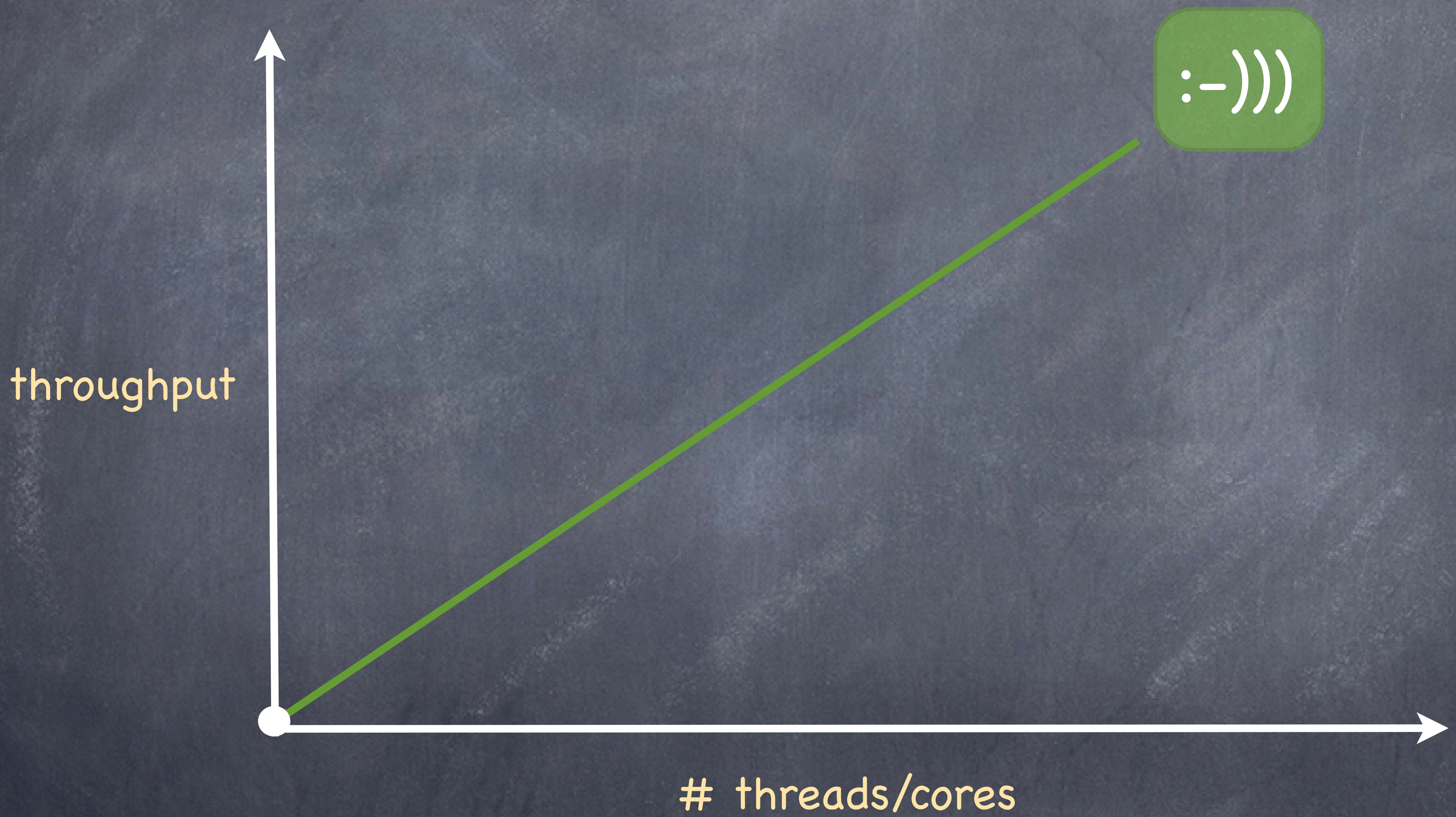
Performance and scalability

Performance and scalability



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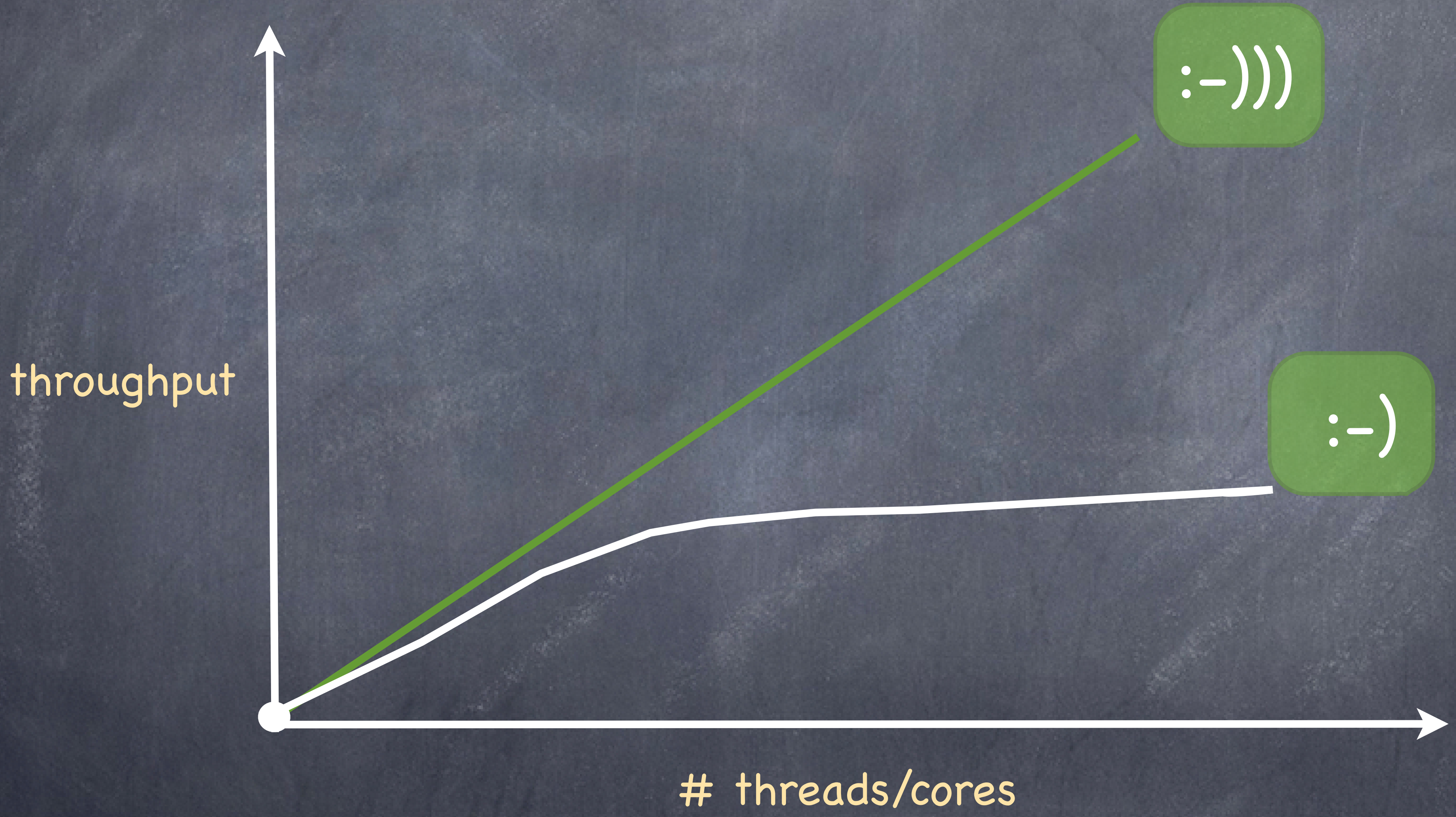

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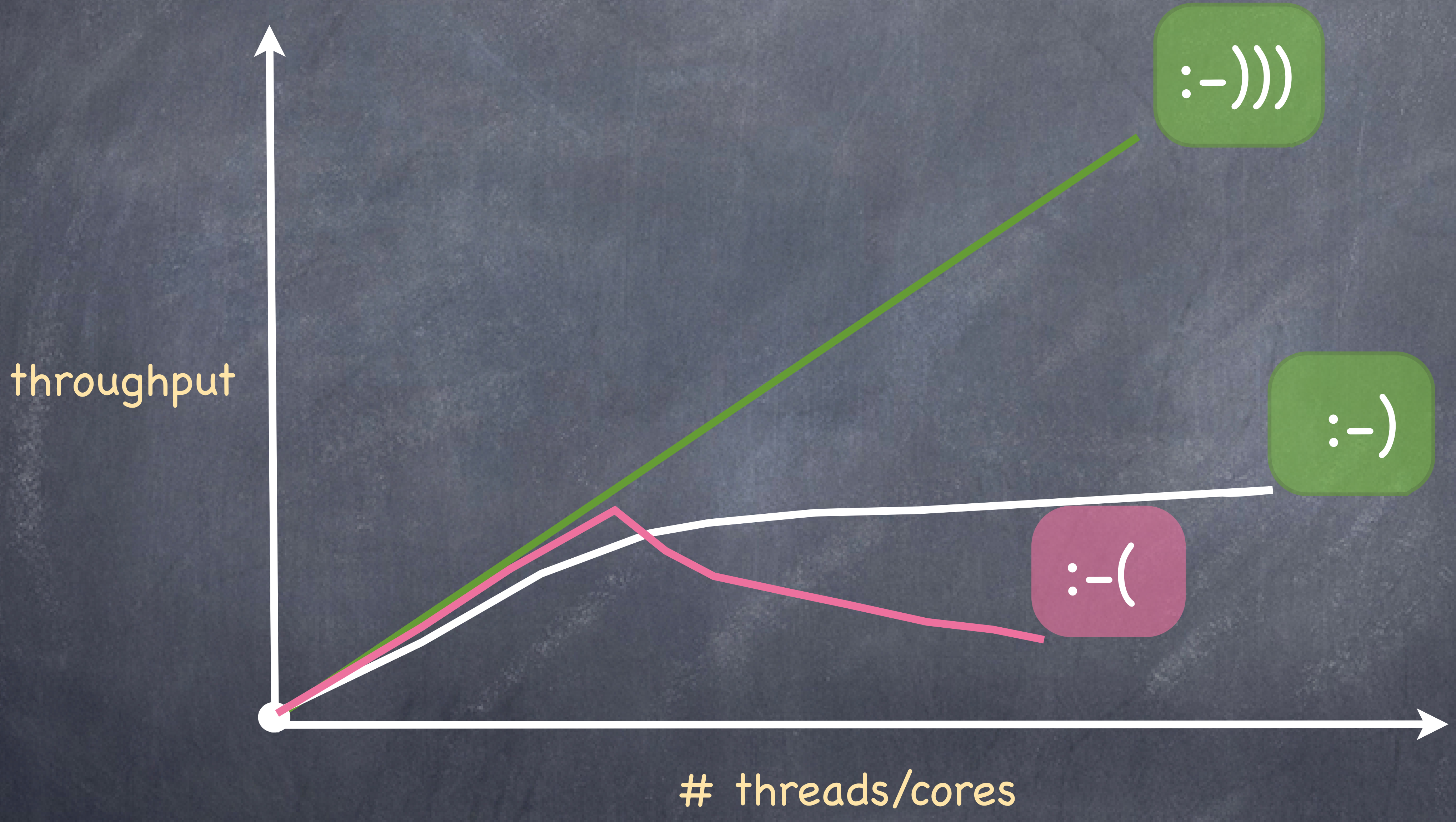

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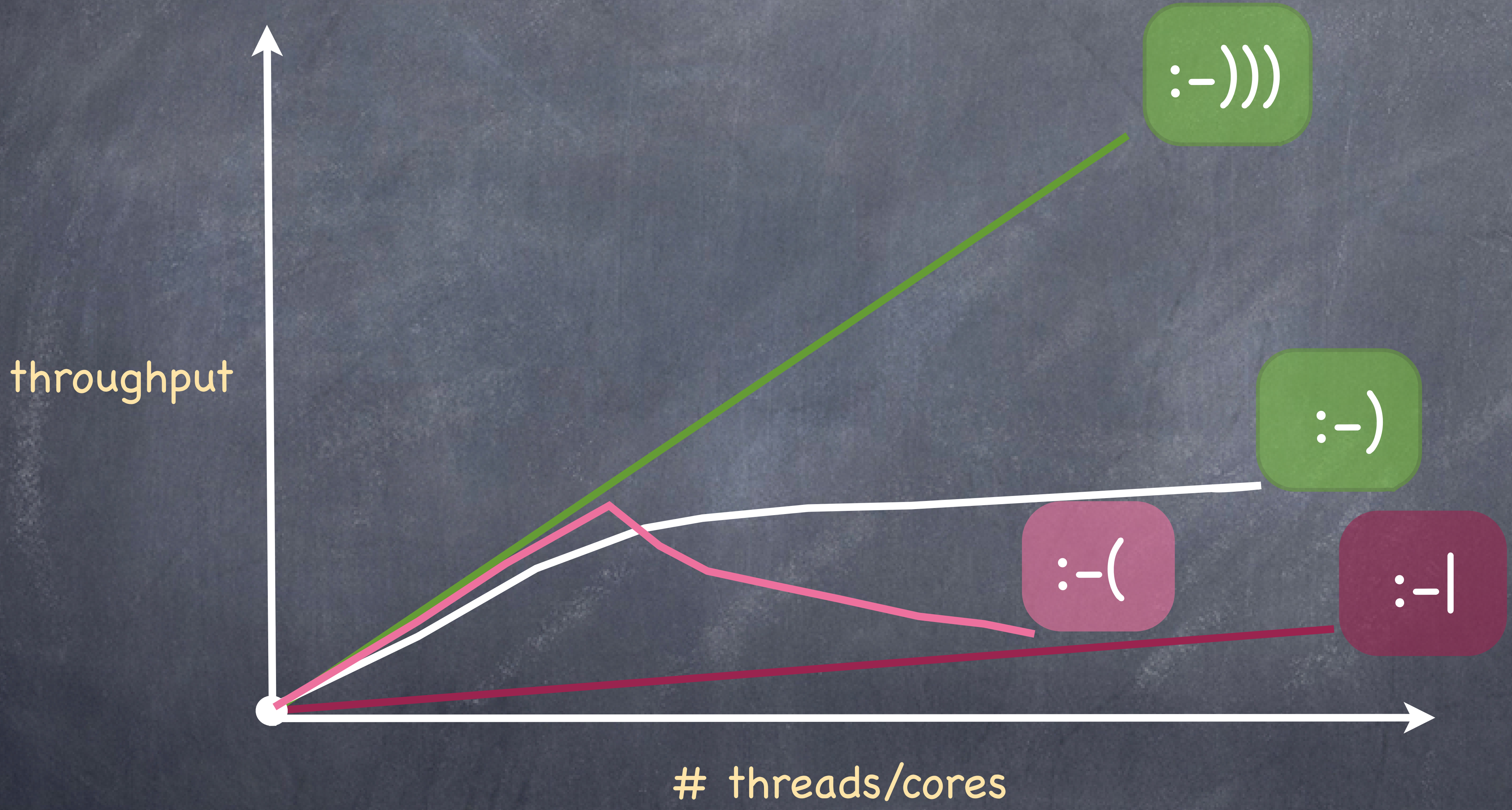

The goal

- Trading correctness for performance
- In a controlled way with quantitative bounds

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measure the error from correct behavior

The goal

Stack - incorrect behavior

push(a)push(b)push(c)pop(a)pop(b)

- Trading correctness for performance
- In a controlled way with quantitative bounds

correct in a relaxed stack
... 2-relaxed? 3-relaxed?

measure the error from correct behavior

Why relax?

- It is theoretically interesting
- Provides potential for better performing concurrent implementations

...

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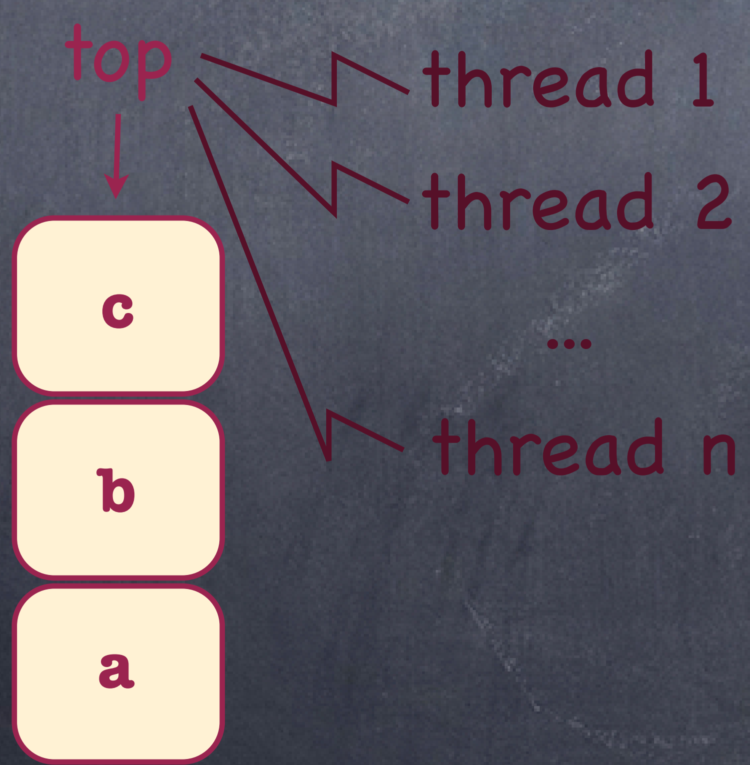
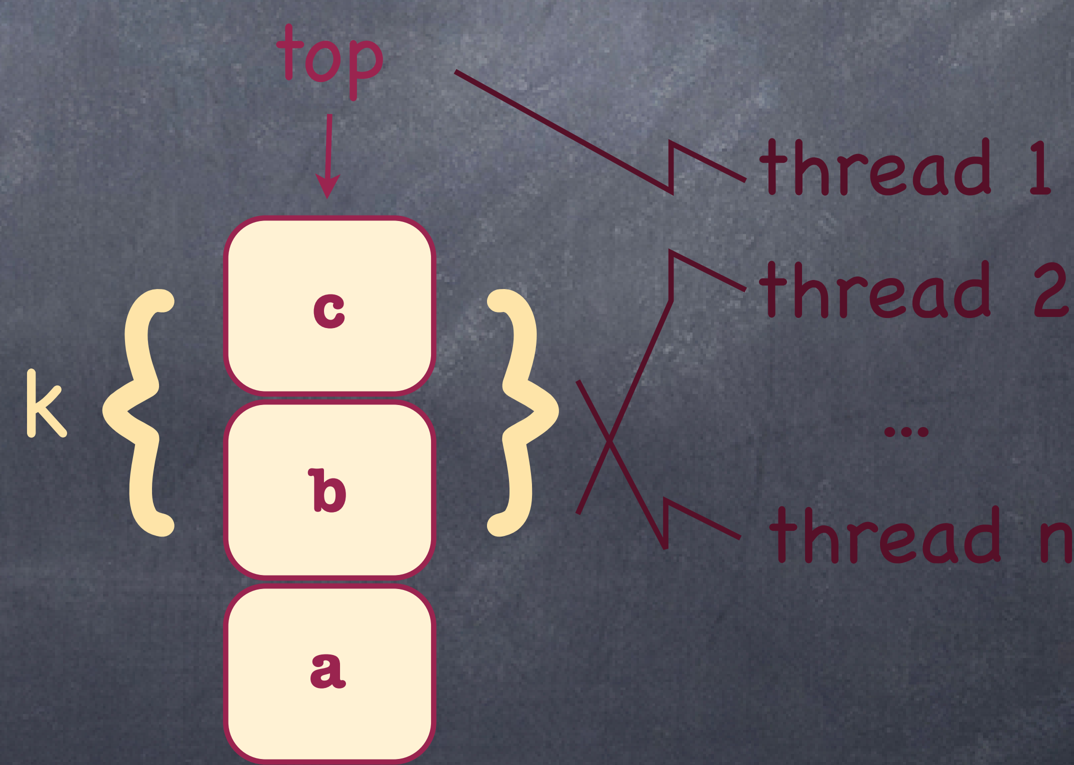
Stack



k-Relaxed stack



What we have

- Framework — for semantic relaxations
- Generic examples — out-of-order / stuttering
- Concrete relaxation examples — stacks, queues, priority queues,.. / CAS, shared counter
- Efficient concurrent implementations — of relaxation instances

Enough introduction

☺

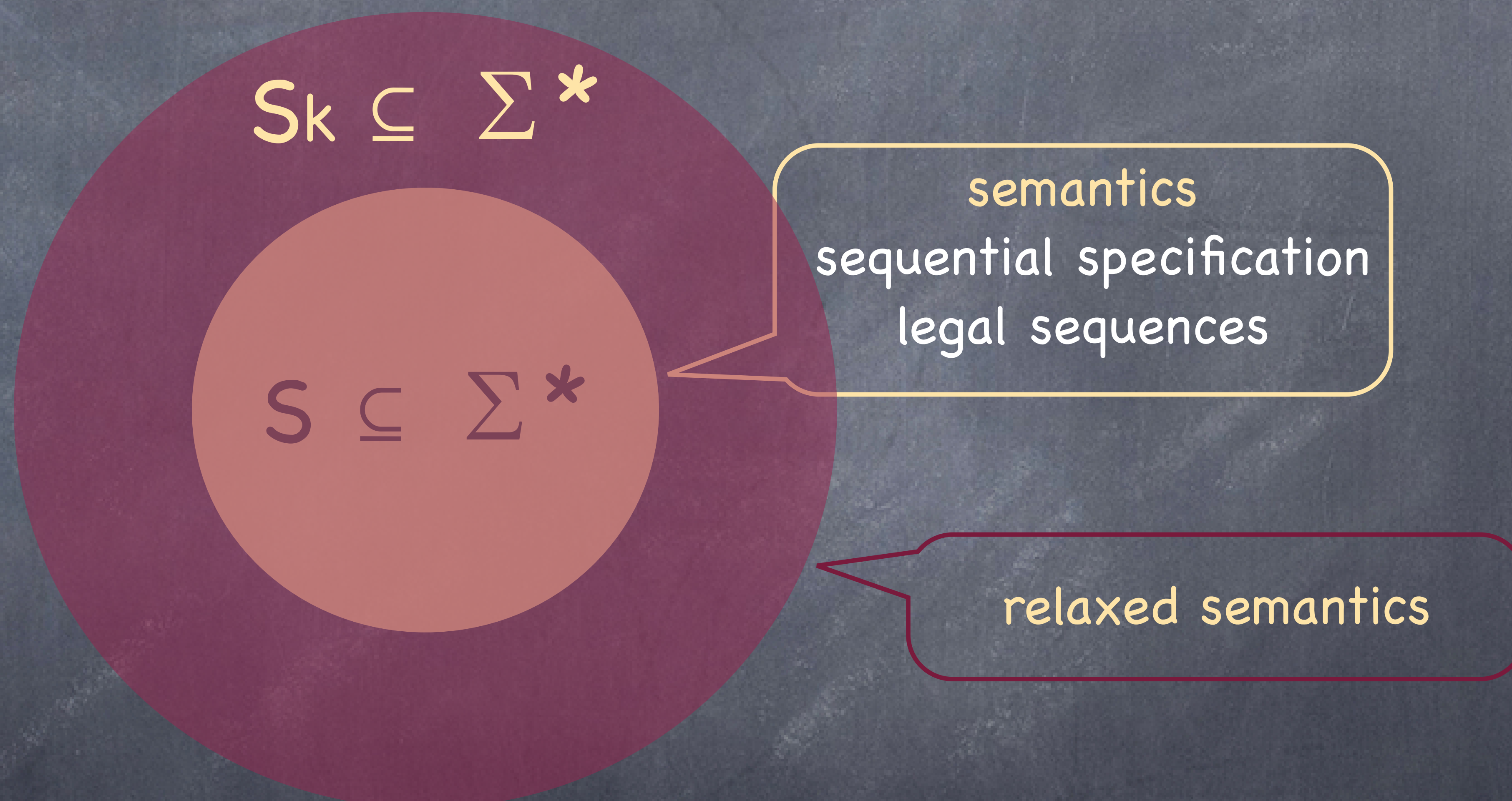
The big picture

$S \subseteq \Sigma^*$

semantics
sequential specification
legal sequences

Σ - methods with arguments

The big picture



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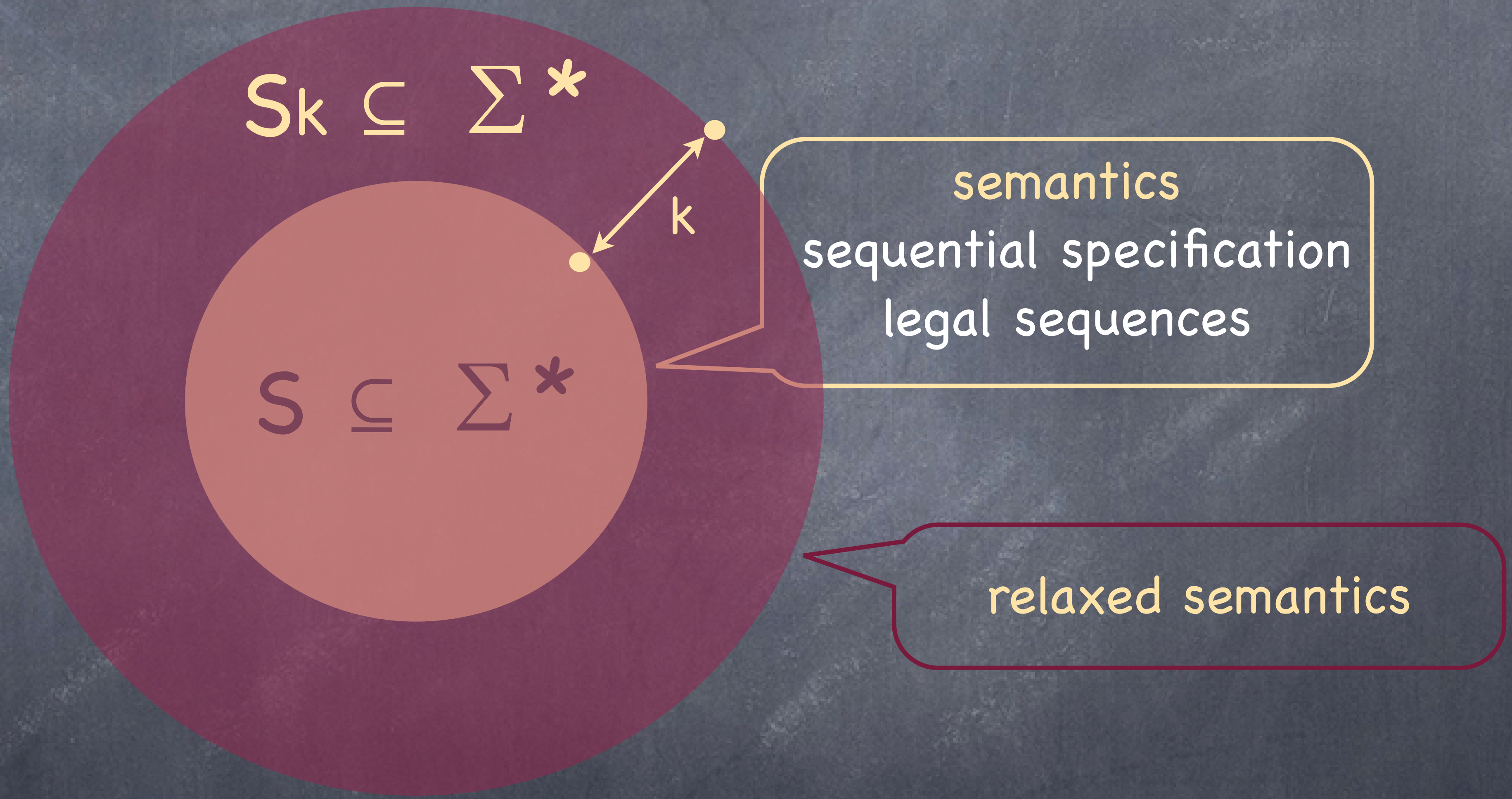

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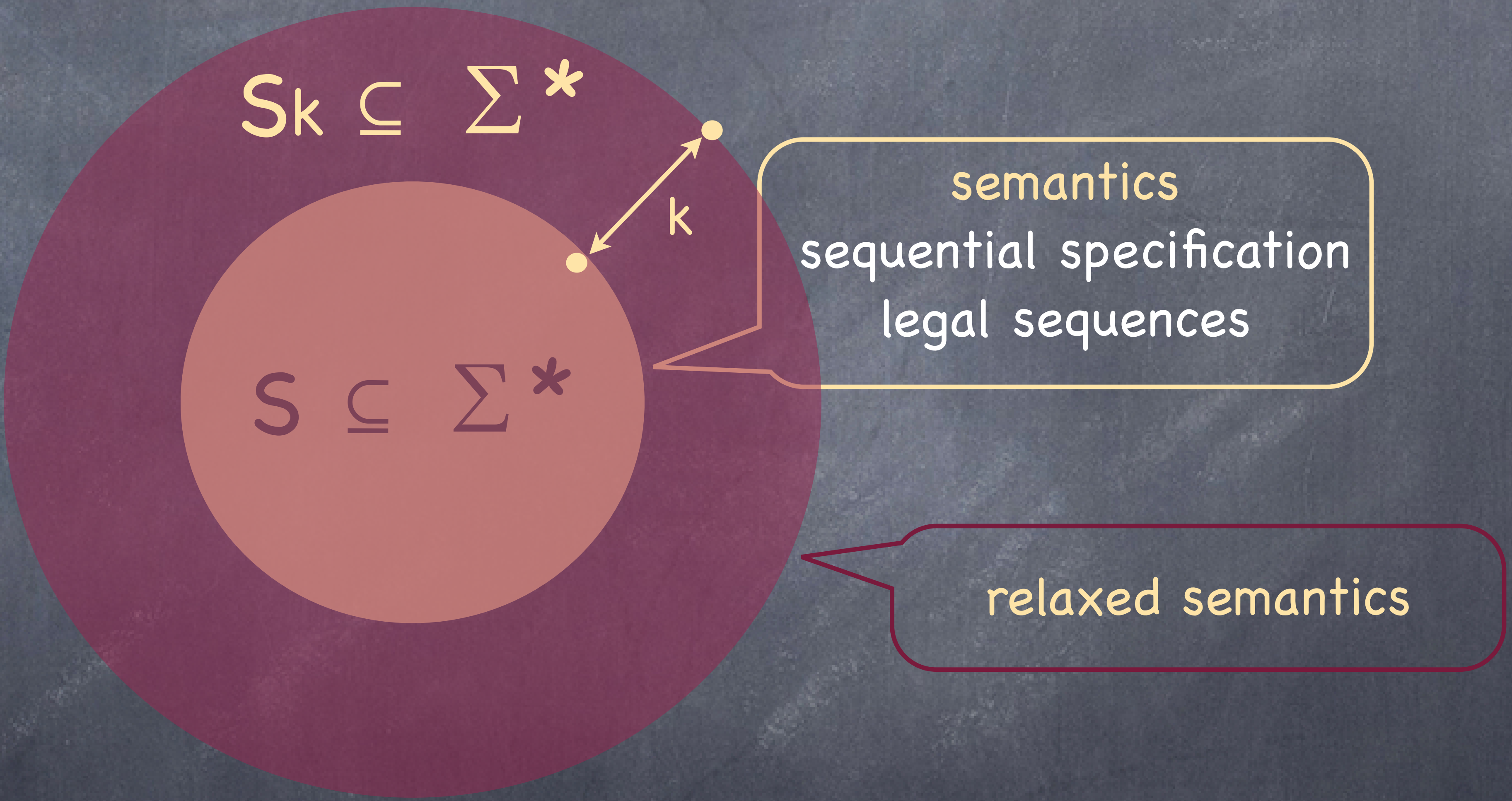

Σ - methods with arguments

distance?

Challenge

There are natural concrete relaxations...

Stack

Each **pop** pops one of the (k+1)-youngest elements

Each **push** pushes .....

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k-out-of-order relaxation

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makes sense also for queues, priority queues, ...

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There are natural concrete relaxations...

Stack

Each **pop** pops one of the (k+1)-youngest elements

Each **push** pushes .....

k-out-of-order relaxation

makes sense also for queues, priority queues, ...

How is it reflected by a distance between sequences?

one distance for all?

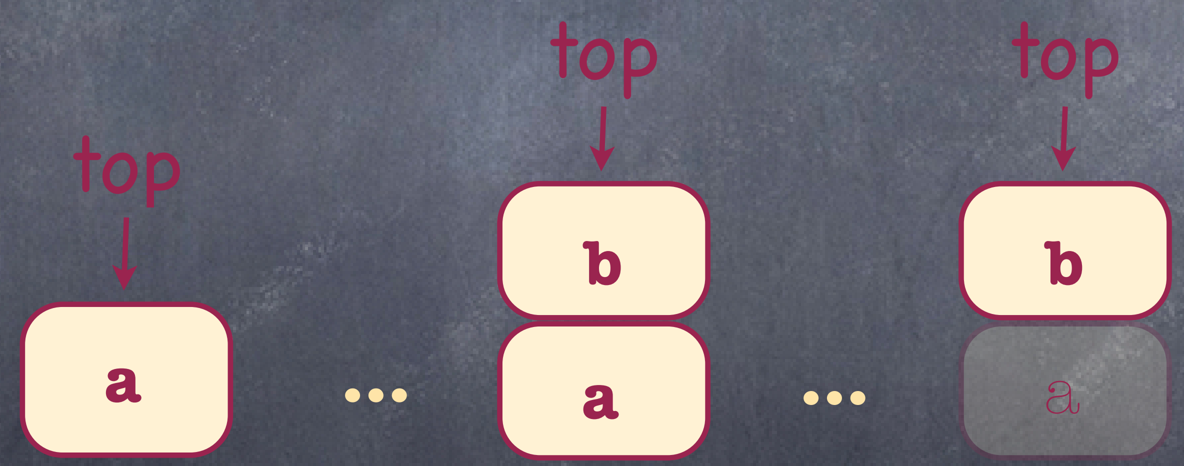
Syntactic distances do not help

push(a)[push(i)pop(i)]^{n}push(b)[push(j)pop(j)]^{m}pop(a)

Syntactic distances do not help

$\texttt{push(a)[push(i)pop(i)]}^n\texttt{push(b)[push(j)pop(j)]}^m\texttt{pop(a)}$

is a 1-out-of-order stack sequence



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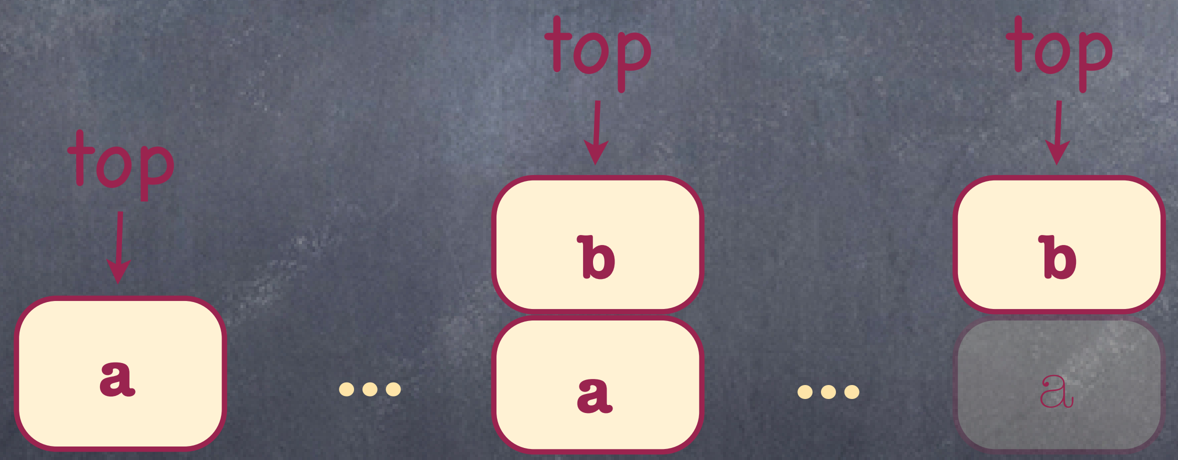

its permutation distance is min(n,m)

Stack example

push(a)push(b)push(c)pop(a)pop(b)

state evolution

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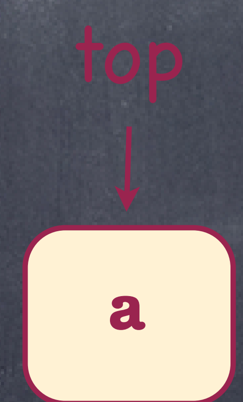

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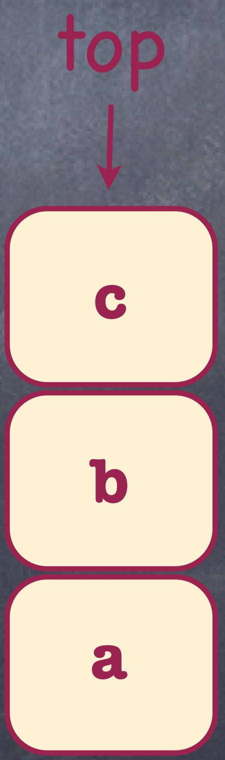

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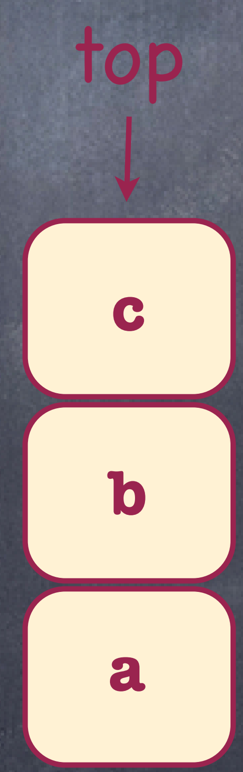

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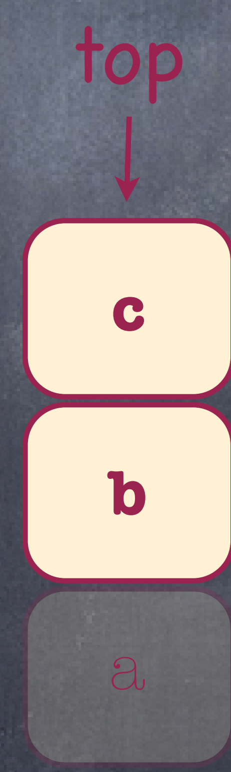

???

How much does this error cost?

Stack example

push(a)push(b)push(c)pop(a)pop(b)

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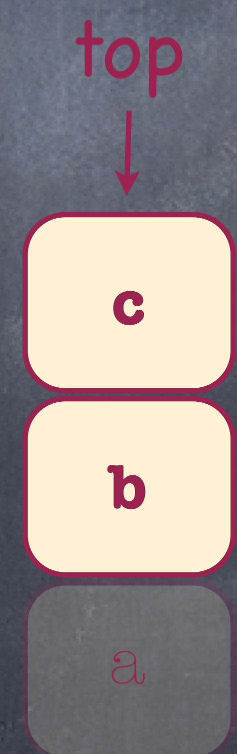

Cost 2

Stack example

push(a)push(b)push(c)pop(a)pop(b)

state evolution

top

c

b

a

Cost 2

???

Stack example

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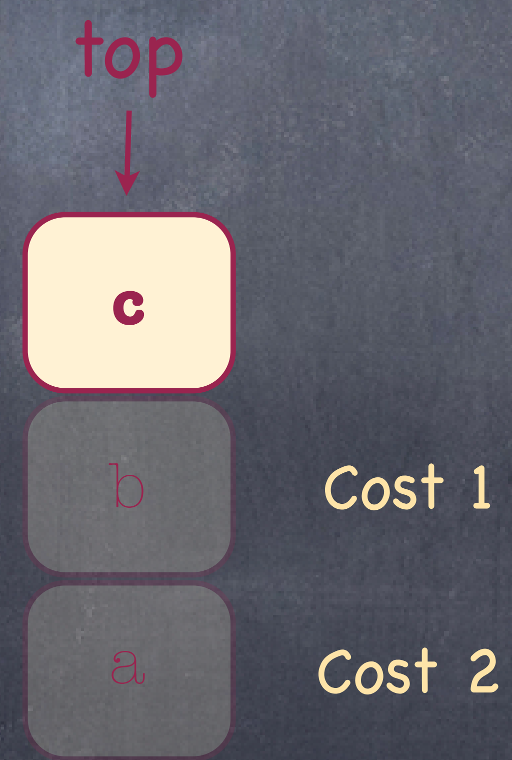

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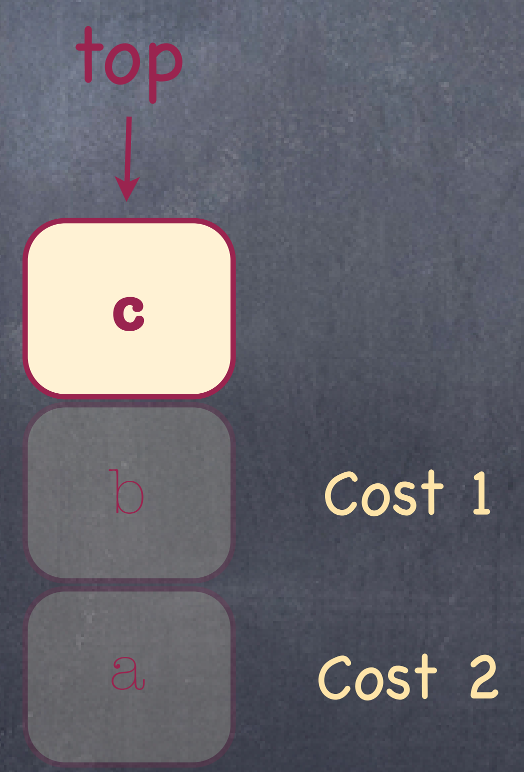

Total cost?

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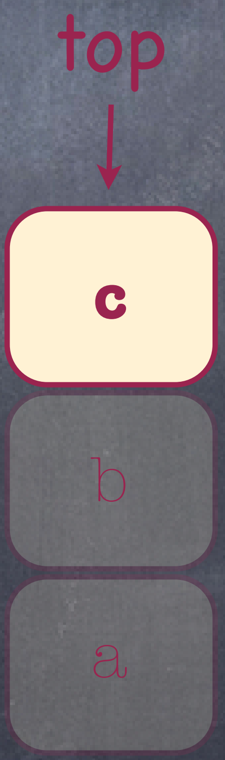

Cost 1

Cost 2

max = 2

sum = 3

Total cost?

Semantic distances need a notion of state

- States are equivalence classes of sequences in S
- Two sequences in S are equivalent if they have an indistinguishable future

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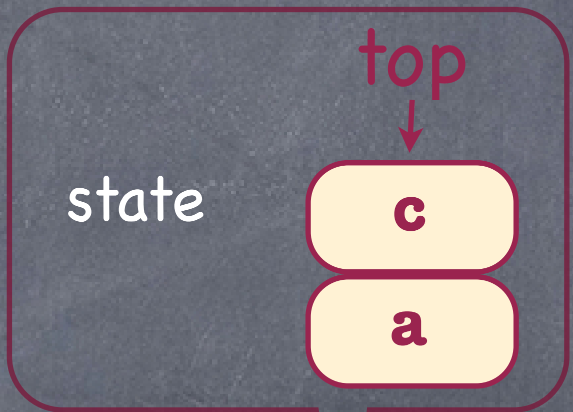

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Semantics goes operational

- $S \subseteq \Sigma^*$ is the sequential specification

states | labels | initial state

- $LTS(S) = (S/_\equiv, \Sigma, \rightarrow, [\varepsilon]_\equiv)$ with

transition relation

$$[s]_\equiv \xrightarrow{m} [sm]_\equiv \quad \Leftrightarrow \quad sm \in S$$

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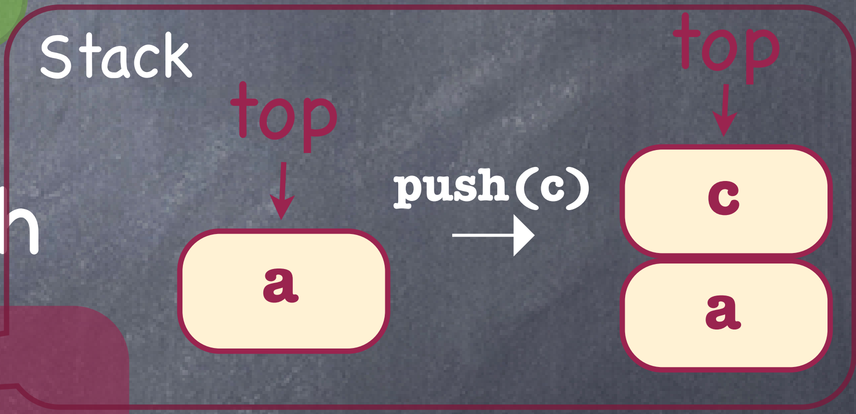

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The framework

- Start from LTS(S)
- Add transitions with transition costs
- Fix a path cost function

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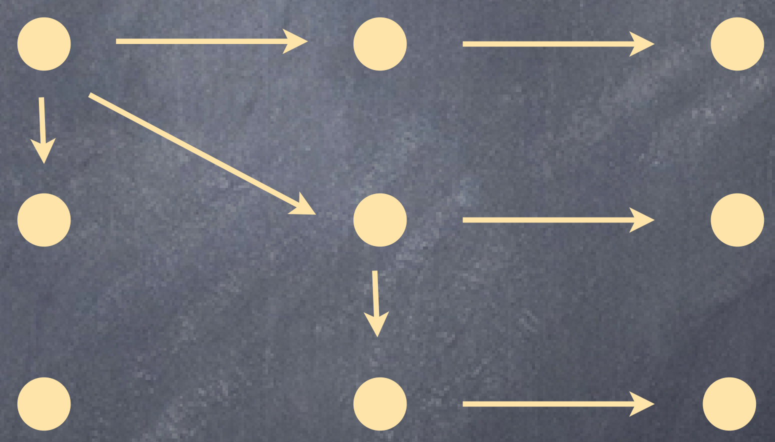
Σ - singleton



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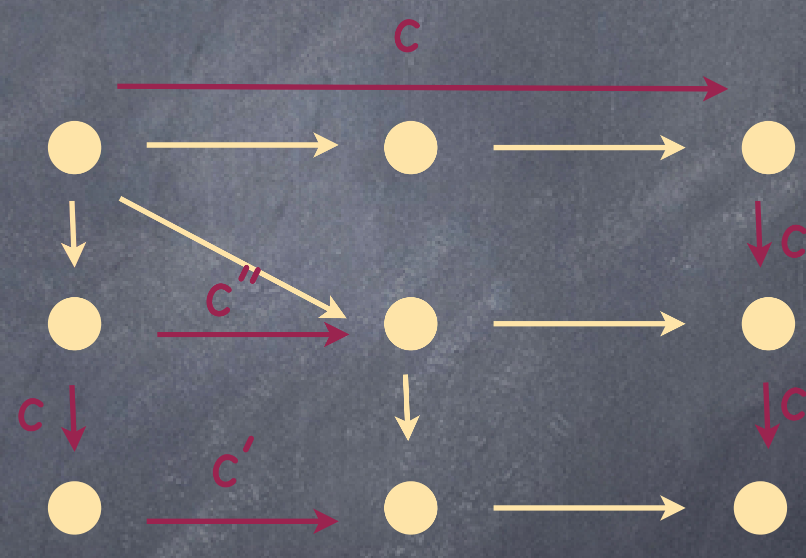

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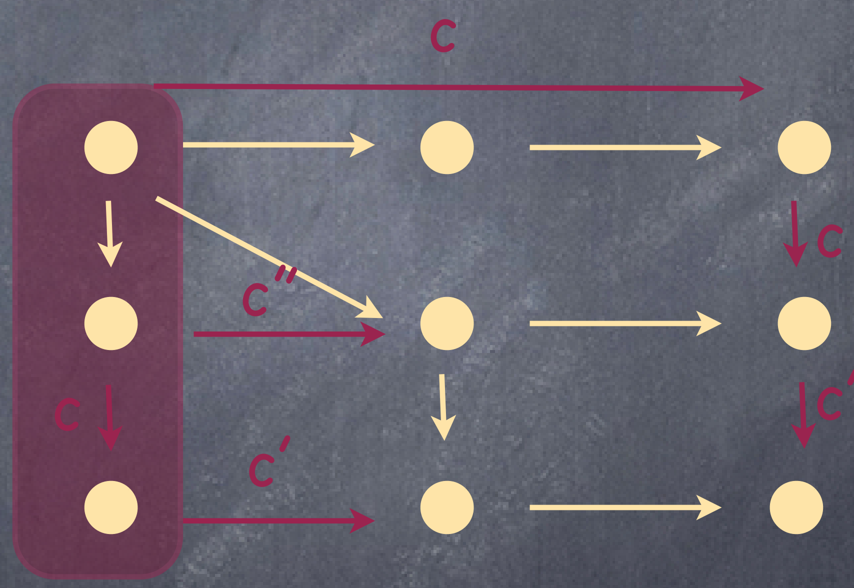

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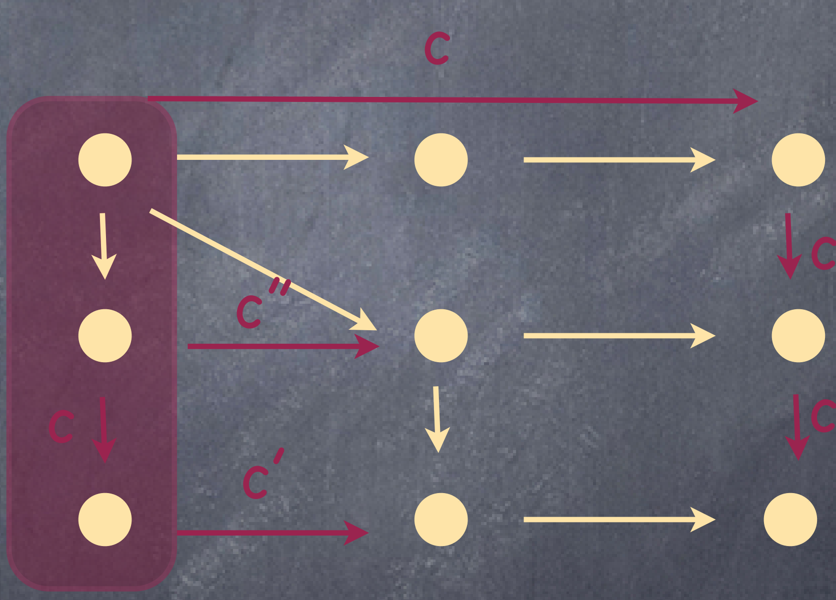

- Fix a path cost function

distance - minimal cost on all paths labelled by the sequence

For the user

- Pick your favorite data structure S
- Add desired incorrect transitions and assign them transition costs
- Choose a path cost function

distance and relaxation follow

For the user

The framework clears the head,
direct concrete relaxations are also possible

- Pick your favorite data structure S
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distance and relaxation follow

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```
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```
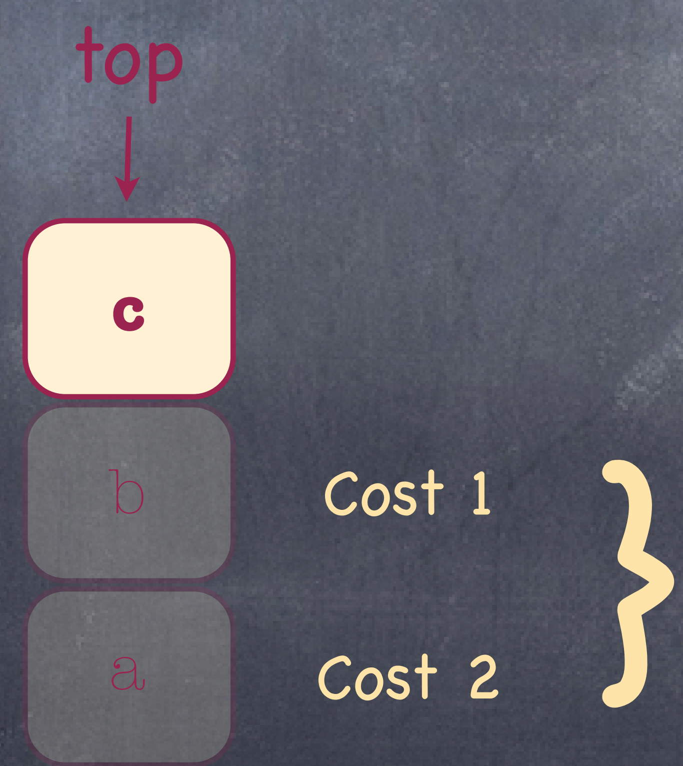

state evolution

Total cost



max = 2
sum = 3

Stack example

- Canonical representative of a state
- Add incorrect transitions with costs
- Possible path cost functions max, sum,...

Stack example

Sequence of **push**'s with no matching **pop**

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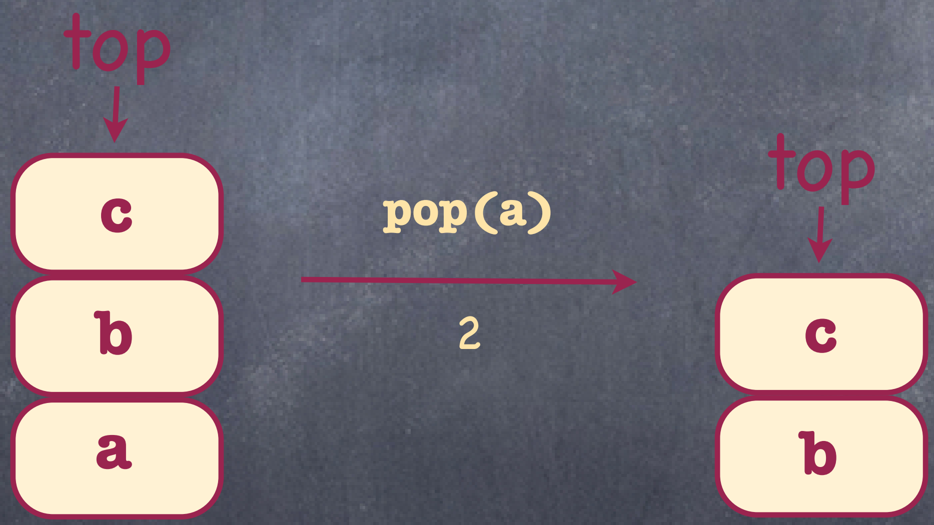

- Possible path cost functions max, sum,...

It's more general...

Generic out-of-order

segment_cost($q \xrightarrow{m} q'$) $= |\mathbf{v}|$

transition cost

where $\mathbf{v}$ is a sequence of minimal length s.t.

(1) $[\mathbf{uvw}]_{\equiv} = q$, $\mathbf{uvw}$ is minimal, $\mathbf{uw}$ is minimal

removing $\mathbf{v}$ enables a transition

(1.2) $[\mathbf{uw}]_{\equiv} \xrightarrow{m} [\mathbf{uw'}]_{\equiv}$, $[\mathbf{uvw'}]_{\equiv} = q'$

(2) $[\mathbf{uw}]_{\equiv} = q$, $\mathbf{uw}$ is minimal, $\mathbf{uvw}$ is minimal

inserting $\mathbf{v}$ enables a transition

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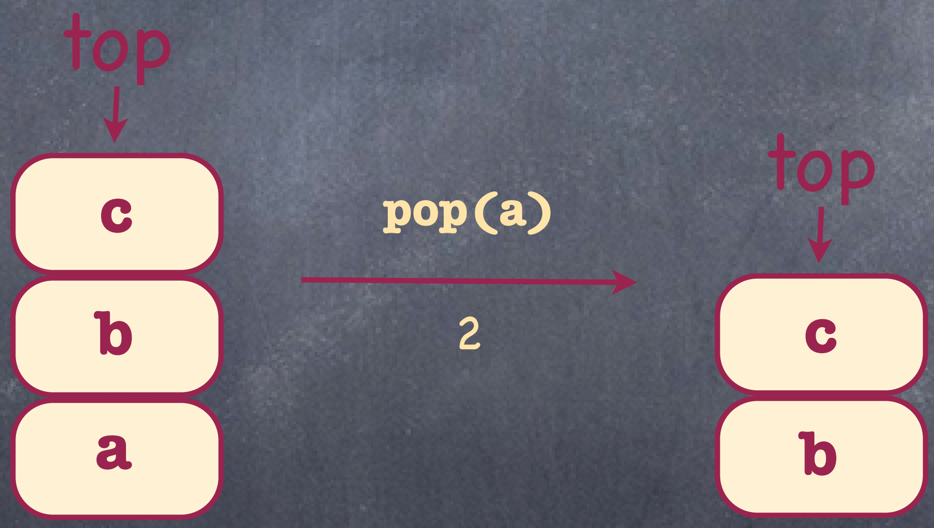

- Possible path cost functions max, sum,...

also ``shrinking window''
restricted out-of-order

Out-of-order queue

Sequence of **enq**'s with no matching **deq**

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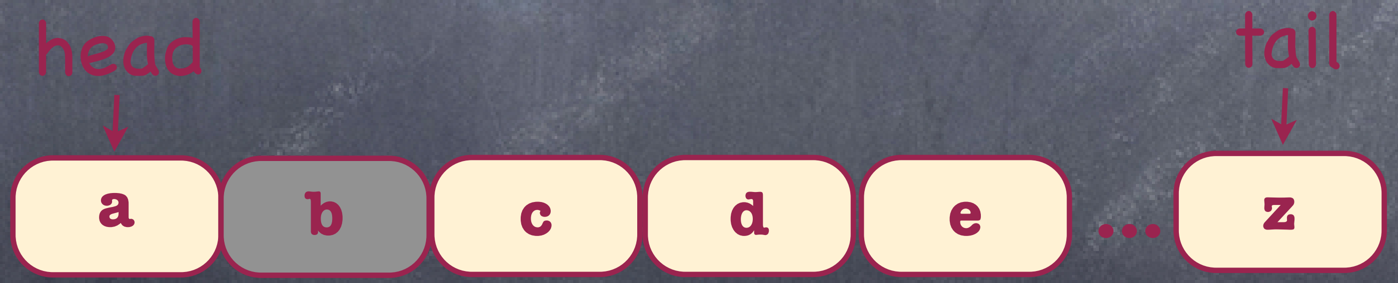
- Possible path cost functions max, sum,...

also "shrinking window" restricted out-of-order

Out-of-order variants

Queue



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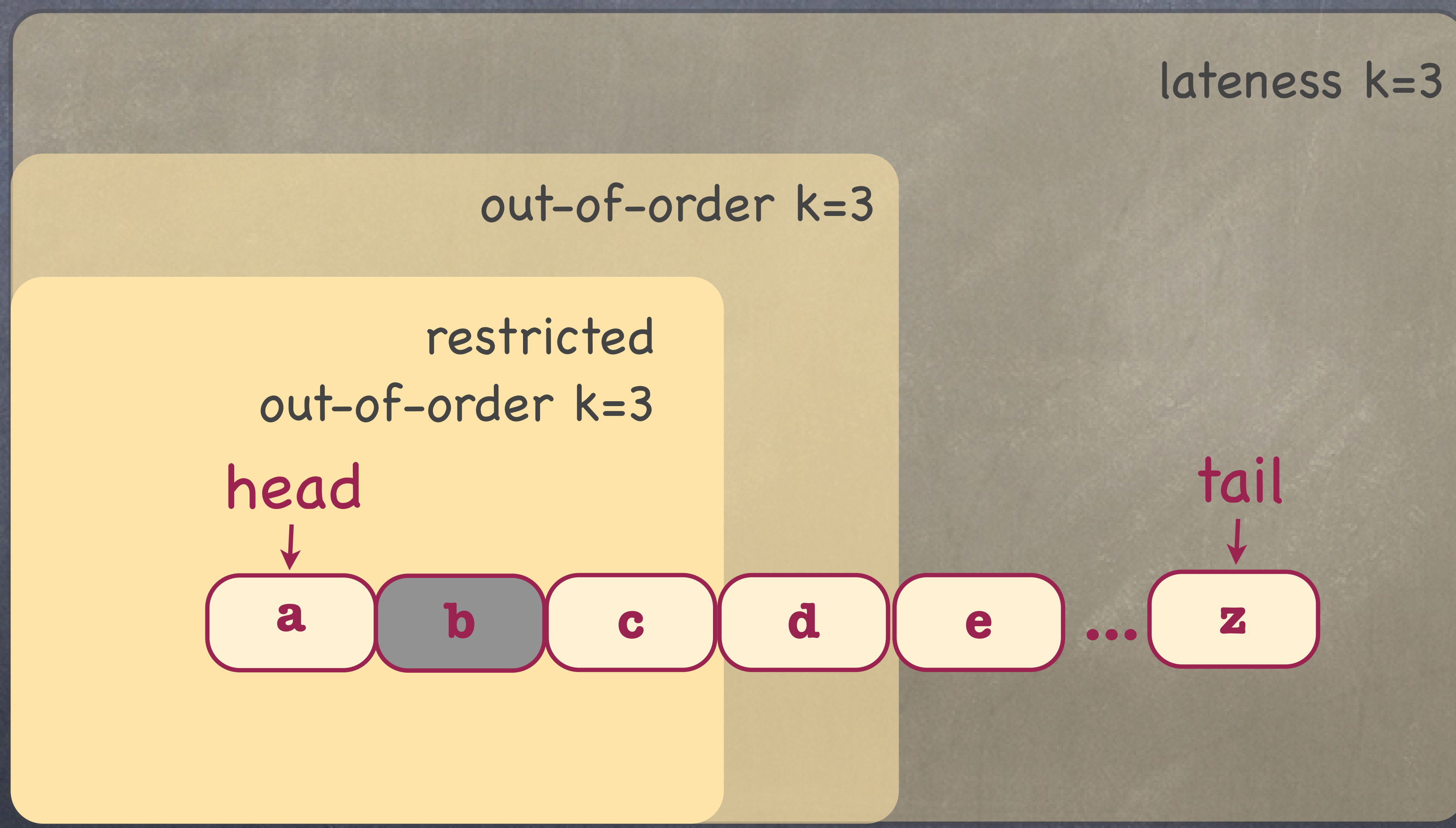

How about implementations? Performance?

Short-term history

- SCAL queues [KPRS'11]
- Quasi linearizability theory and implementations [AKY'10]
- Some straightforward implementations [HKPSS'12]
- Efficient lock-free segment queue [KLP'12]

(almost) all implement restricted out-of-order

Short-term history

distributed, one k-queue

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performs very well

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Lessons learned

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The way from sequential specification to concurrent implementation is hard

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Being relaxed not necessarily means better performance

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Being relaxed not necessarily means better performance

Well-performing implementations of relaxed specifications do exist!

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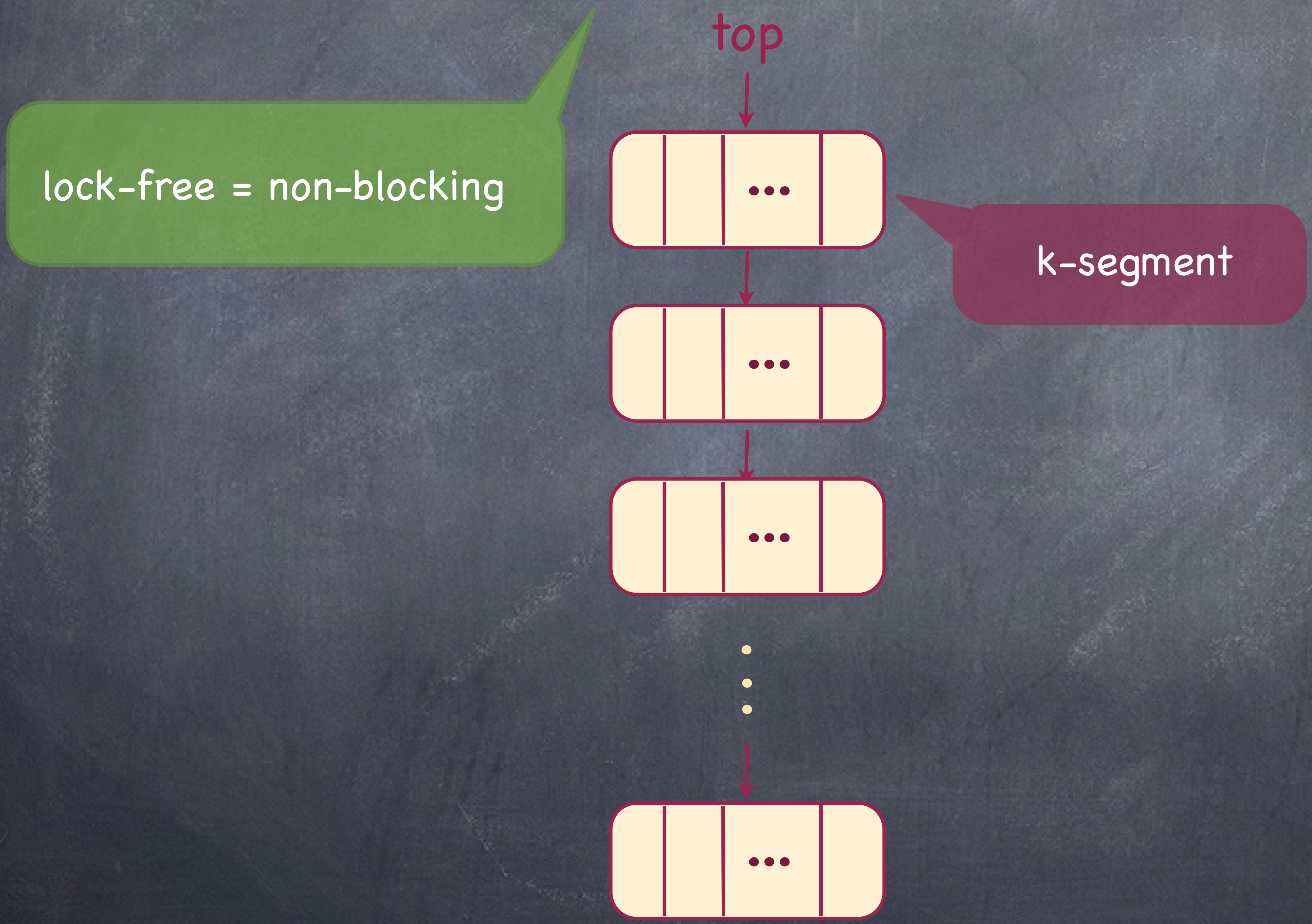
Well-performing implementations of relaxed specifications do exist!

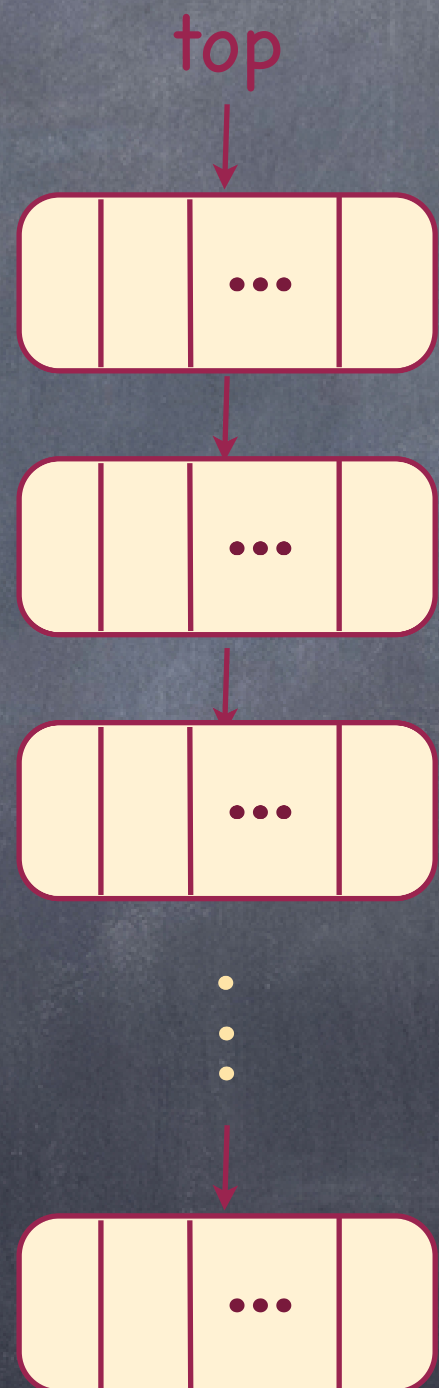
Let's see them!

Restricted-out-of-order k-Stack



Restricted-out-of-order k-Stack

lock-free = non-blocking



k-segment

```
loop:
  read consistent state
  try to add/remove an item (*)
  if successful:
    return
  else:
    try to repair the stack
    goto loop (retry)
```

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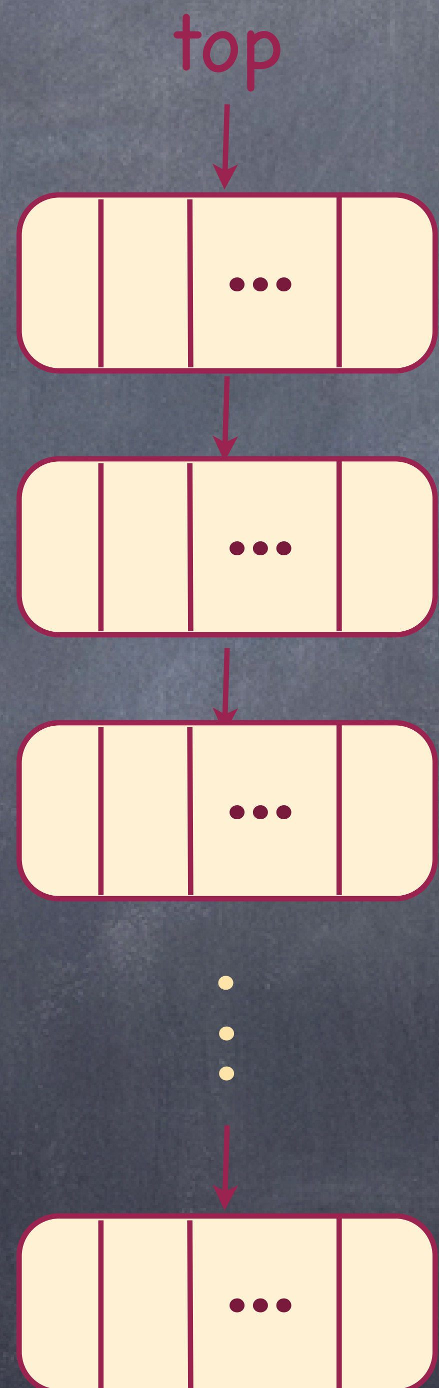

k-segment

add/remove segment

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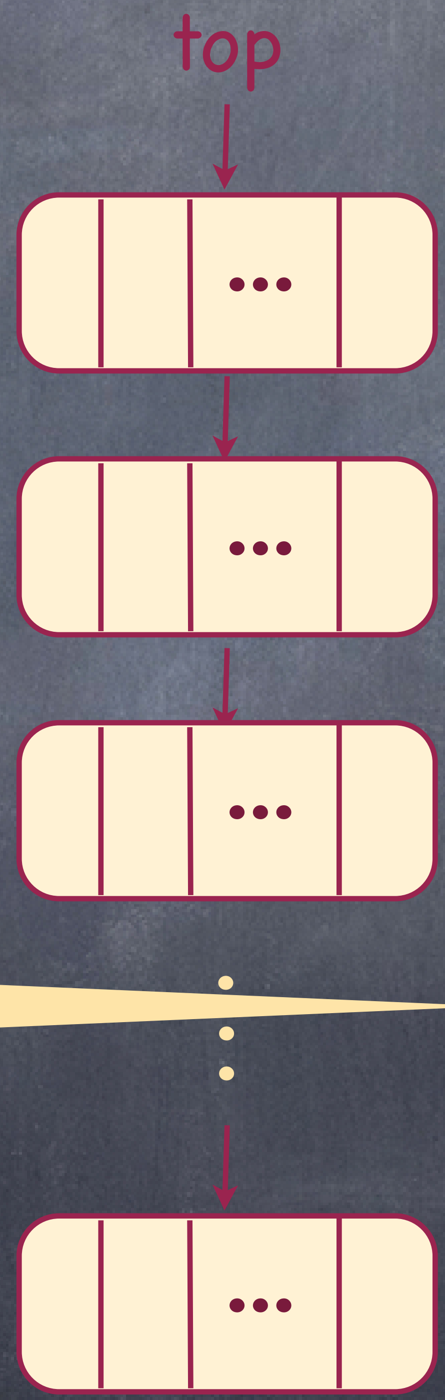
Restricted-out-of-order k-Stack



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k-segment

add/remove segment

CAS - based

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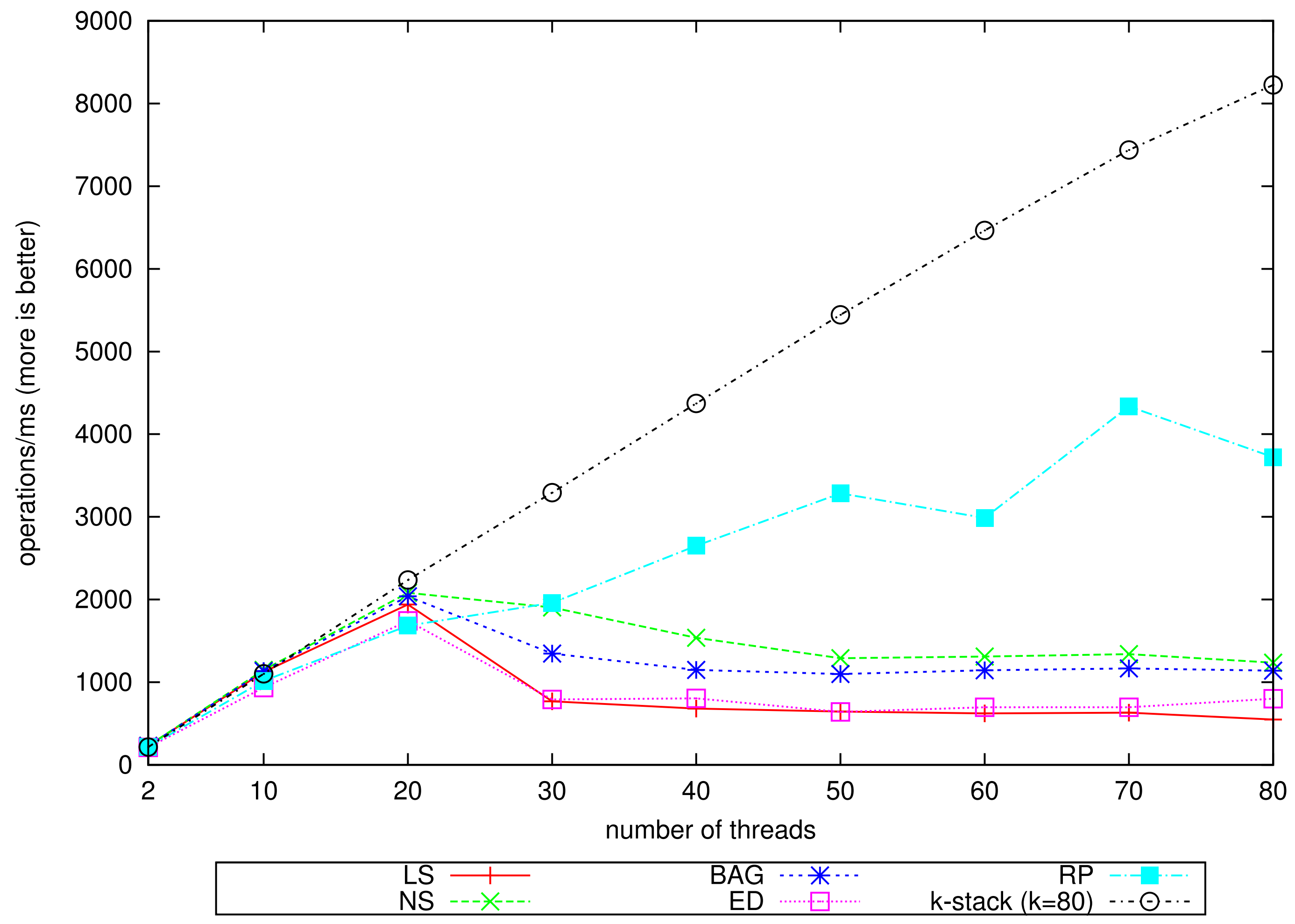
Stack

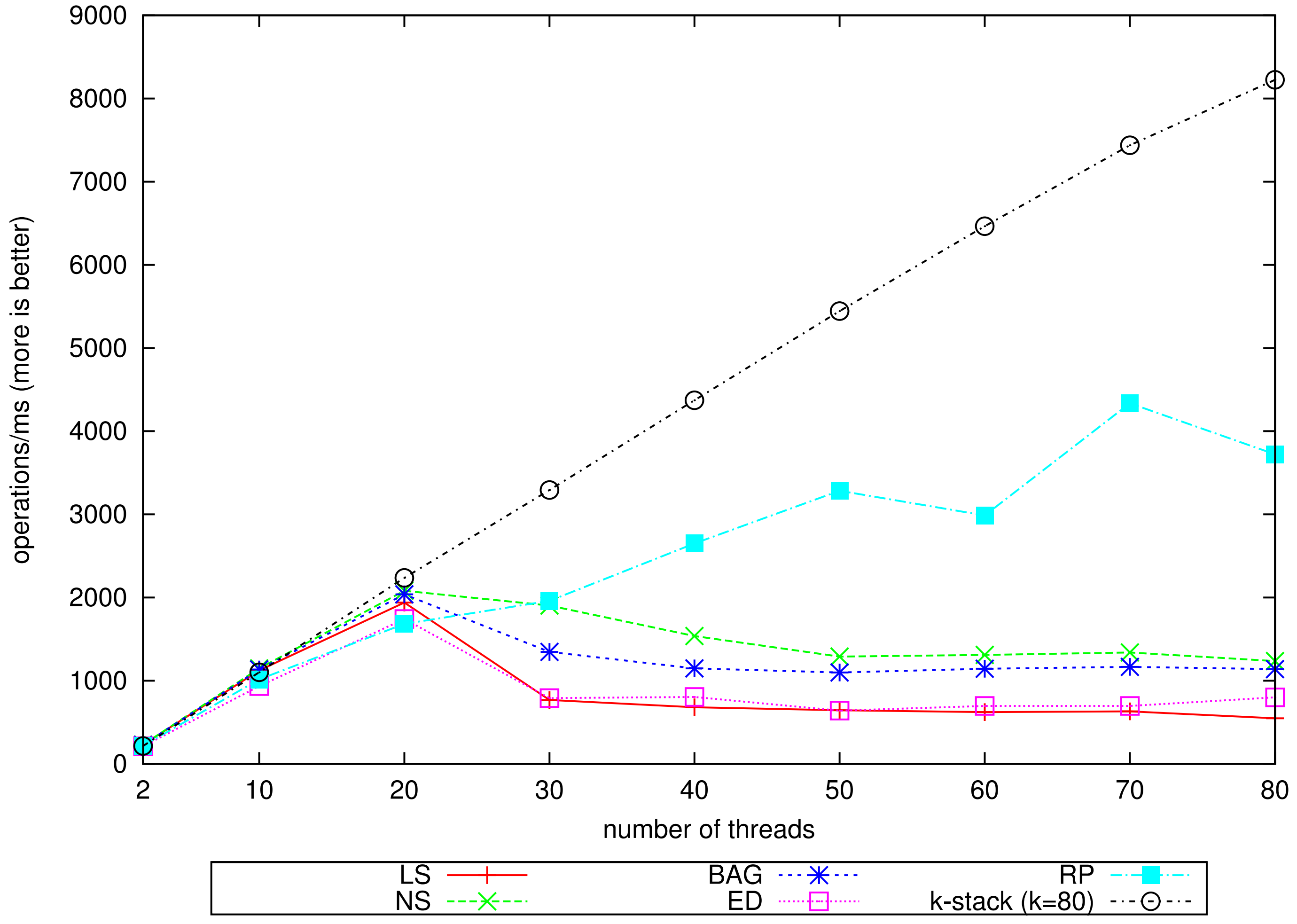
Scalability comparison



Stack

Scalability comparison

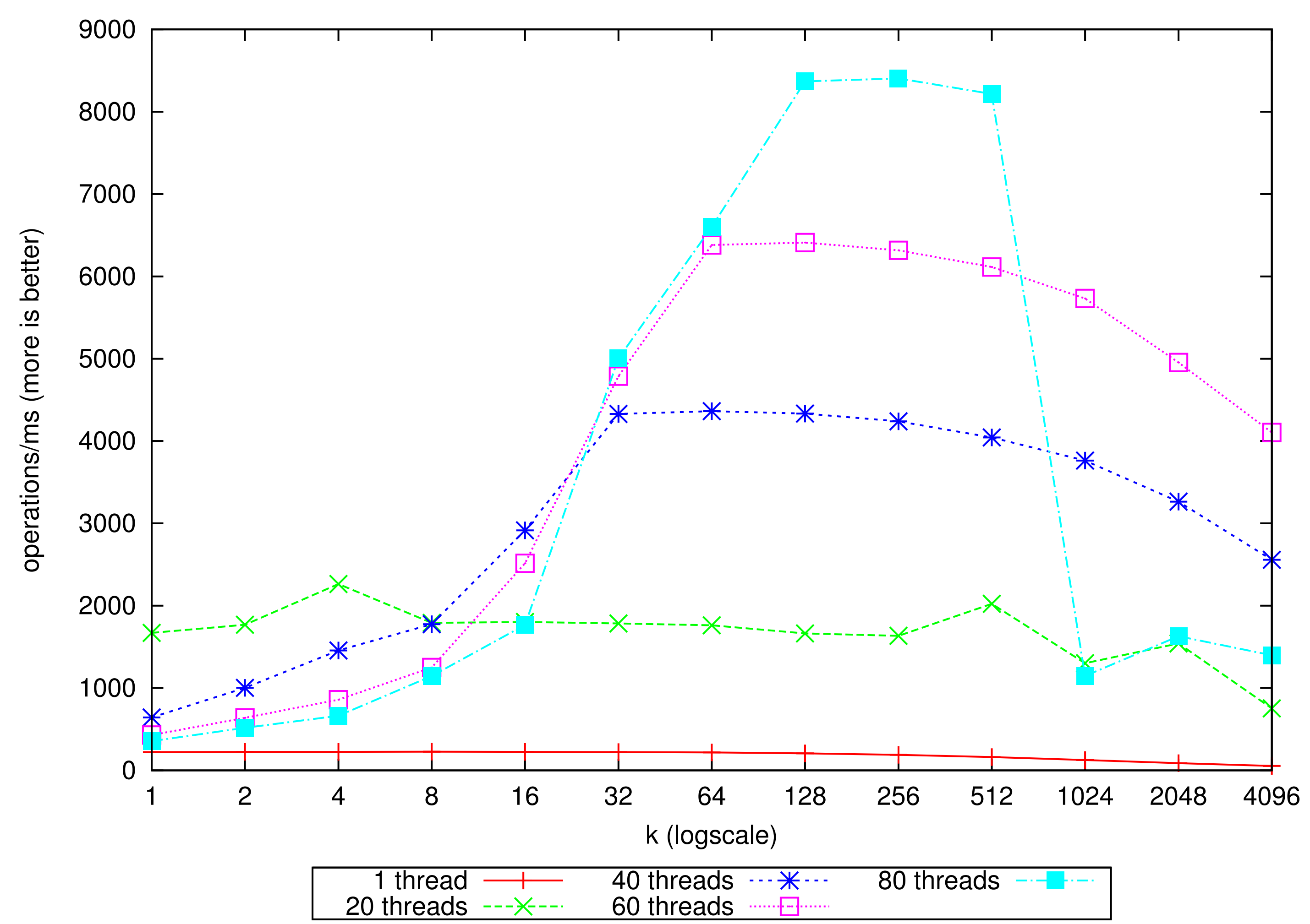
"80"-core machine



k-Stack

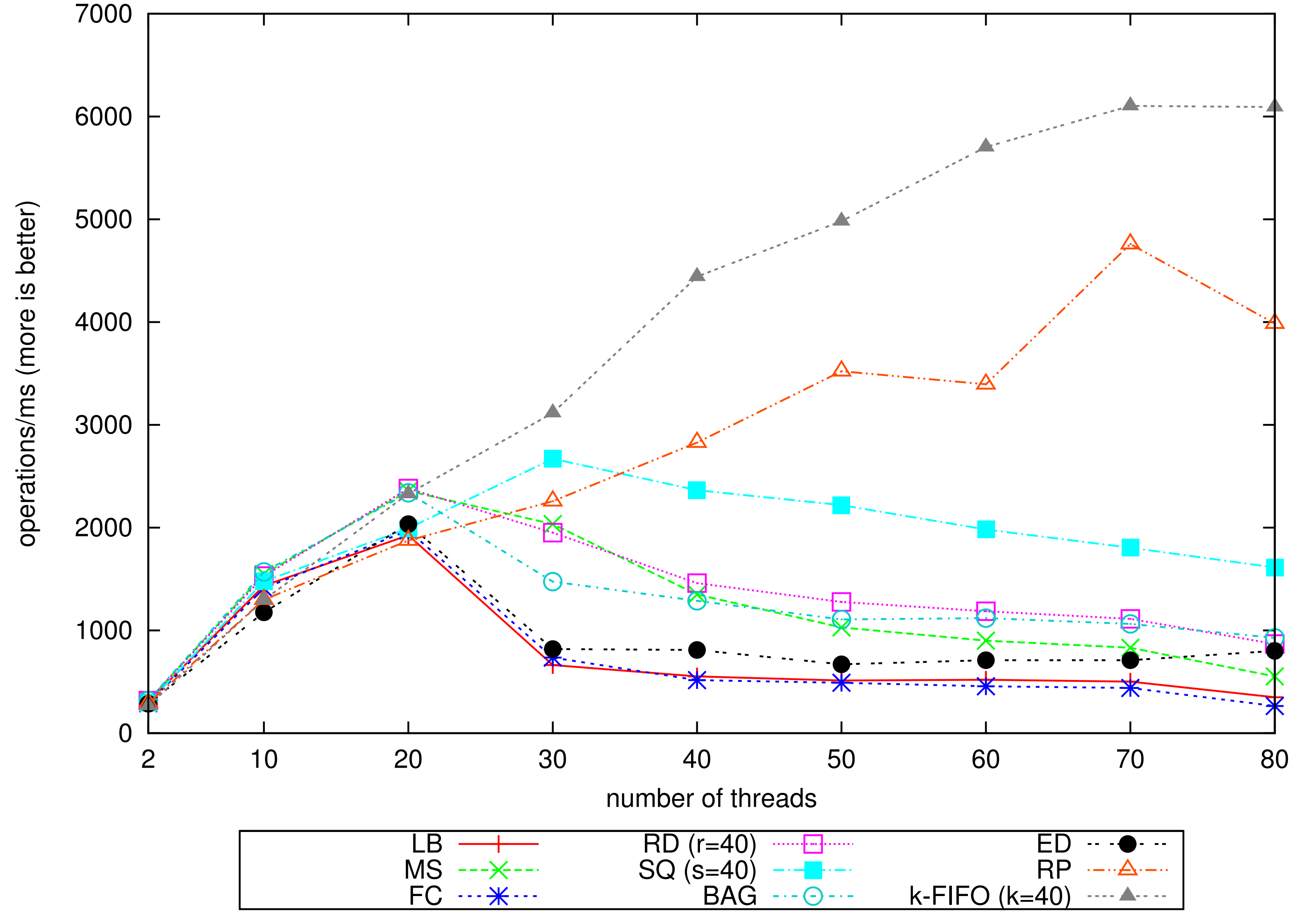
The more relaxed, the better

lock-free segment stack



Queue

Scalability comparison



Queue

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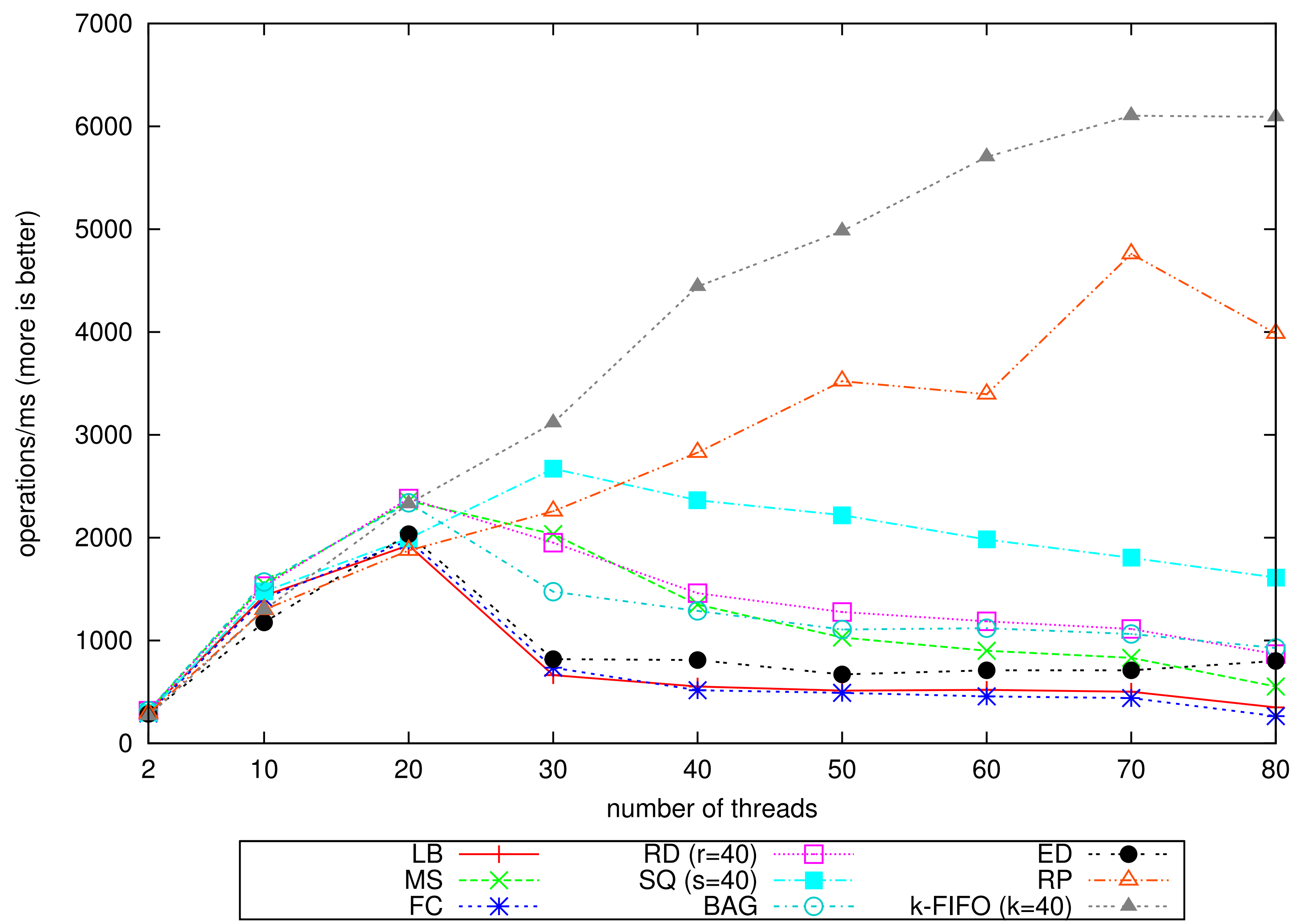

k-Queue

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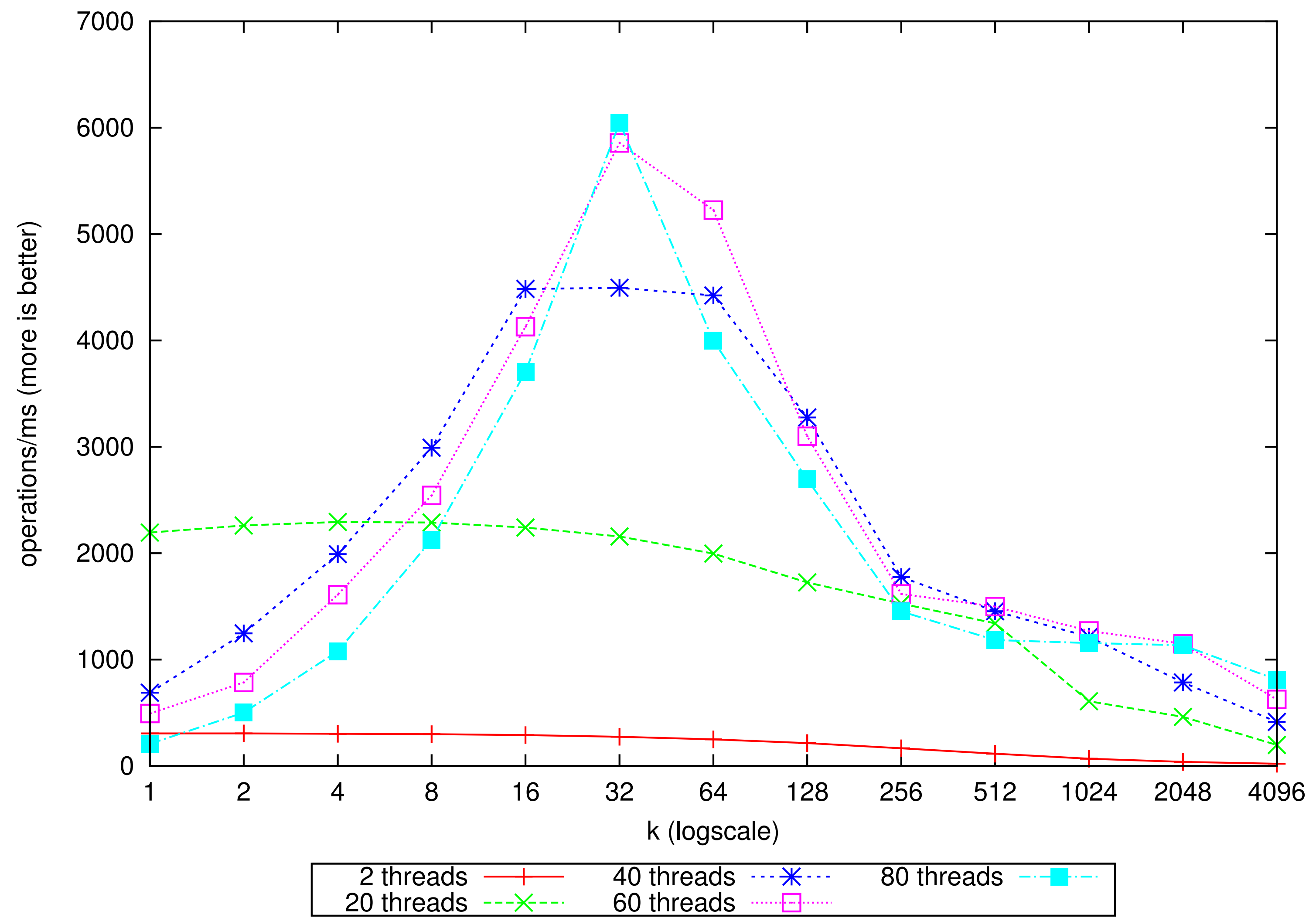
lock-free segment queue



Conclusions

Contributions

Framework for quantitative relaxations
generic relaxations, concrete examples,
efficient implementations exist

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all kinds of

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Difficult open problem

How to get from theory to practice?

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Difficult open problem

THANK YOU

How to get from theory to practice?

For the future

- Study applicability
- Learn from efficient implementations

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which applications tolerate relaxation ?

maybe there is nothing to tolerate!

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  - which applications tolerate relaxation ?
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  - towards synthesis
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