Concurrent Data Structures: Semantics and Relaxations

Ana Sokolova

UNIVERSITY of SALZBURG

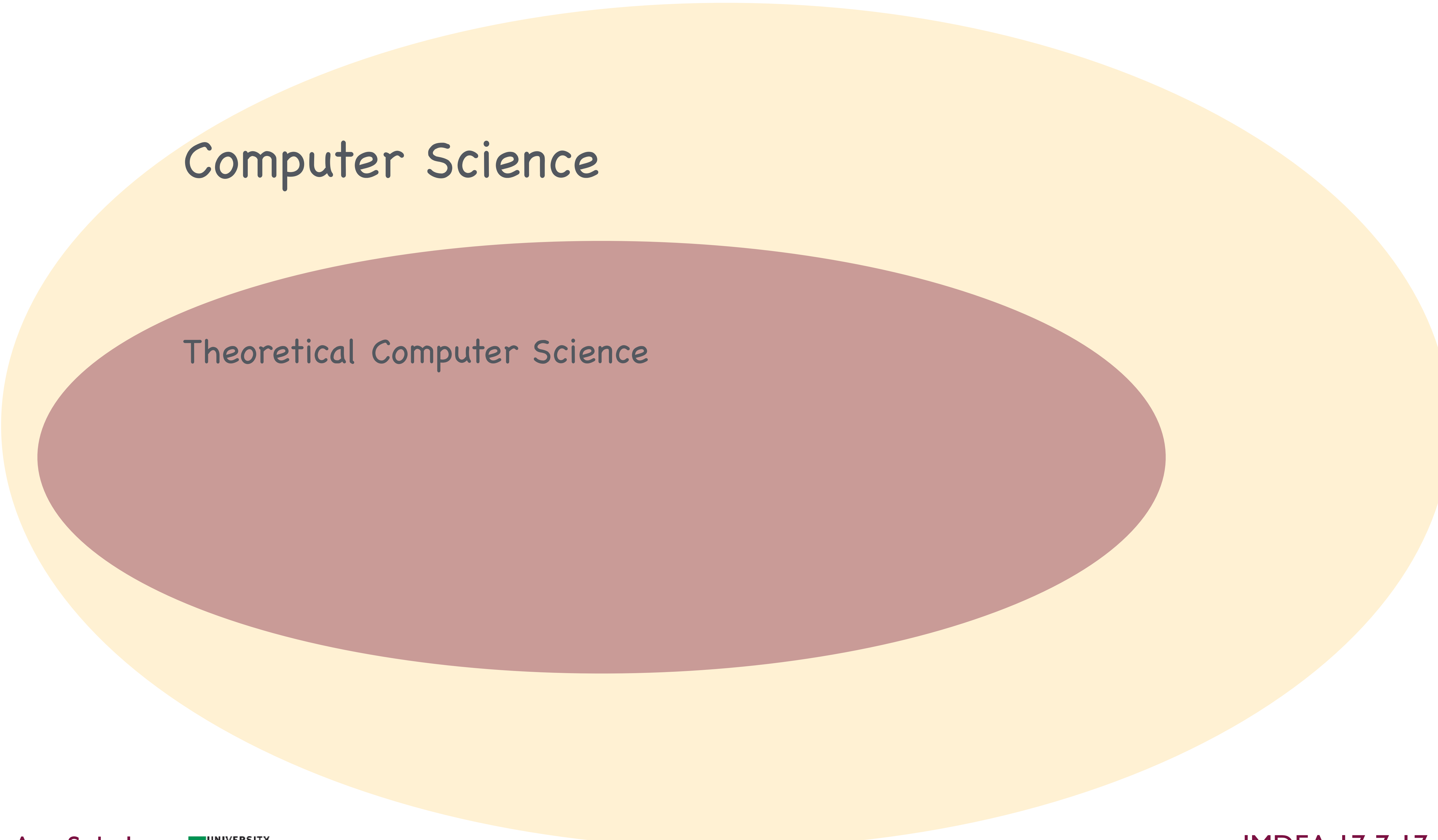
IMDEA, Madrid, 17.7.17

Background big picture

Background big picture

Computer Science

Background big picture



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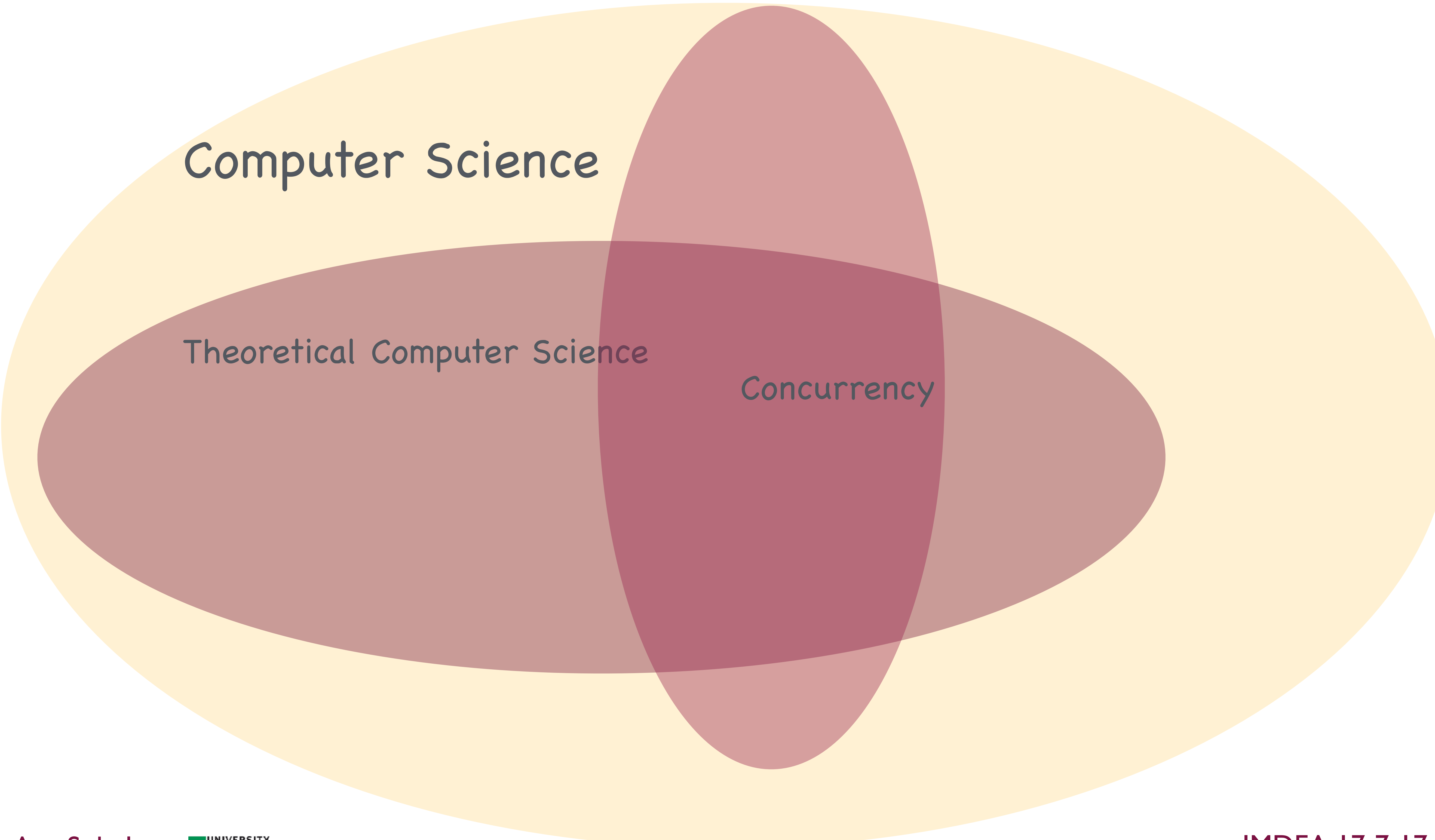

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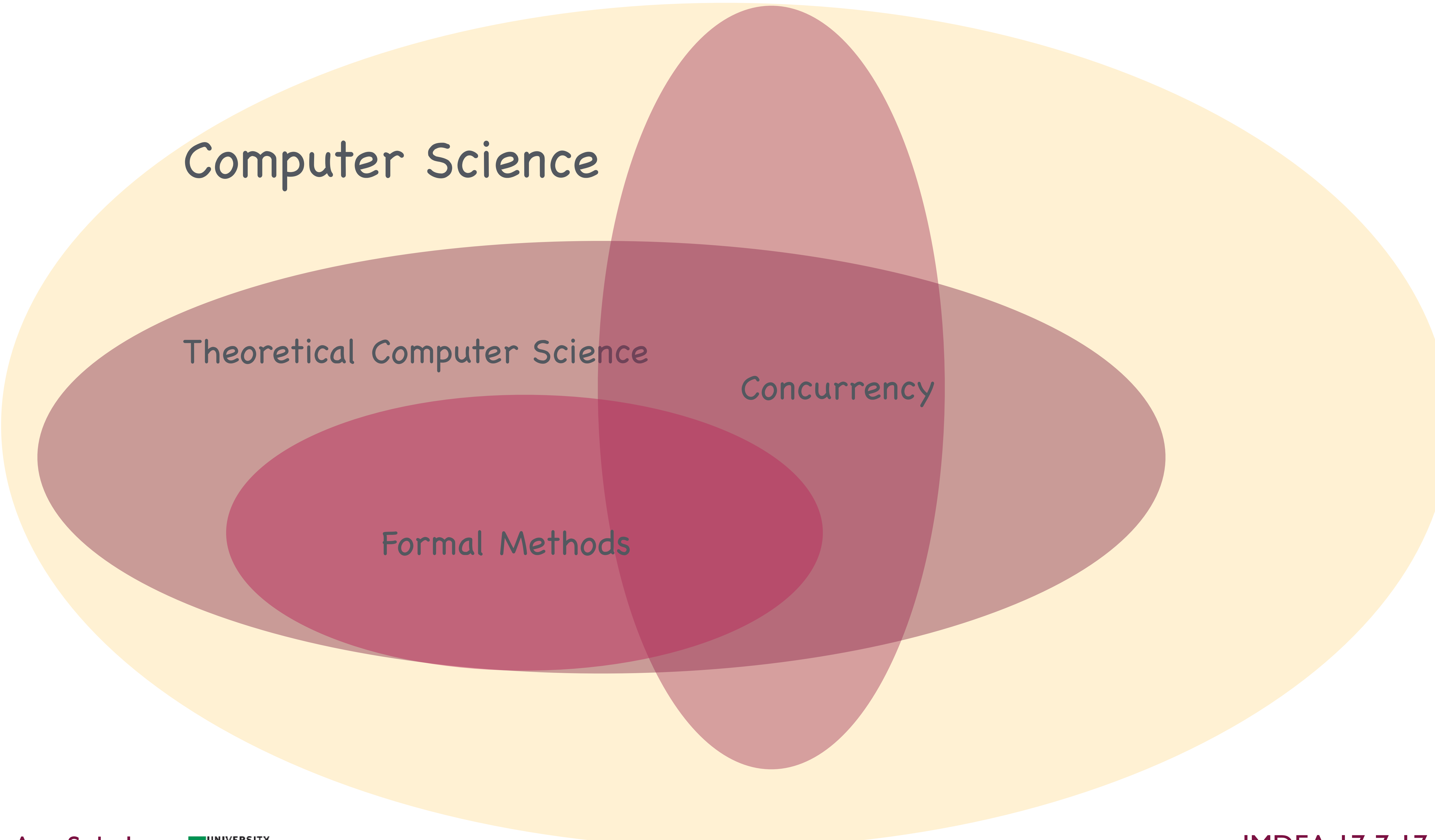

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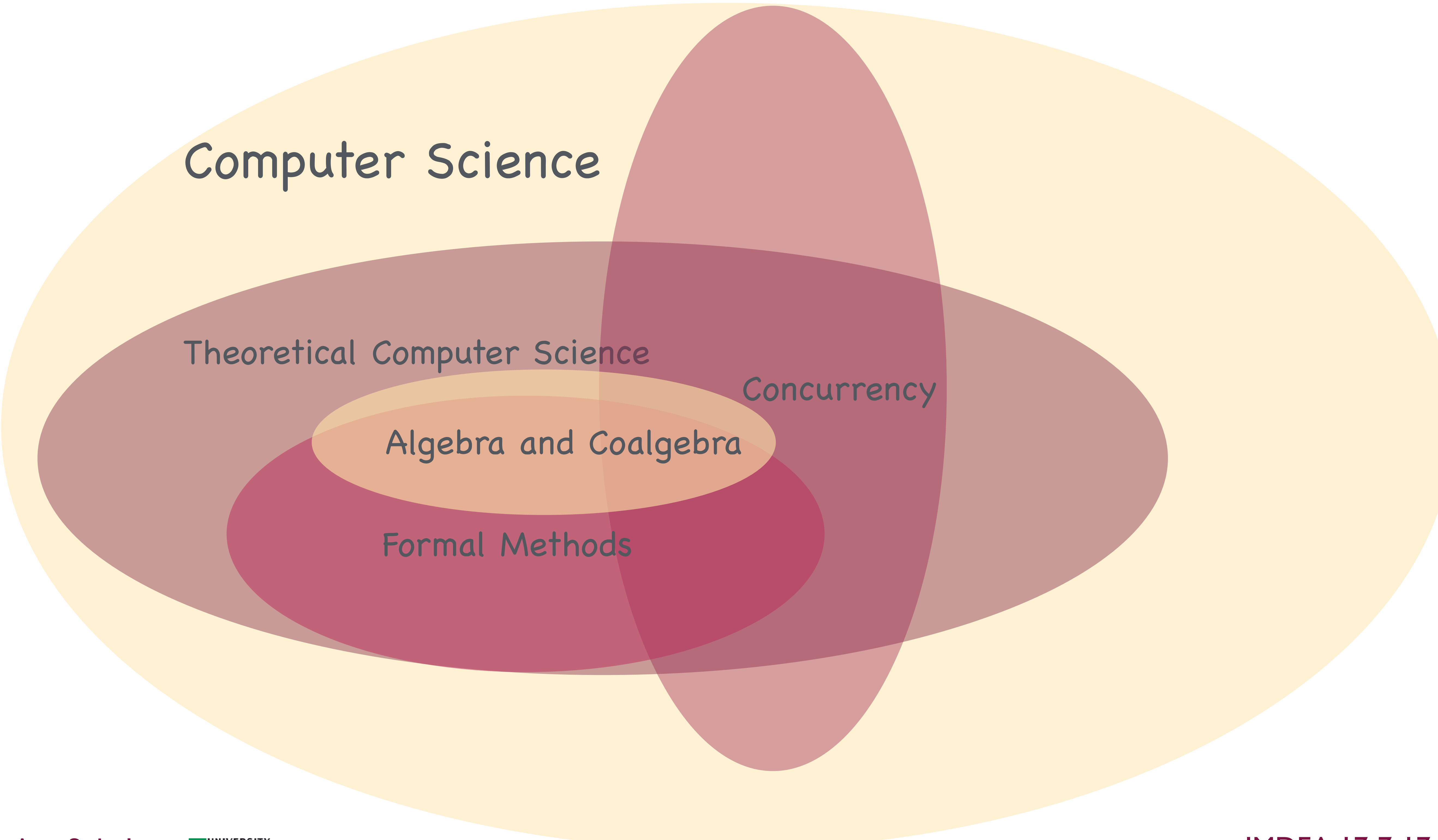

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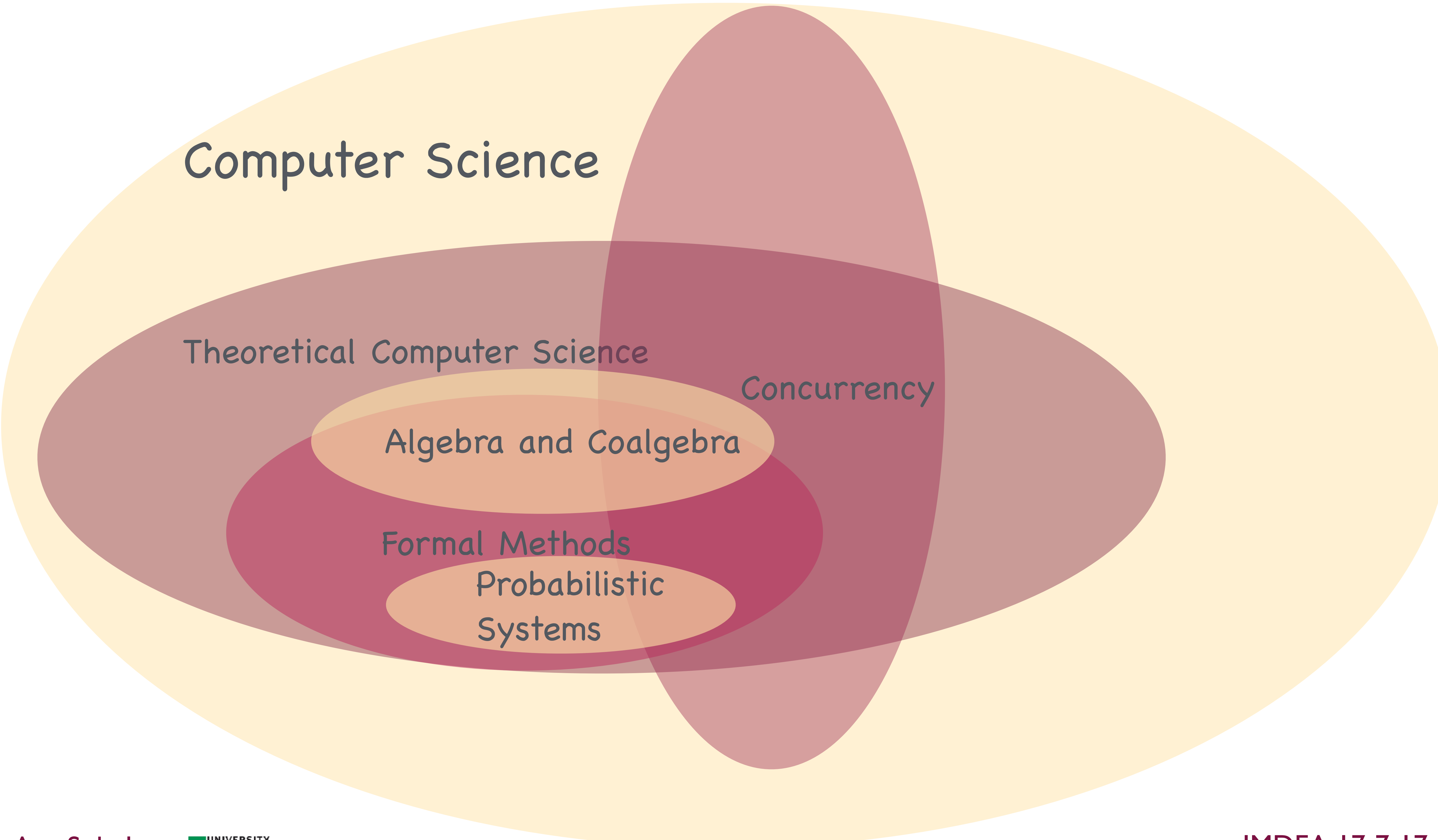

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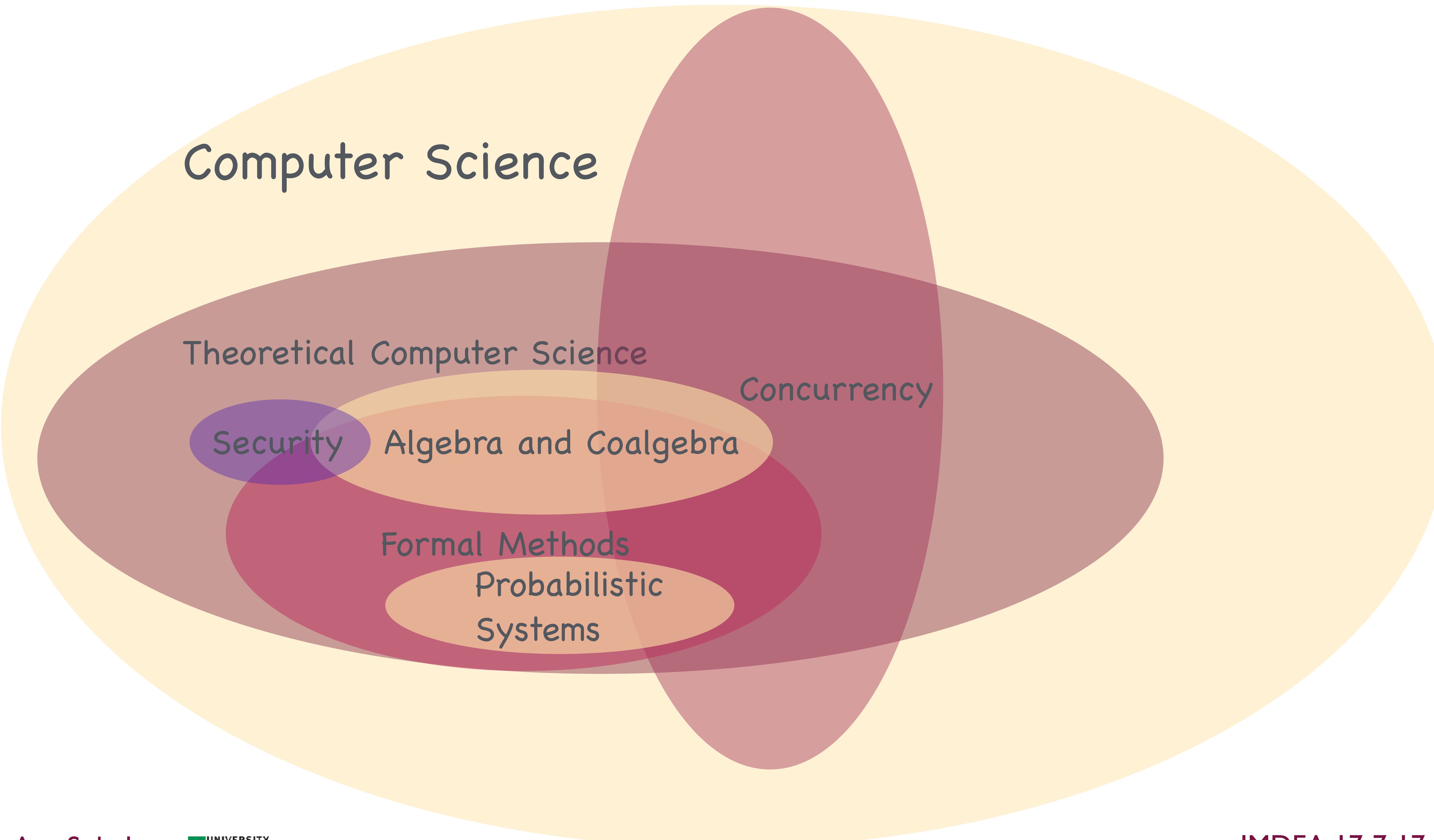

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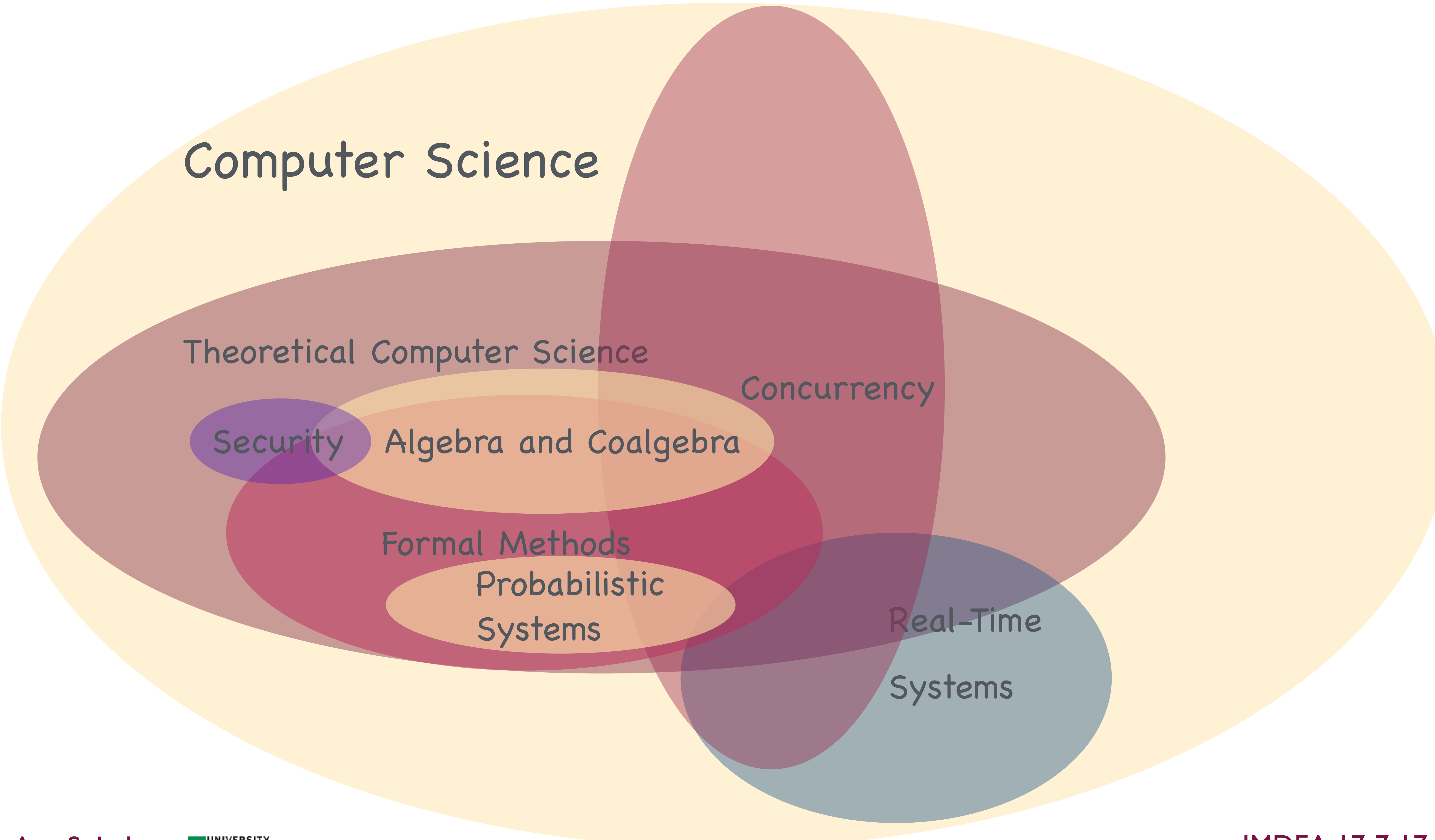

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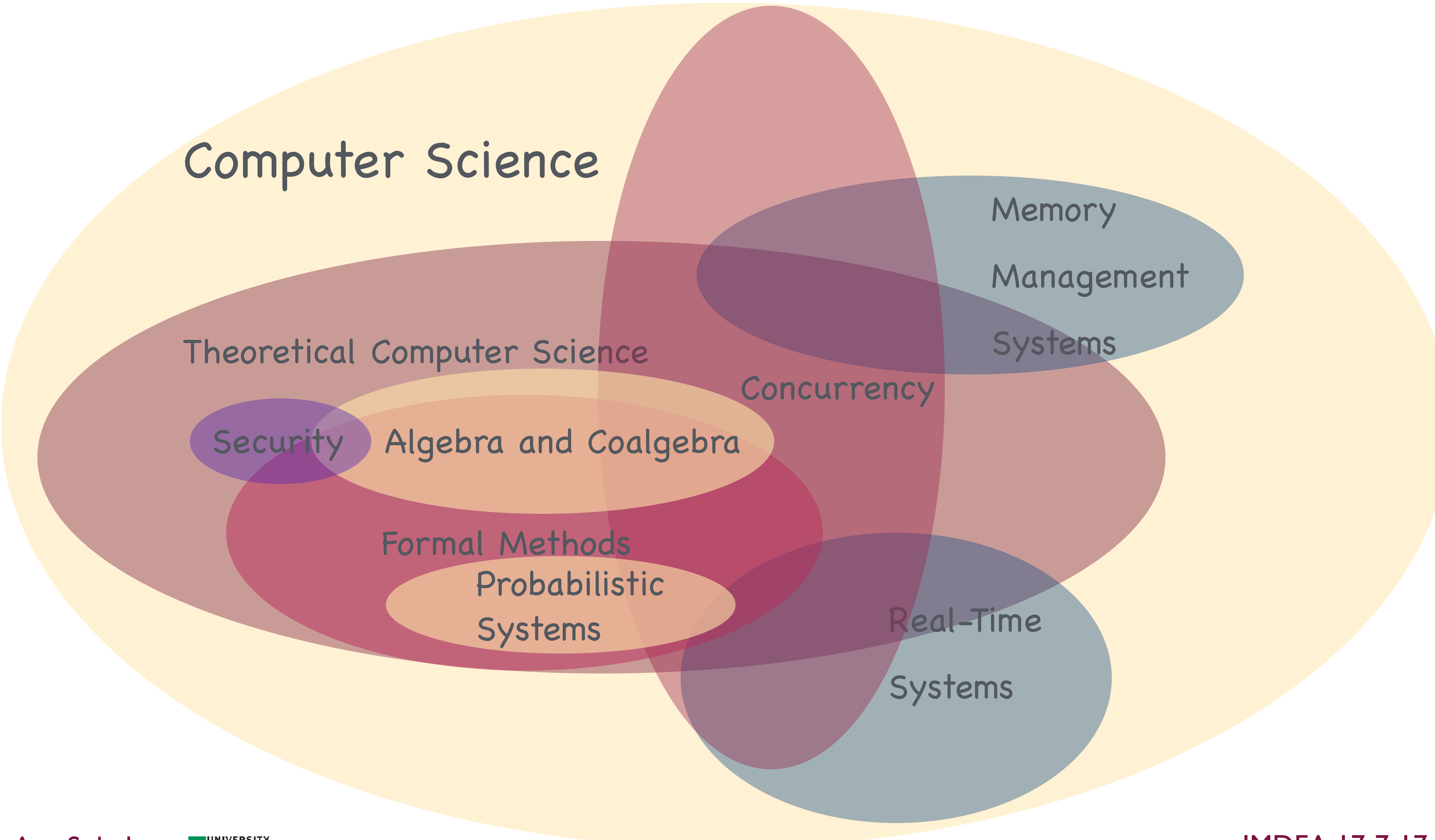

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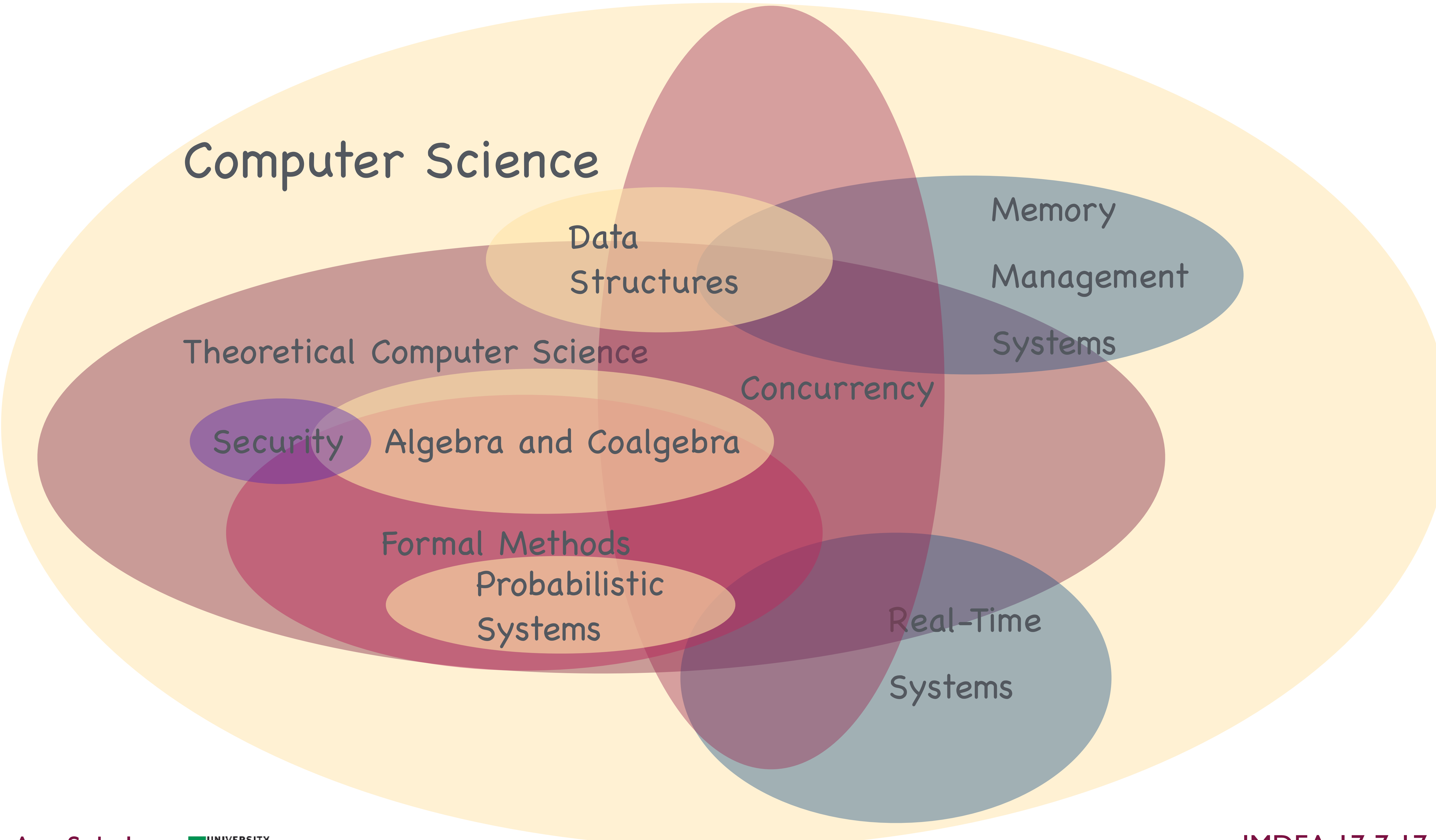

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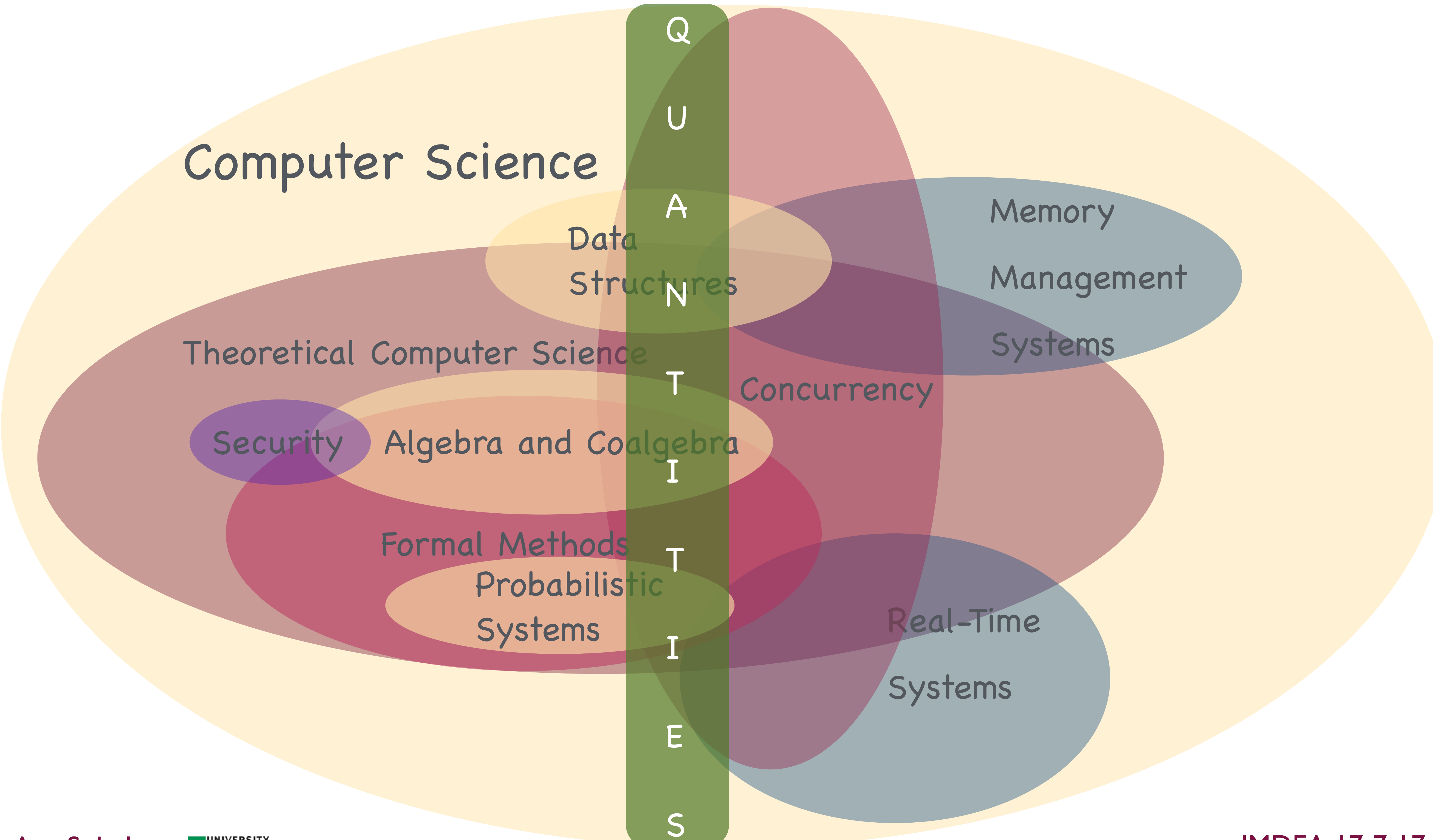
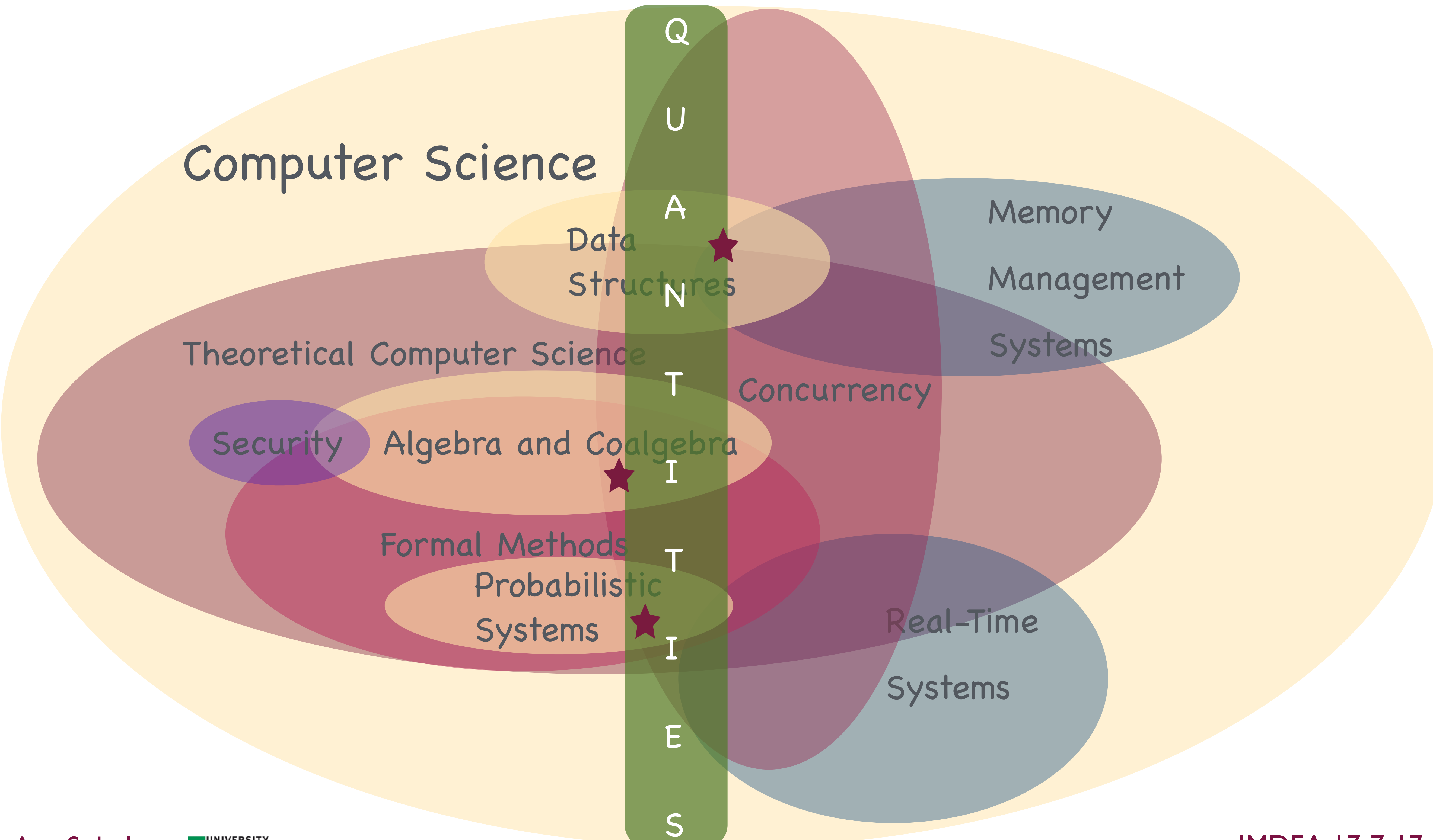


Favourites



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Computer Science
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Concurrent Data Structures: Semantics and Relaxations

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Concurrent Data Structures: Correctness and Performance

Semantics of concurrent data structures

Semantics of concurrent data structures

e.g. pools, queues, stacks

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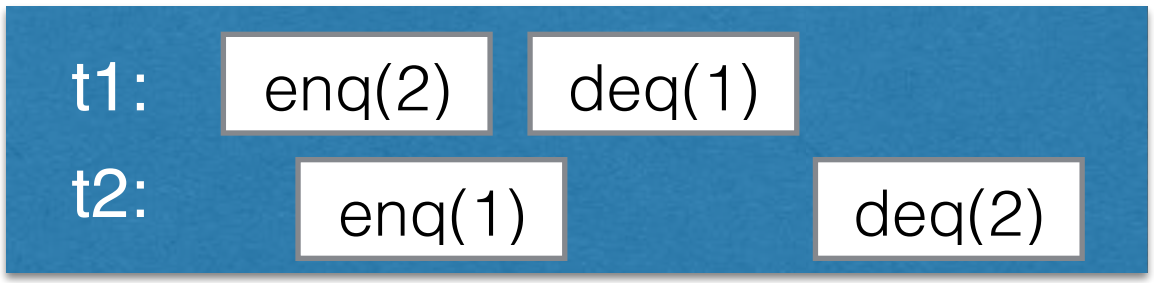

e.g. pools, queues, stacks

Semantics of concurrent data structures

t1: enq(2) deq(1)

t2: enq(1) deq(2)

e.g. pools, queues, stacks

- Sequential specification = set of legal sequences

- Consistency condition = e.g. linearizability / sequential consistency

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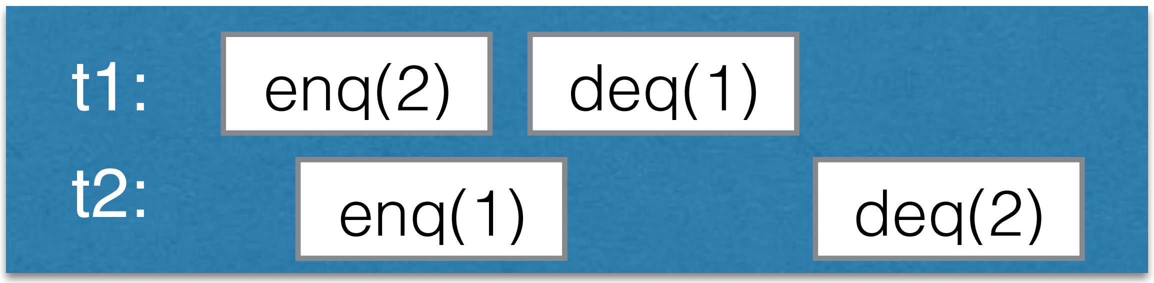

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e.g. the concurrent history above is a linearizable queue concurrent history

Consistency conditions

Linearizability [Herlihy,Wing '90]

↓

Sequential Consistency [Lamport'79]

Consistency conditions

there exists a legal sequence that preserves precedence

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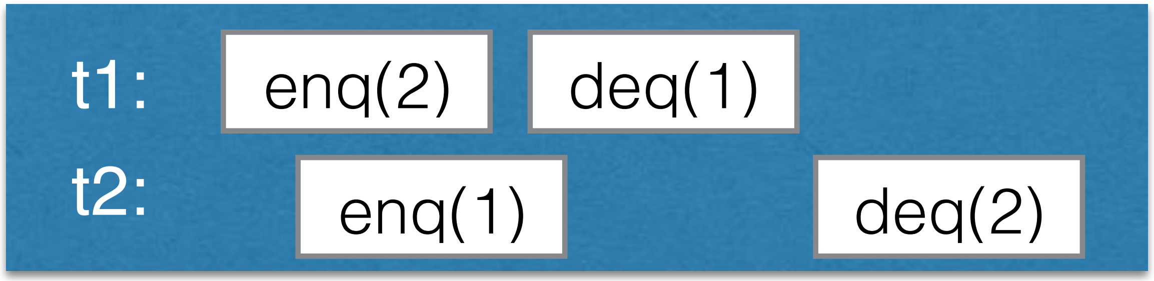

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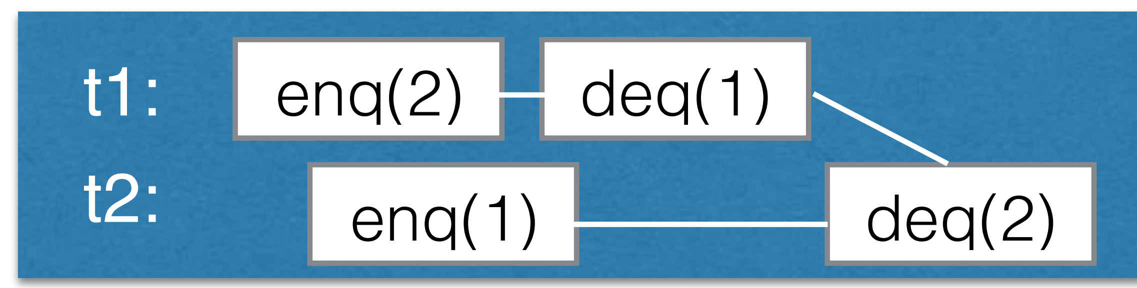

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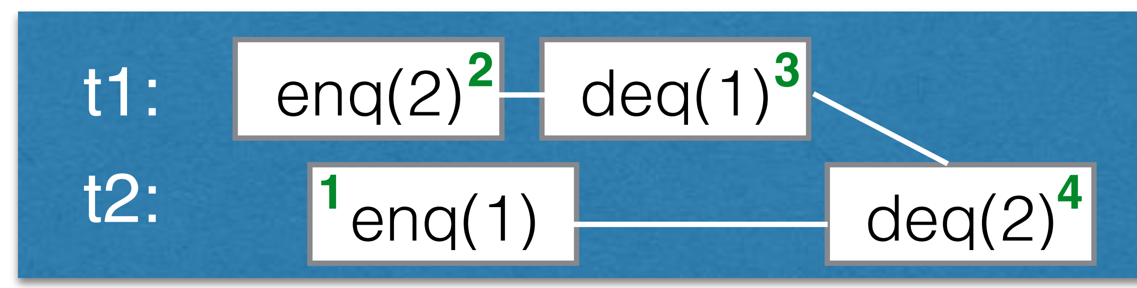

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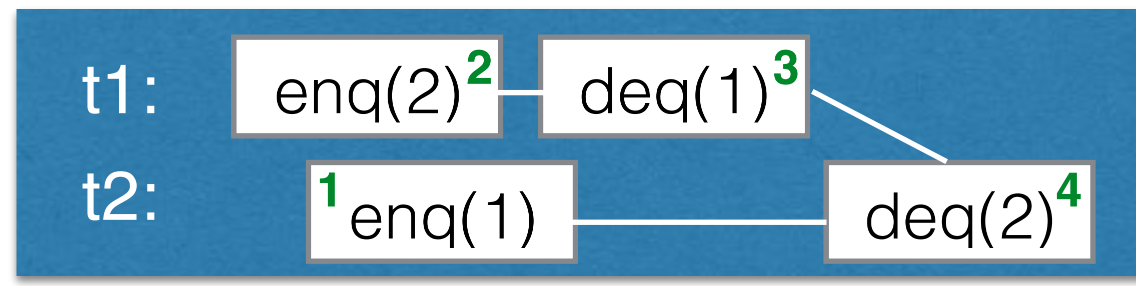

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there exists a legal sequence that preserves per-thread precedence (program order)

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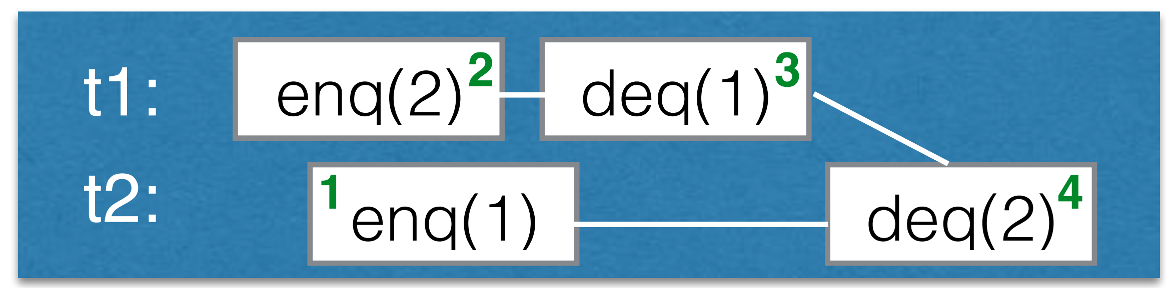

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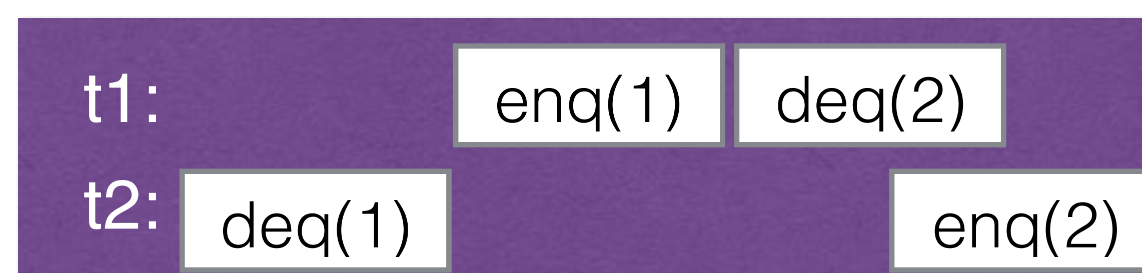

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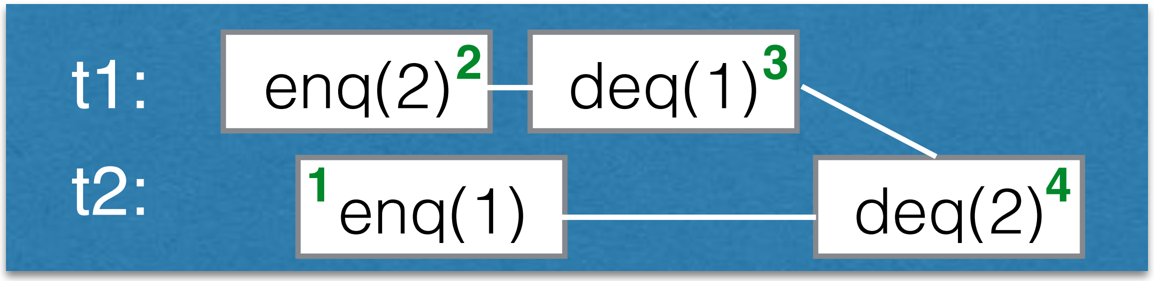

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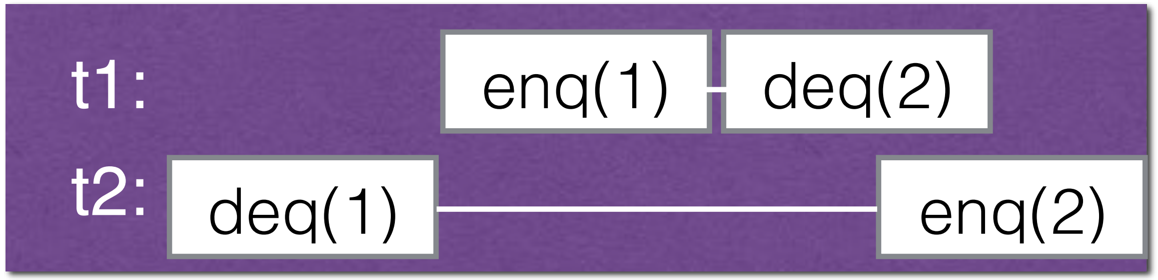

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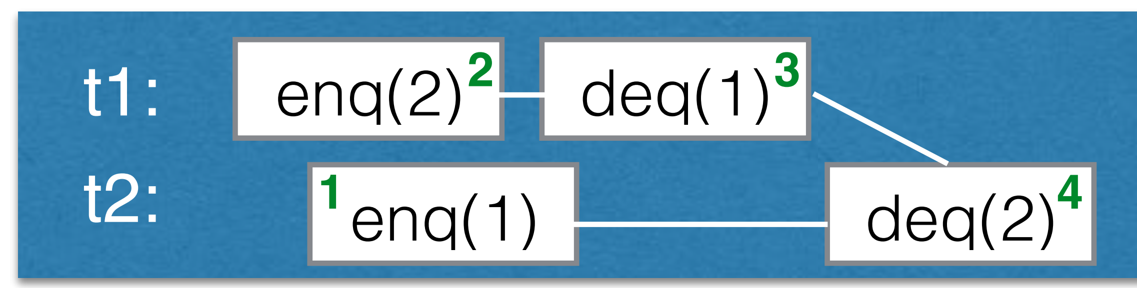

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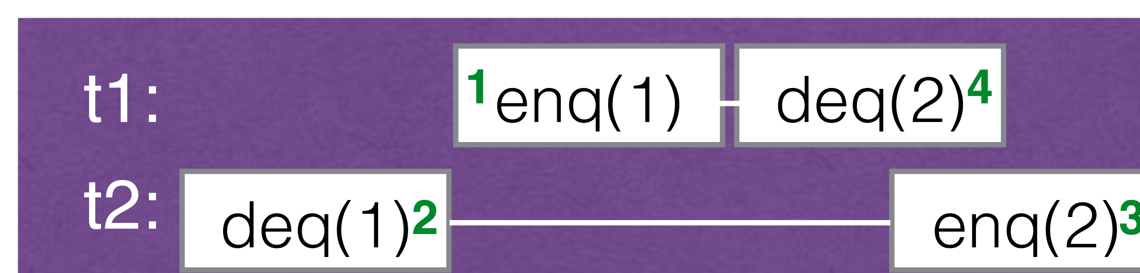
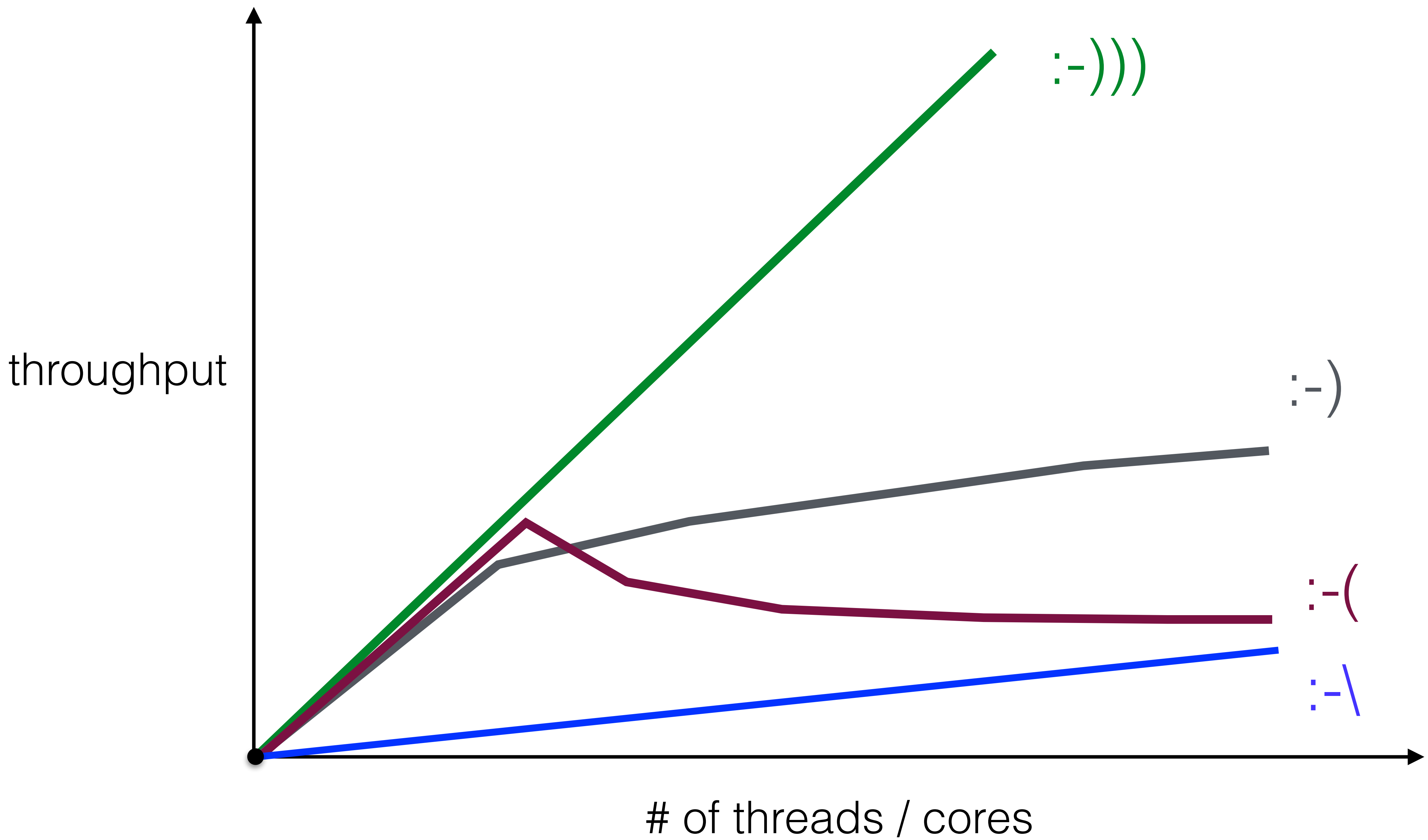


Performance and scalability



Relaxations allow trading

correctness
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provide the potential for better-performing implementations

Relaxing the semantics

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- Sequential specification = set of legal sequences
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Quantitative relaxations
POPL13

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Local linearizability
CONCUR16

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too weak

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for queues/stacks only
(feel free to ask for more)

too weak

Local linearizability
CONCUR16

Relaxing the sequential specification

Relaxing the sequential specification

Quantitative relaxations (POPL13)

Goal

- trade correctness for performance
- in a controlled way with quantitative bounds

measure the
error from correct
behaviour

Goal

Stack - incorrect behavior

push(a)push(b)push(c)pop(a)pop(b)

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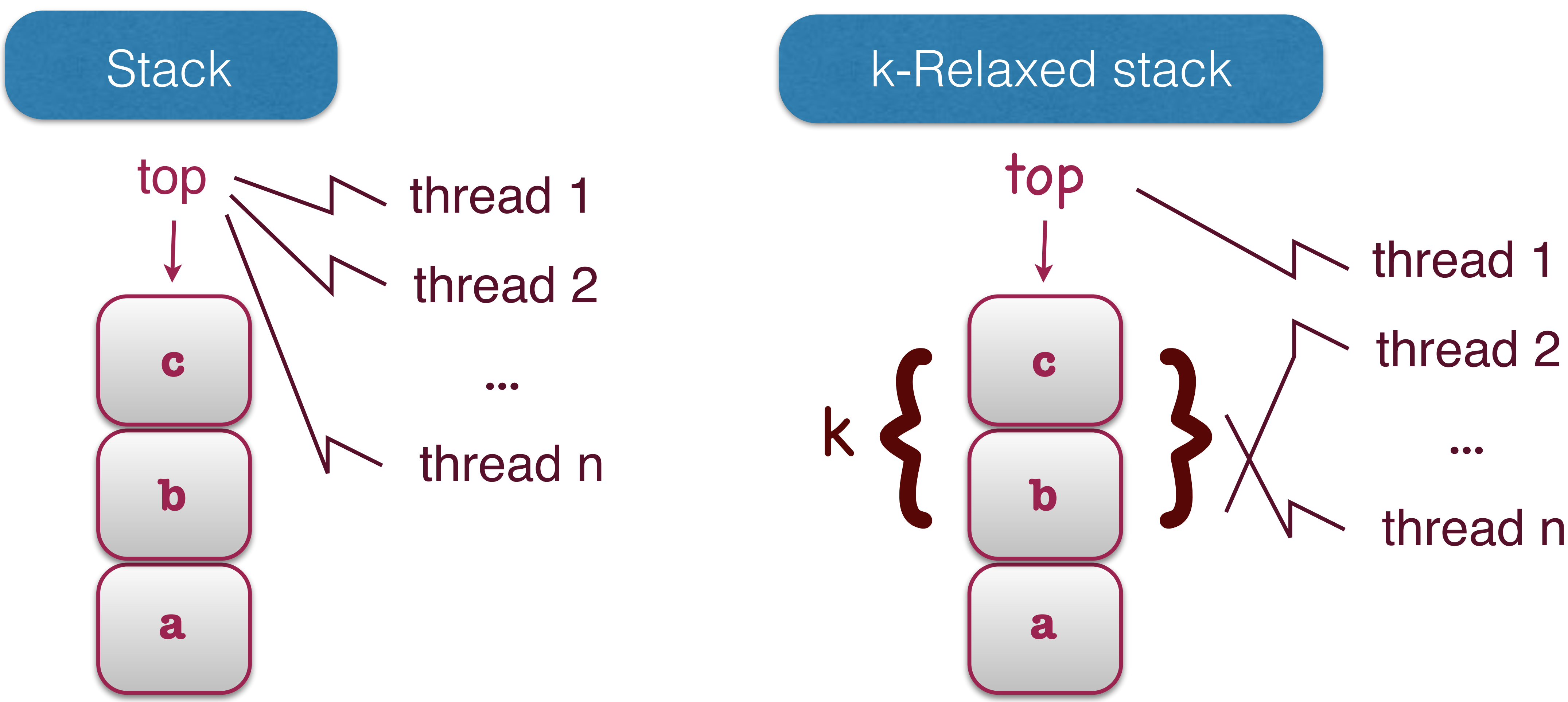
correct in a relaxed stack
... 2-relaxed? 3-relaxed?

How can relaxing help?



What we have

- Framework
- Generic examples
- Concrete relaxation examples
- Efficient concurrent implementations

for semantic relaxations

out-of-order / stuttering

stacks, queues, priority queues,.. / CAS, shared counter

of relaxation instances

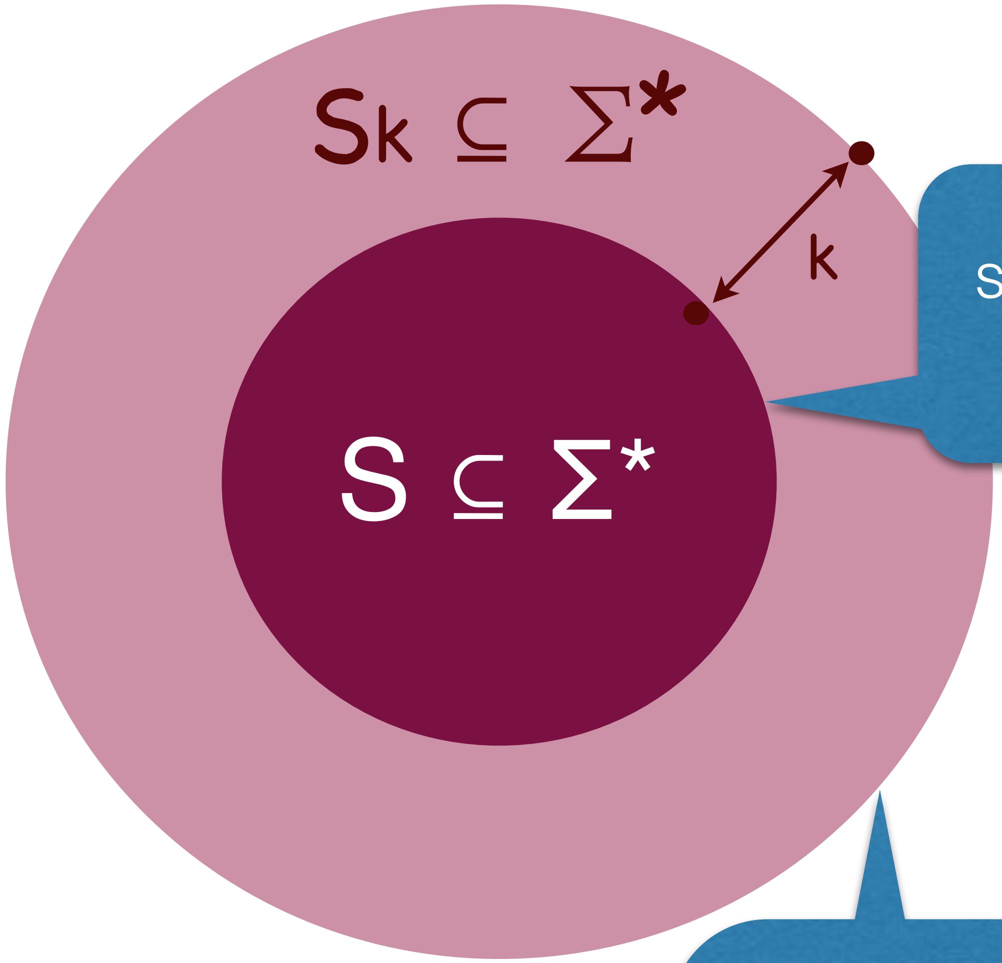
The big picture

$S \subseteq \Sigma^*$

sequential specification
legal sequences

Σ - methods with arguments

The big picture



sequential specification
legal sequences

relaxed sequential specification
sequences at distance up to k from S

Σ - methods with arguments

Syntactic distances do not help

push(a)[push(i)pop(i)]^{n}push(b)[push(j)pop(j)]^{m}pop(a)

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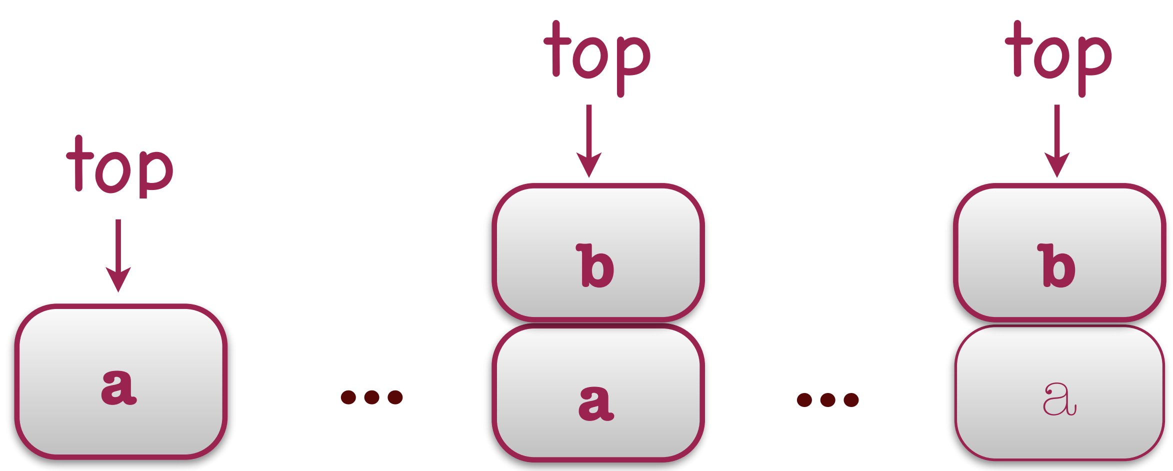
is a 1-out-of-order stack sequence



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$$\mathrm{push}(a)[\mathrm{push}(i)\mathrm{pop}(i)]^n\mathrm{push}(b)[\mathrm{push}(j)\mathrm{pop}(j)]^m\mathrm{pop}(a)$$

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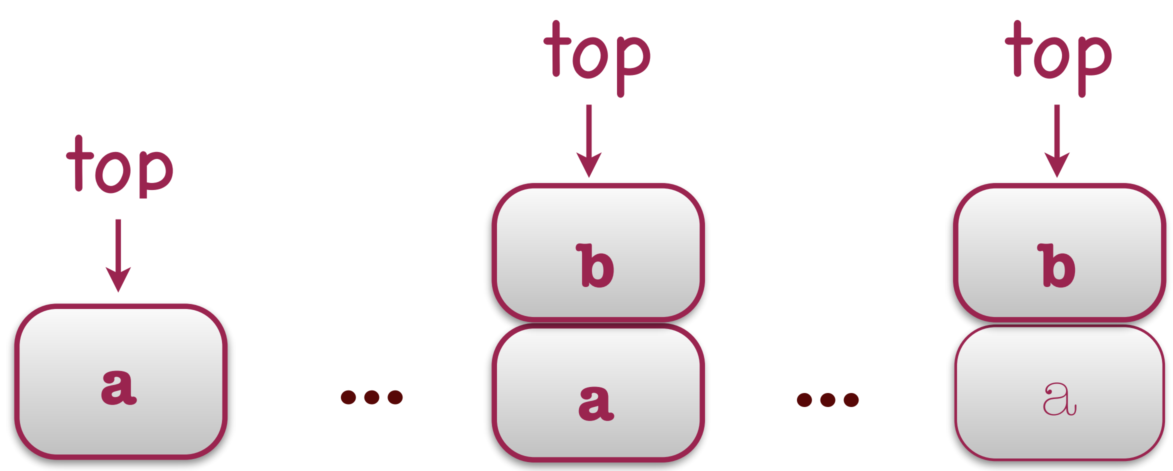

its permutation distance is min(2n,2m)

Semantic distances need a notion of state

- States are equivalence classes of sequences in S

- Two sequences in S are equivalent iff they have an indistinguishable future

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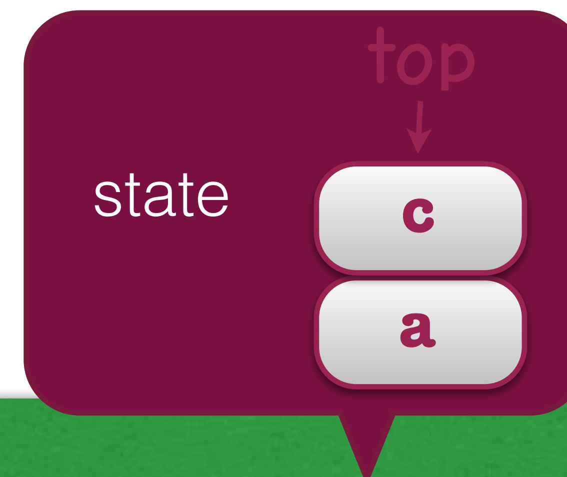

- States are equivalence classes of sequences in S

example: for stack

$$\mathrm{push(a)push(b)pop(b)push(c)} \equiv \mathrm{push(a)push(c)}$$

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$$x \equiv y \quad \Leftrightarrow \quad \forall u \in \Sigma^*.\,(xu \in S \;\Leftrightarrow\; yu \in S)$$

Semantics goes operational

$S \subseteq \Sigma^*$ is the sequential specification

states

labels

initial state

$LTS(S) = (S/_{\equiv}, \Sigma, \rightarrow, [\varepsilon]_{\equiv})$ with

transition relation

$$[s]_{\equiv} \xrightarrow{m} [sm]_{\equiv} \quad \Leftrightarrow \quad sm \in S$$

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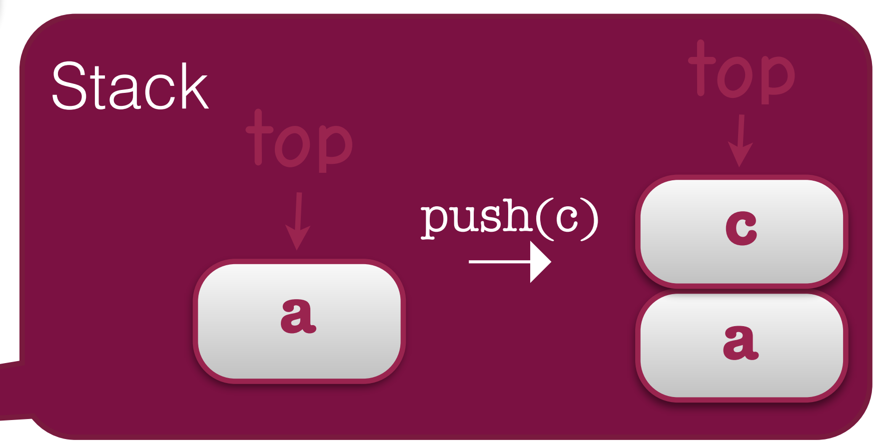

The relaxation framework

- Start from LTS(S)
- Add transitions with transition costs
- Fix a path cost function

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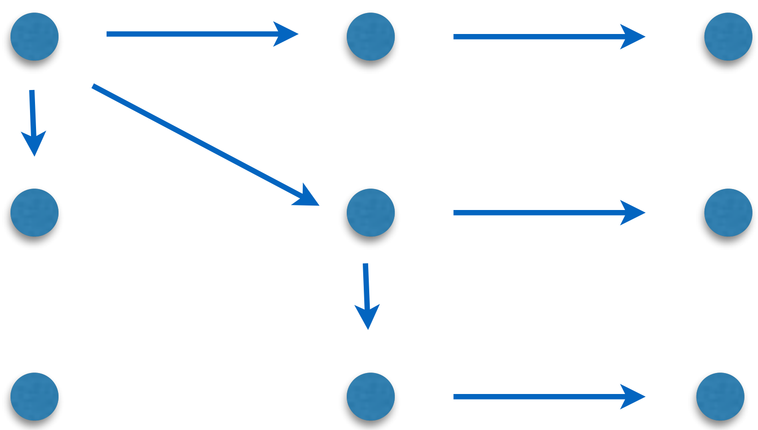

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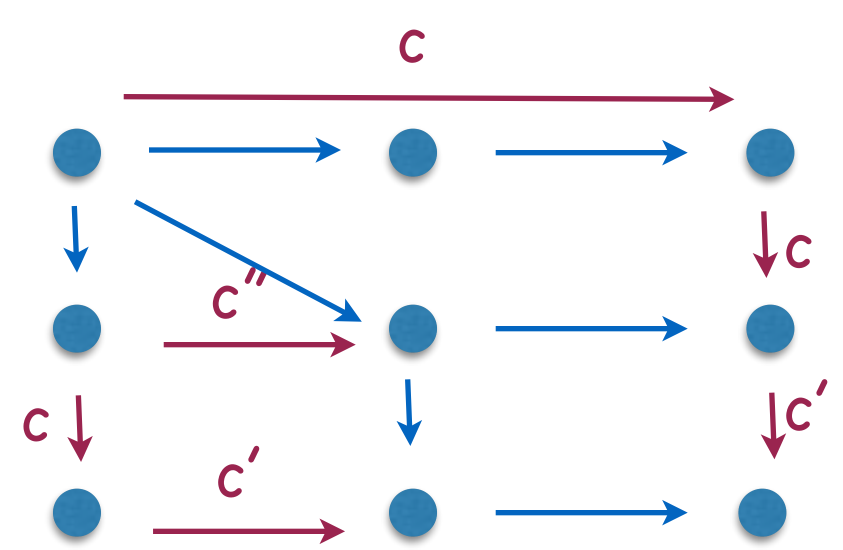

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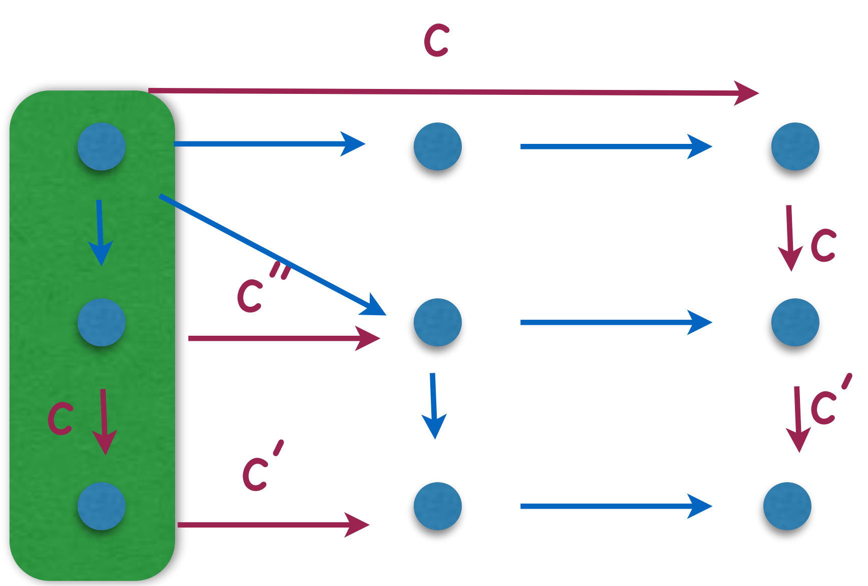

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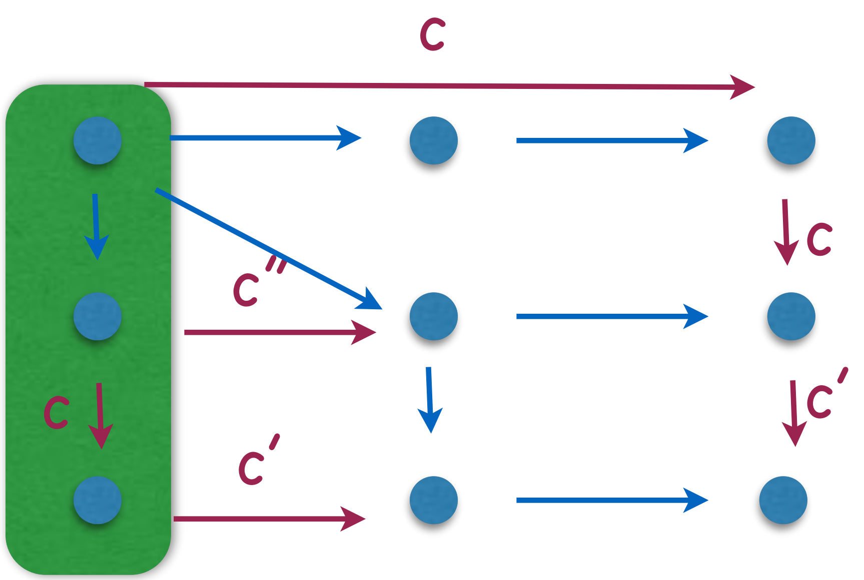


distance - minimal cost on all paths labelled by the sequence

Generic out-of-order

$\texttt{segment\_cost}(\, q \xrightarrow{m} q' \,) = |\mathbf{v}|$

transition cost

Where **v** is a sequence of minimal length s.t.

removing **v** enables a transition

or

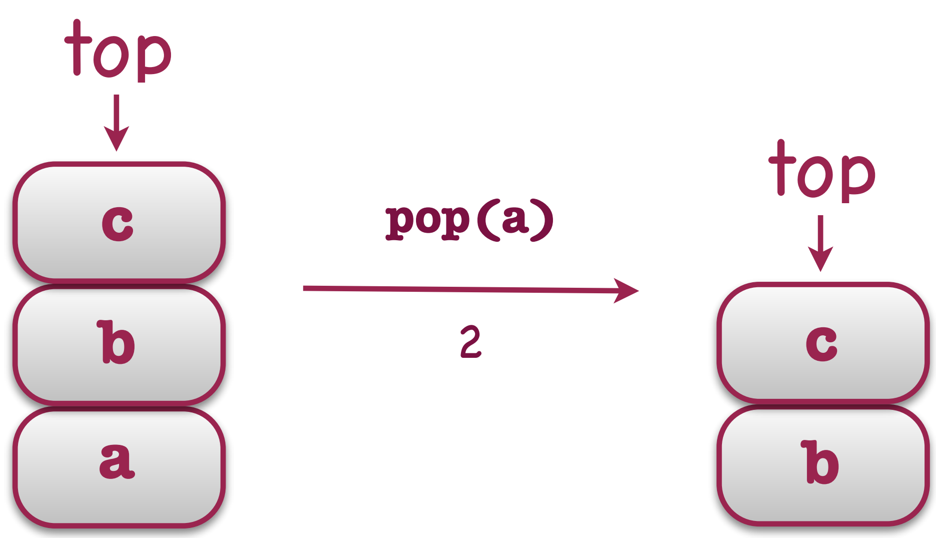
inserting **v** enables a transition

goes with different path costs

Out-of-order stack

Sequence of **push**'s with no matching **pop**

- Canonical representative of a state
- Add incorrect transitions with segment-costs



- Possible path cost functions max, sum,...

also more advanced

Relaxing the Consistency Condition

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Local Linearizability (CONCUR16)

Local Linearizability main idea

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- Partition a history into a set of local histories
- Require linearizability per local history

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Already present in some shared-memory consistency conditions (not in our form of choice)

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Local sequential consistency… is also possible

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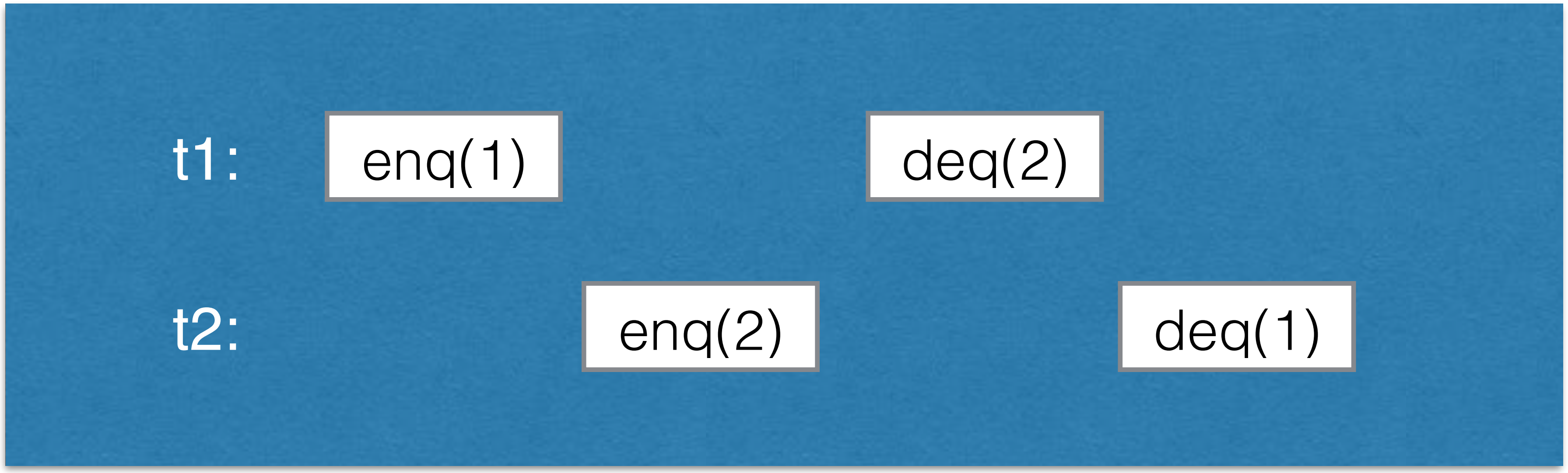
no global witness

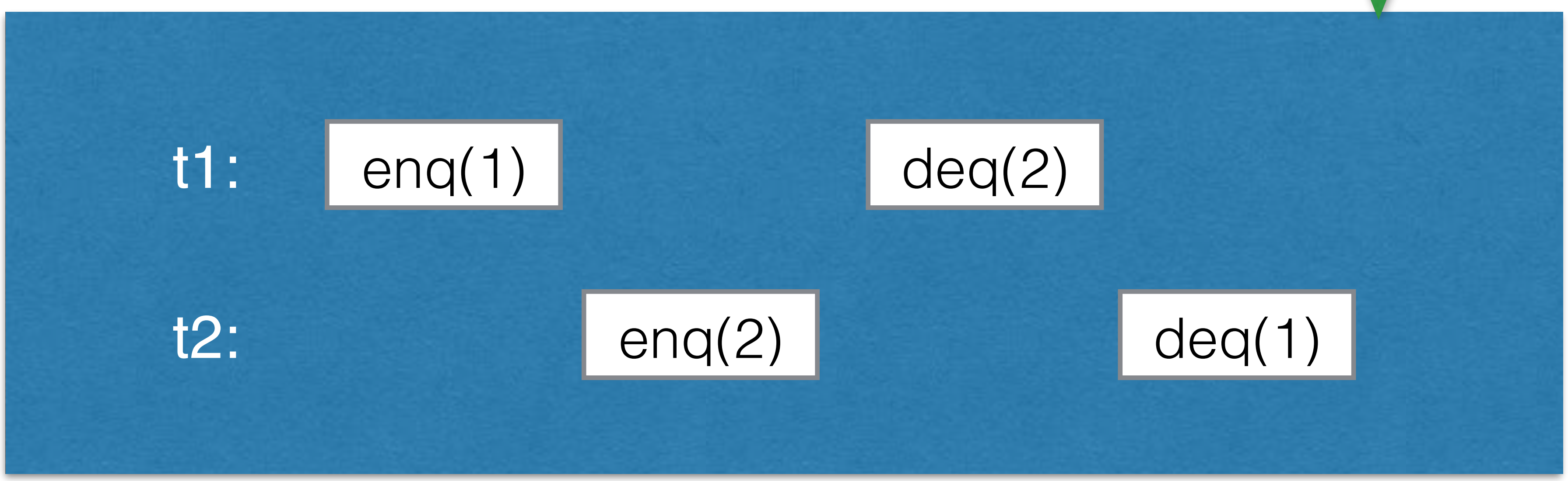
Local sequential consistency… is also possible

Local Linearizability (queue) example



Local Linearizability (queue) example

(sequential) history not linearizable



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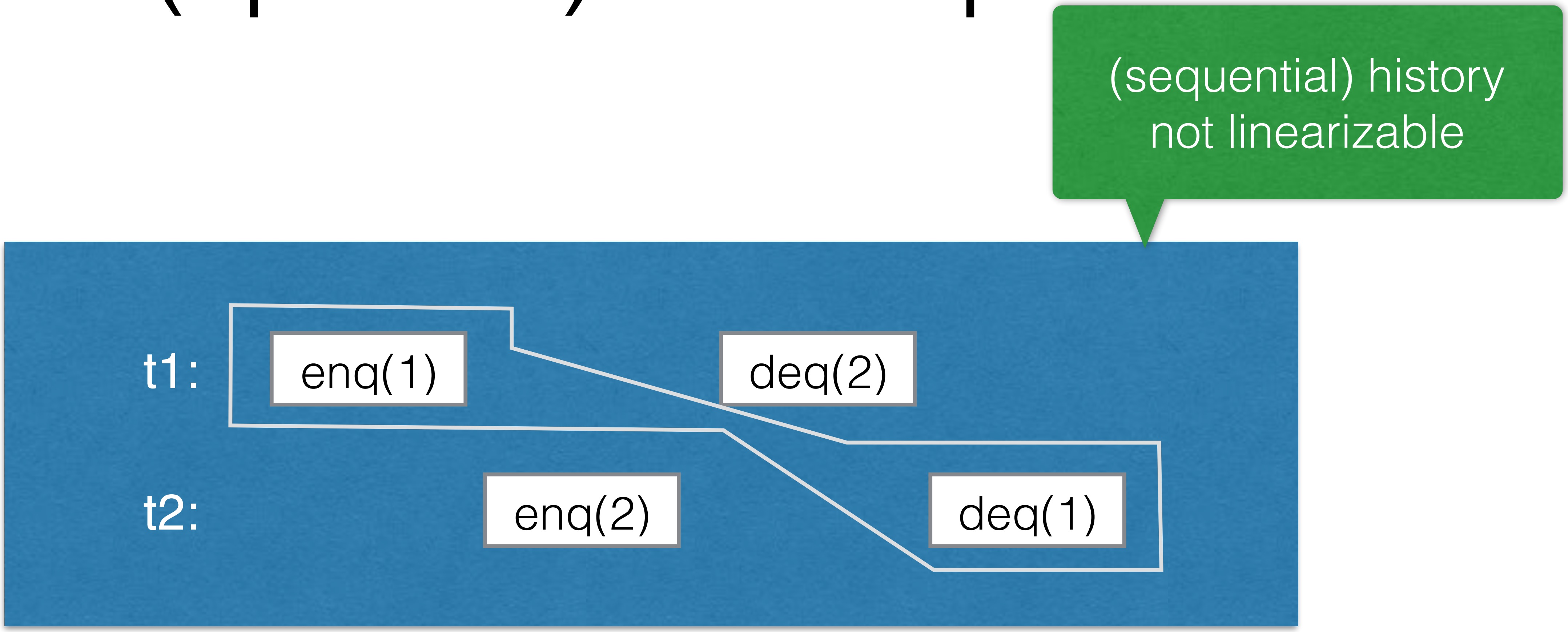

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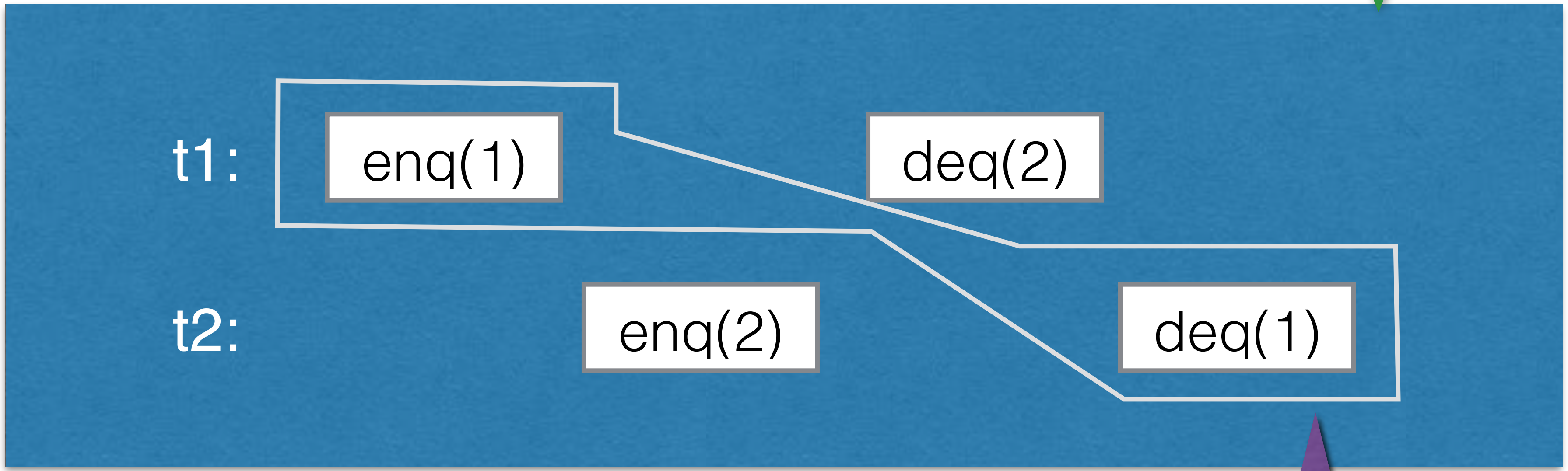

t1-induced history, linearizable

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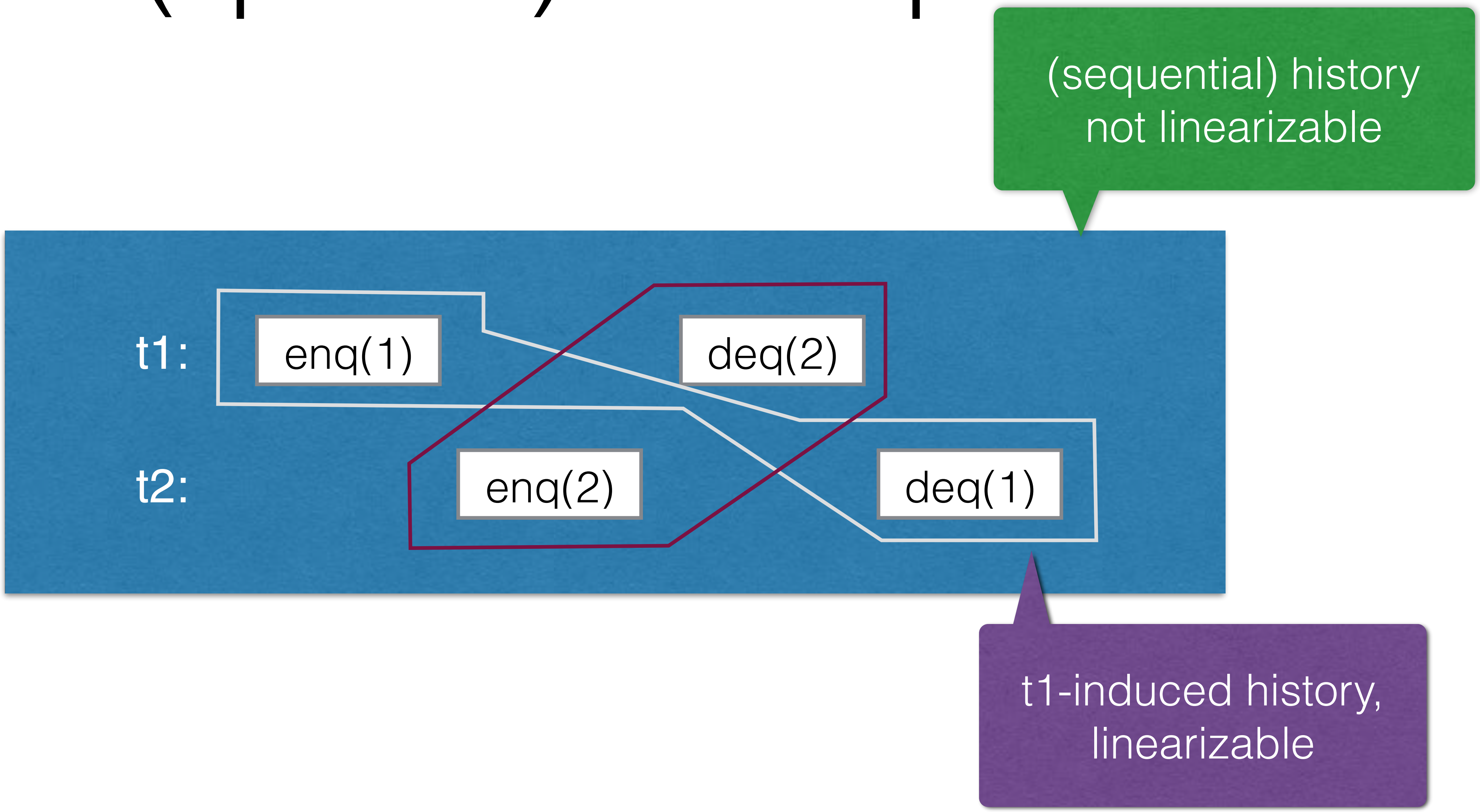

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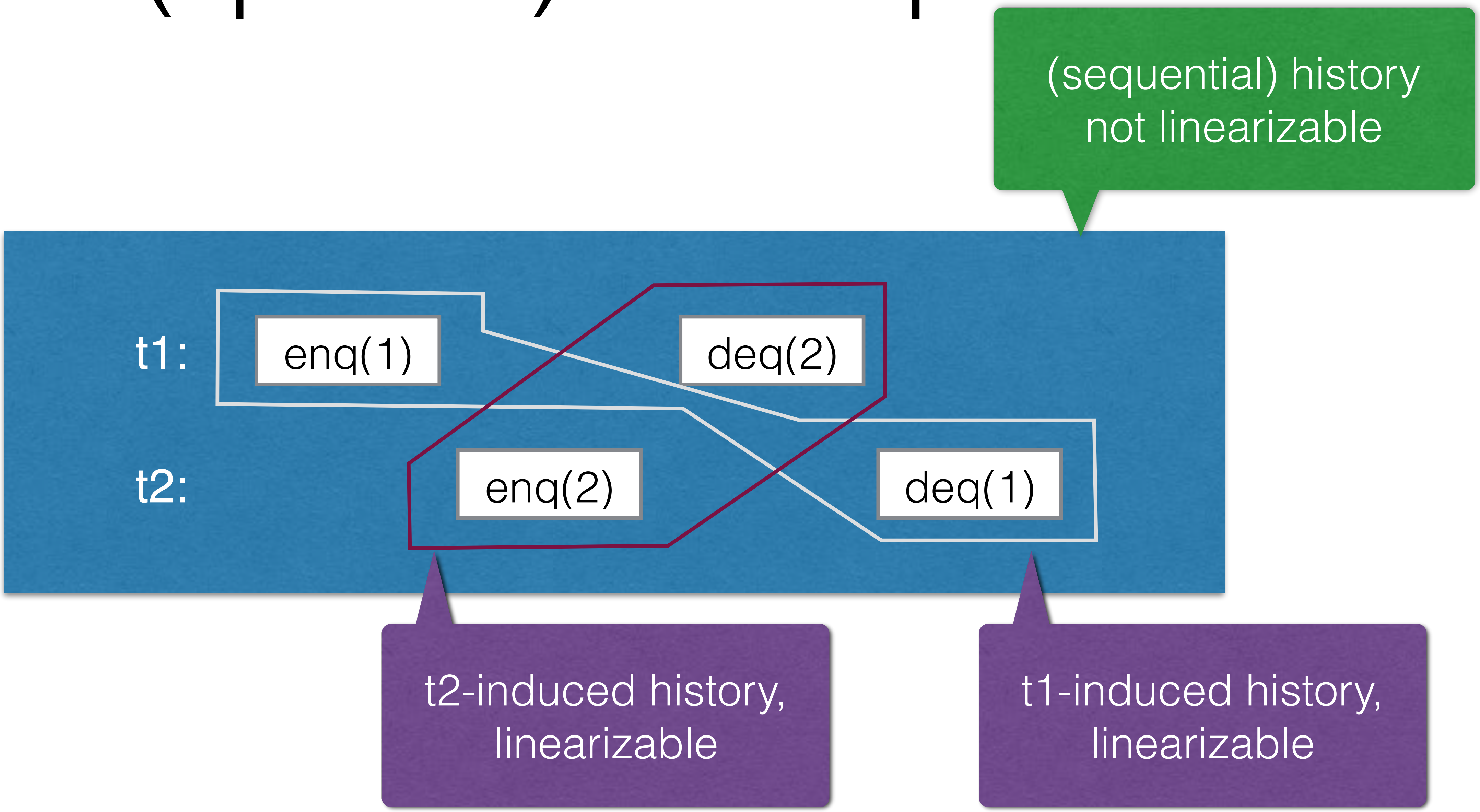

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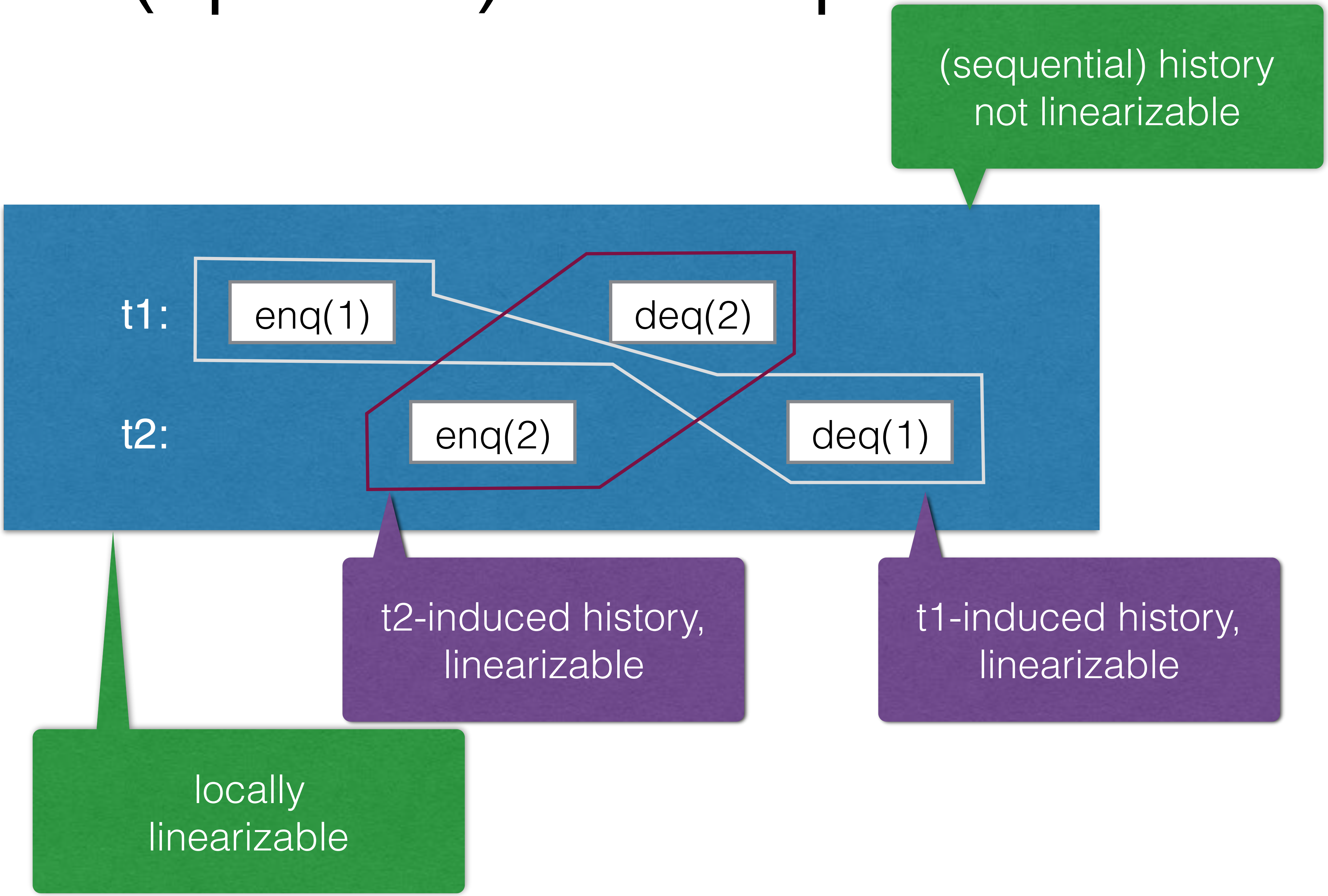

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Queue signature $\Sigma = \{enq(x) \mid x \in V\} \cup \{deq(x) \mid x \in V\} \cup \{deq(empty)\}$

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For a history **h** with a thread T, we put

$$I_T = \{enq(x)^T \in \mathbf{h} \mid x \in V\}$$

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out-methods of thread T are dequeues (performed by any thread) corresponding to enqueues that are in-methods

h is locally linearizable iff every thread-induced history

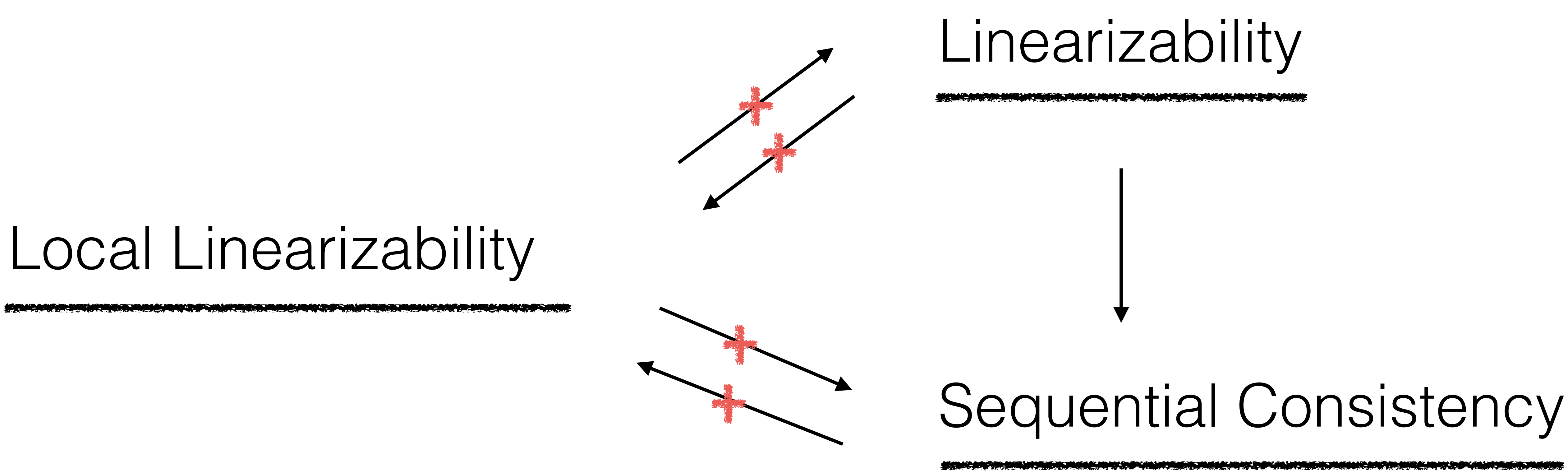
$$\mathbf{h}_T = \mathbf{h} \mid (I_T \cup O_T)$$

is linearizable.

Where do we stand?

Where do we stand?

In general



Where do we stand?

For queues (and most container-type data structures)

Linearizability

Local Linearizability

Sequential Consistency

Properties

Properties

Local linearizability is compositional

Properties

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like linearizability
unlike sequential consistency

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h (over multiple objects) is locally linearizable
iff
each per-object subhistory of **h** is locally linearizable

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Local linearizability is modular /
"decompositional"

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Local linearizability is modular /
"decompositional"

uses decomposition into smaller
histories, by definition

Properties

Local linearizability is compositional

like linearizability
unlike sequential consistency

h (over multiple objects) is locally linearizable
iff
each per-object subhistory of **h** is locally linearizable

Local linearizability is modular / "decompositional"

uses decomposition into smaller histories, by definition

↓

may allow for modular verification

Generic Implementations

Generic Implementations

Your favorite linearizable data structure implementation

Generic Implementations

Your favorite linearizable data structure implementation

Φ

Generic Implementations

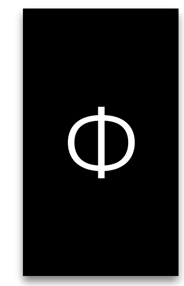
Your favorite linearizable data structure implementation

Φ

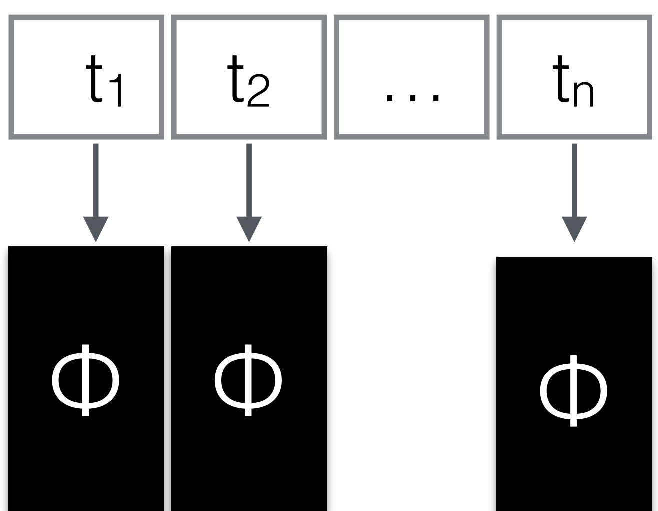
turns into a locally linearizable implementation by:

Generic Implementations

Your favorite linearizable data structure implementation

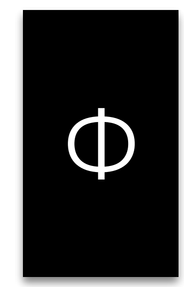


turns into a locally linearizable implementation by:

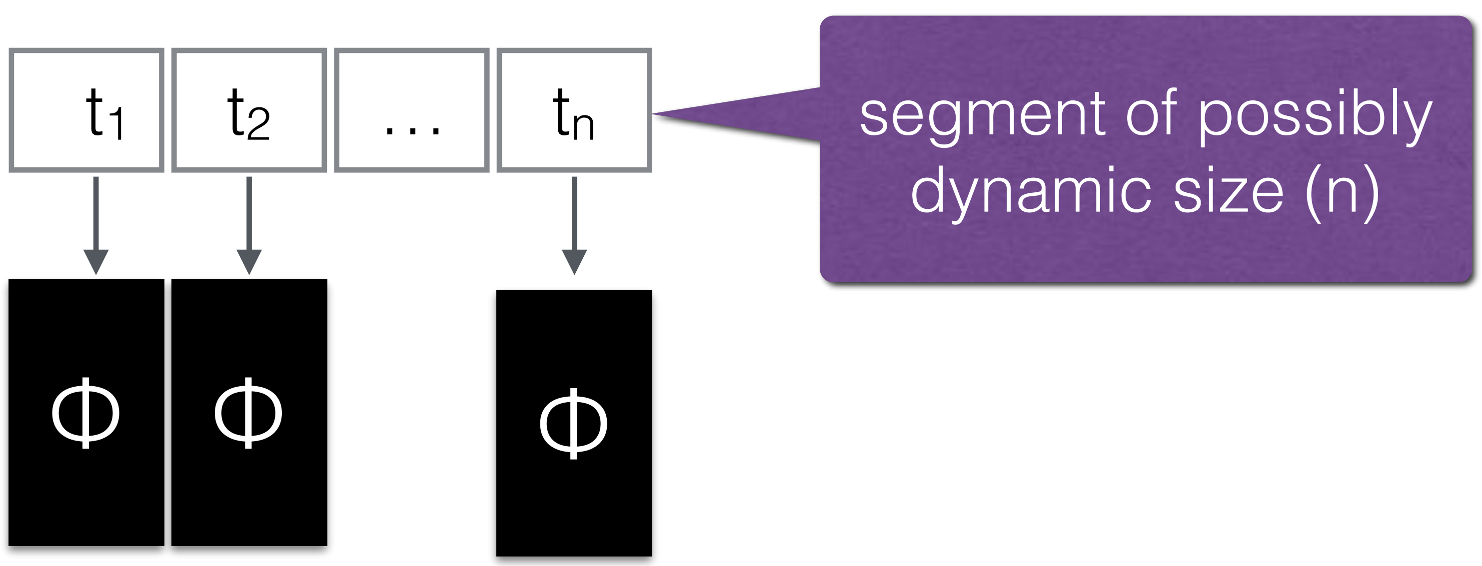


Generic Implementations

Your favorite linearizable data structure implementation

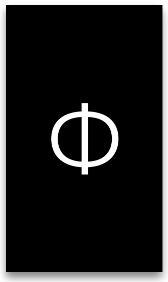


turns into a locally linearizable implementation by:

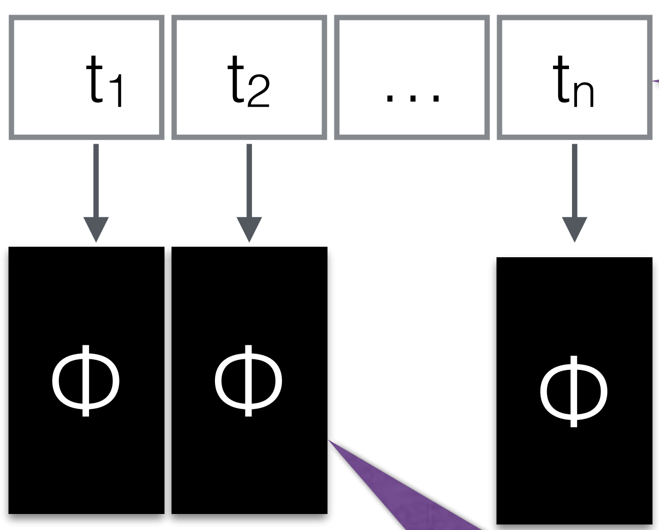


Generic Implementations

Your favorite linearizable data structure implementation



turns into a locally linearizable implementation by:



segment of possibly dynamic size (n)

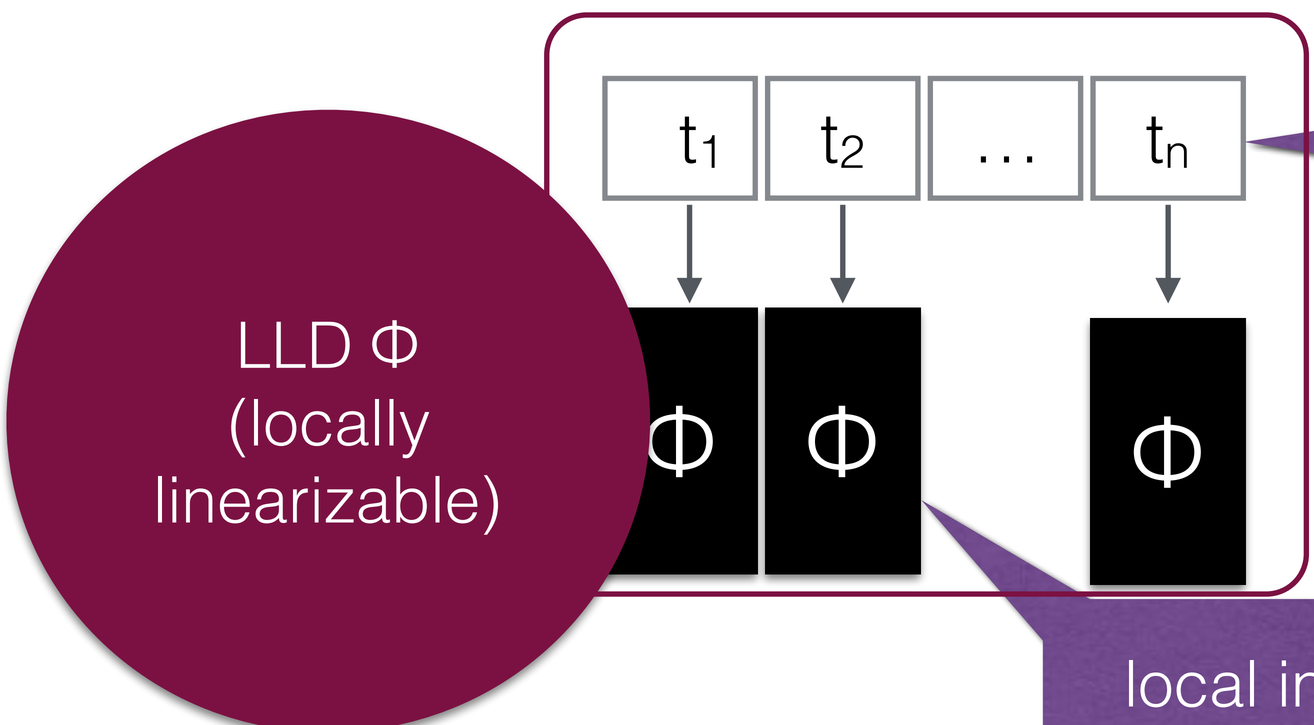
local inserts / global (randomly distributed) removes

Generic Implementations

Your favorite linearizable data structure implementation

Φ

turns into a locally linearizable implementation by:



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Generic Implementations

Your favorite linearizable data structure implementation

Φ

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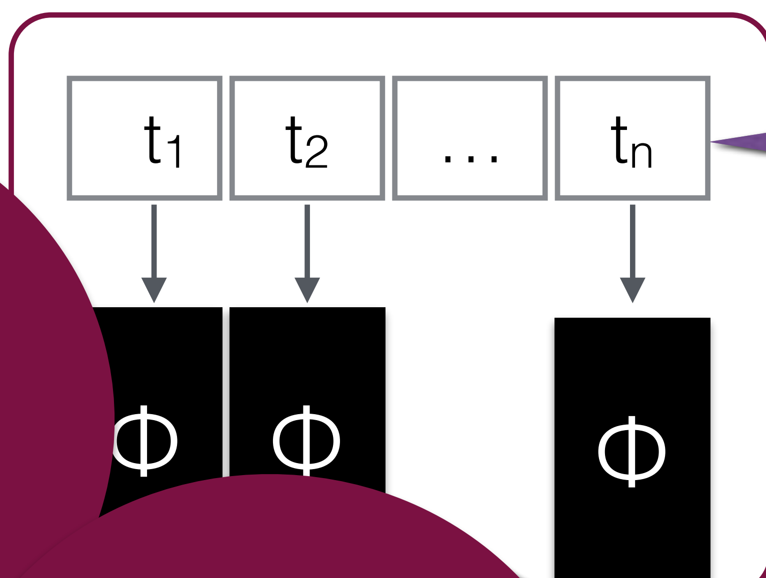


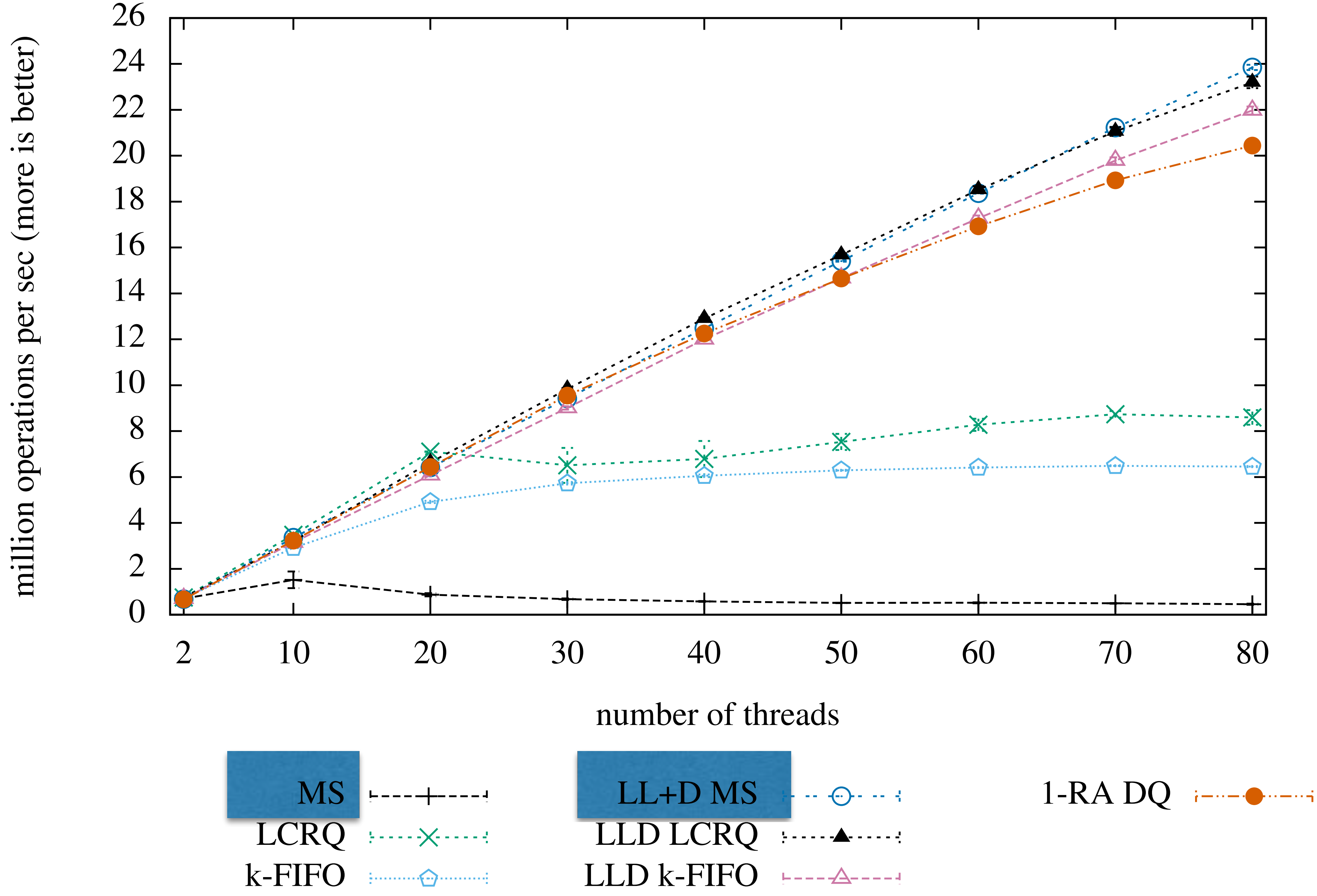
segment of possibly dynamic size (n)

local inserts / global (randomly distributed) removes

LLD Φ
(locally linearizable)

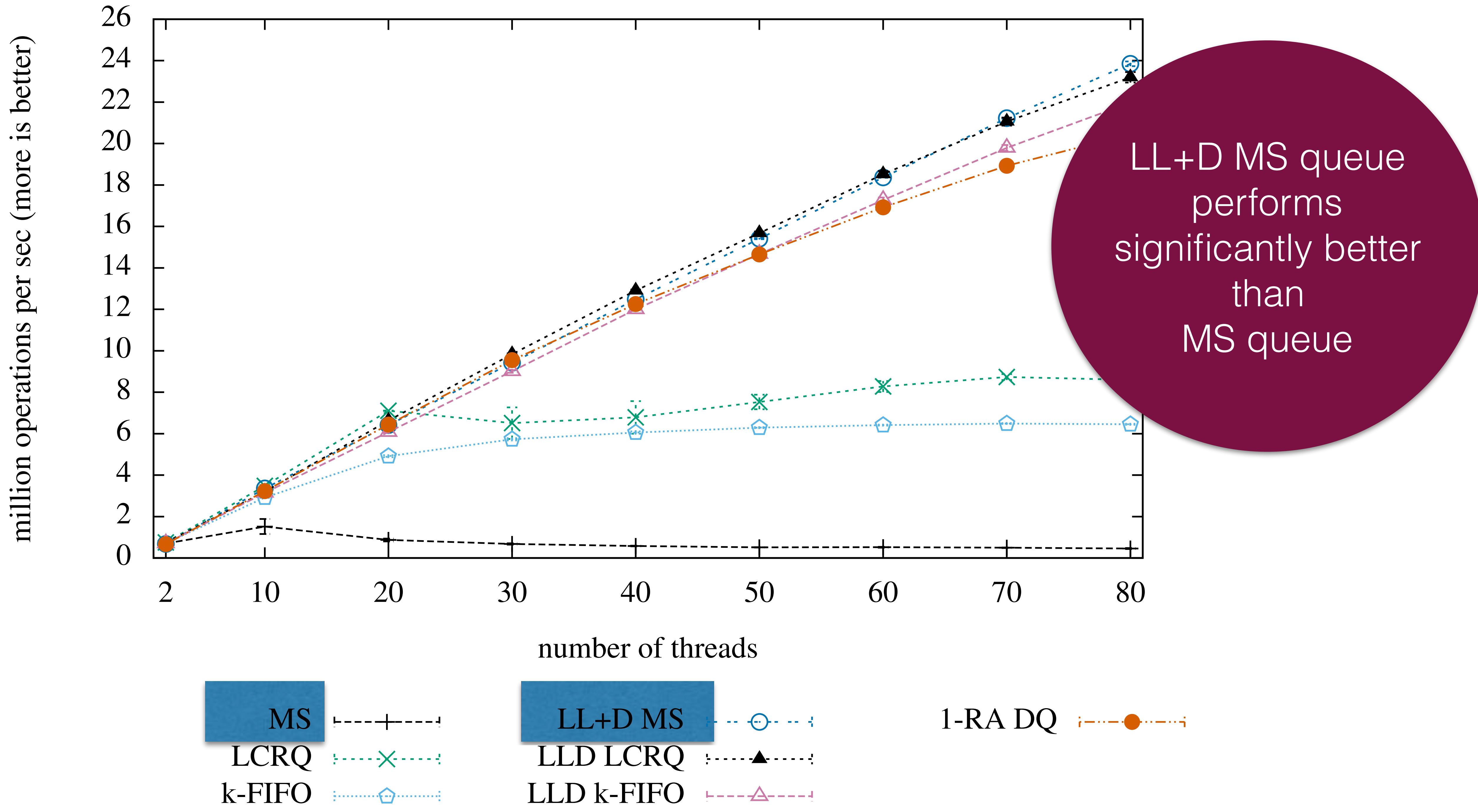
LL+D Φ
(also pool linearizable)

Performance



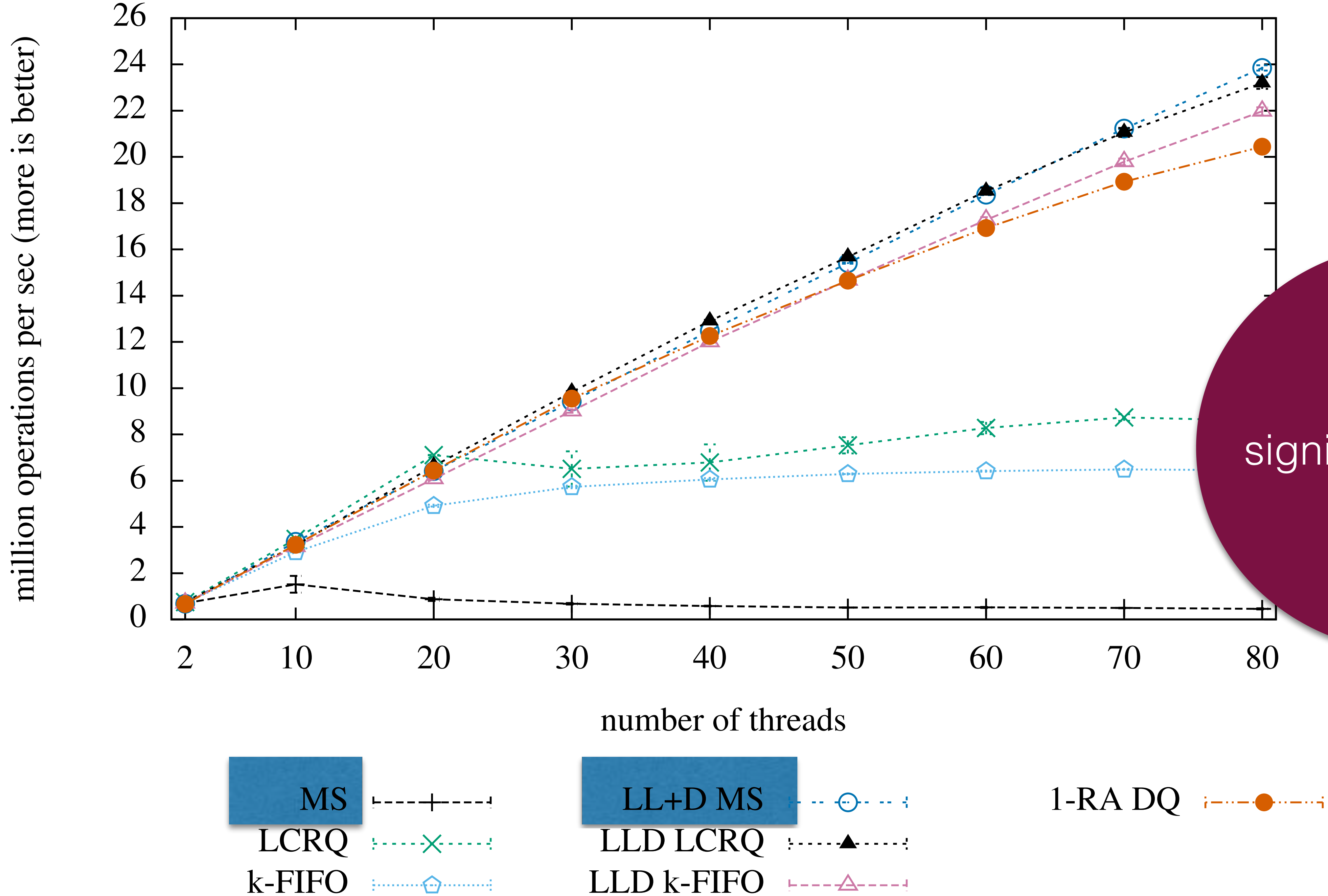
(a) Queues, LL queues, and "queue-like" pools

Performance



(a) Queues, LL queues, and “queue-like” pools

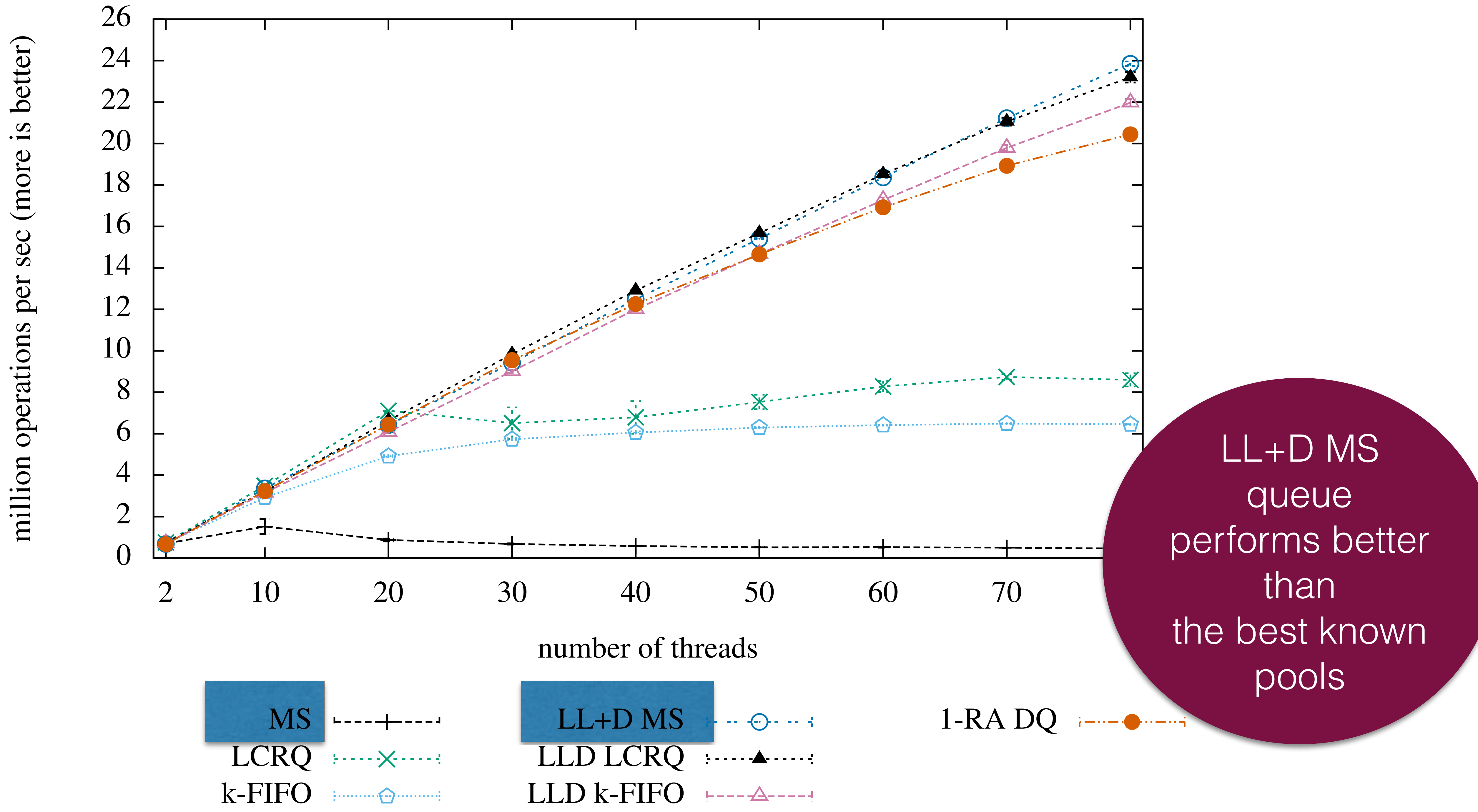
Performance



(a) Queues, LL queues, and "queue-like" pools

LLD Φ performs significantly better than Φ

Performance



(a) Queues, LL queues, and "queue-like" pools

Thank You!

and many thanks to my dear coauthors

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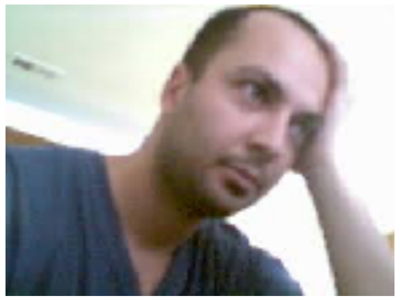

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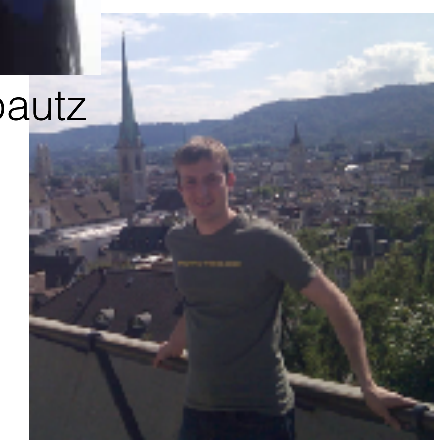

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