

Concurrent Data Structures

Semantics and Quantitative Relaxations

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Semantics of sequential data structures

e.g. pools, queues, stacks

- Sequential specification – set of legal sequences

Stack – legal sequence

`push(a)push(b)pop(b)`

Semantics of concurrent data structures

Stack - legal sequence

`push(a)push(b)pop(b)`

- Sequential specification - set of legal sequences

linearizable
wrt seq.spec.

- Consistency condition - e.g. linearizability

Stack - concurrent history

`begin-push(a)begin-push(b) end-push(a) end-push(b)begin-pop(b)end-pop(b)`

Consistency conditions

There exists a sequential witness that preserves precedence

linearizability

T1 ¹ push(a) ³ pop(b)

T2 ² push(b)

There exists a sequential witness that preserves precedence across quiescent states

There exists a sequential witness that preserves per-thread precedence

sequential consistency

quiescent consistency

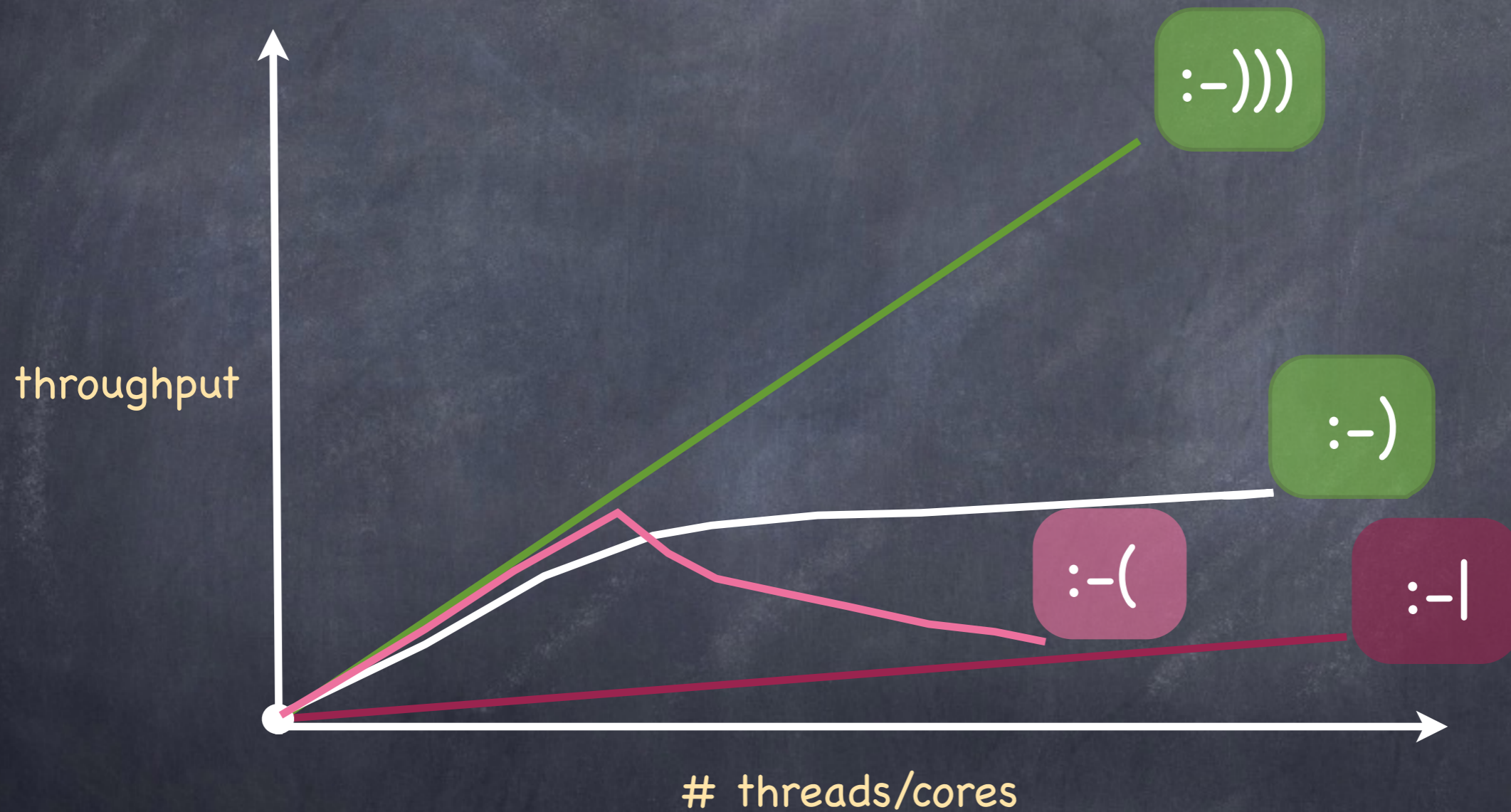
T1 ¹ push(a) ³ pop(b)

T2 ² push(b)

T1 ¹ push(a)

T2 ³ pop(b) ² push(b)

Performance and scalability



Relaxations allow

Stack – incorrect behavior

```
push(a)push(b)push(c)pop(a)pop(b)
```

- Trading correctness for performance
- In a controlled way with quantitative bounds

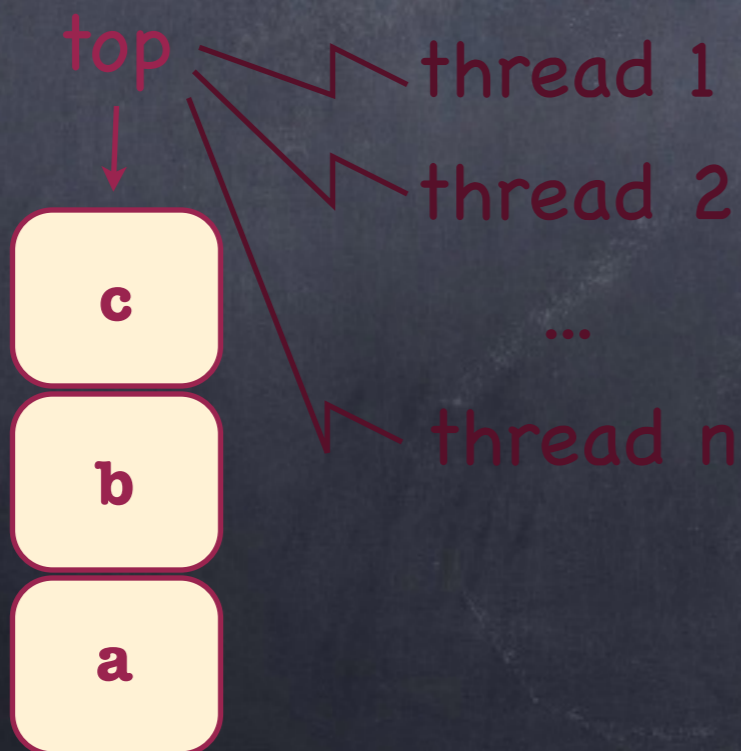
correct in a relaxed stack
... 2-relaxed? 3-relaxed?

measure the error from
correct behavior

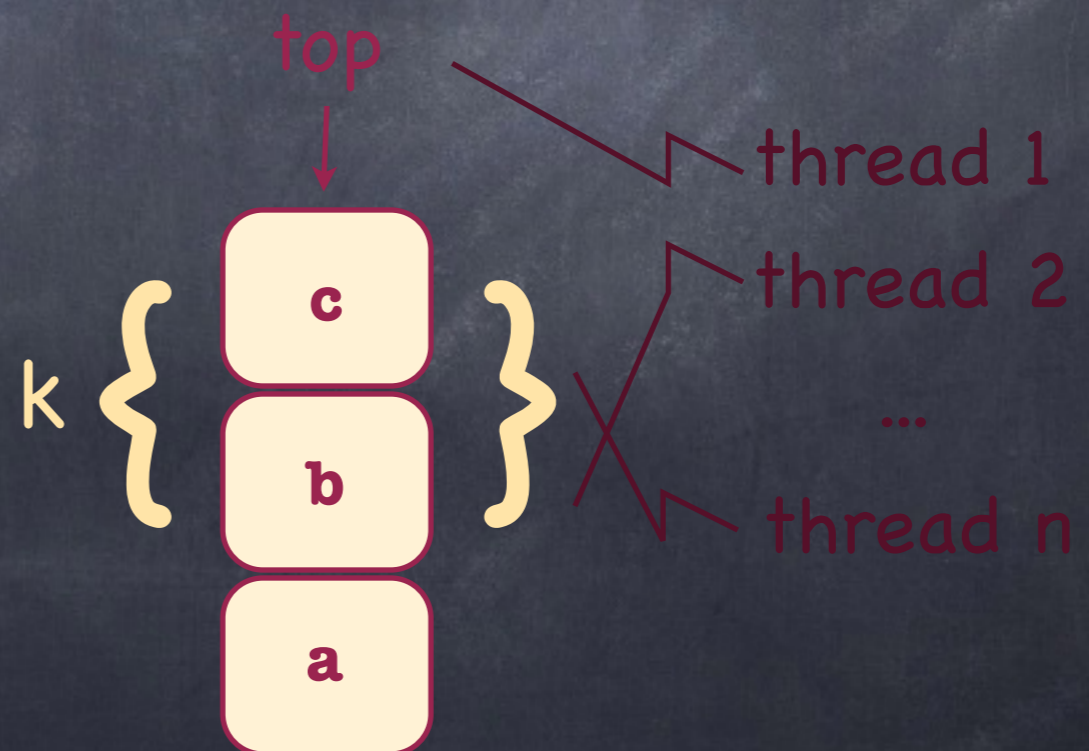
Why relax?

- It is interesting
- Provides potential for **better performing** concurrent implementations

Stack



k-Relaxed stack



Relaxations of concurrent data structures

Quantitative relaxations
Henzinger, Kirsch, Payer, Sezgin, S.
POPL 2013

- Sequential specification – set of legal sequences
- Consistency condition – e.g. linearizability

(Quantitative) relaxations
Dodds, Sezgin, S.
work in progress

What we have

• Framework

for semantic relaxations

• Generic examples

out-of-order / stuttering

• Concrete relaxation examples

stacks, queues, priority queues,.. / CAS, shared counter

• Efficient concurrent implementations

of relaxation instances

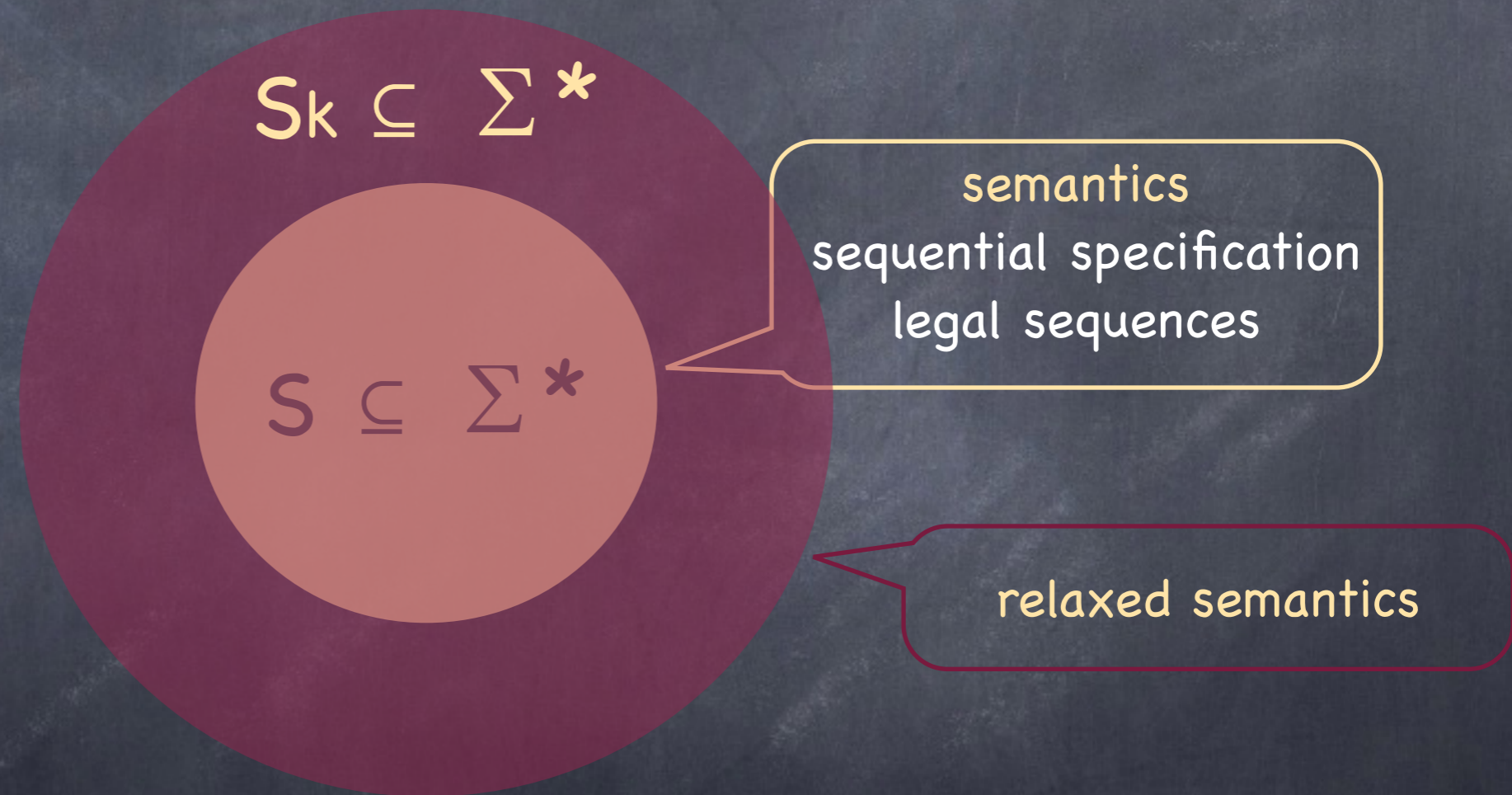
The big picture



semantics
sequential specification
legal sequences

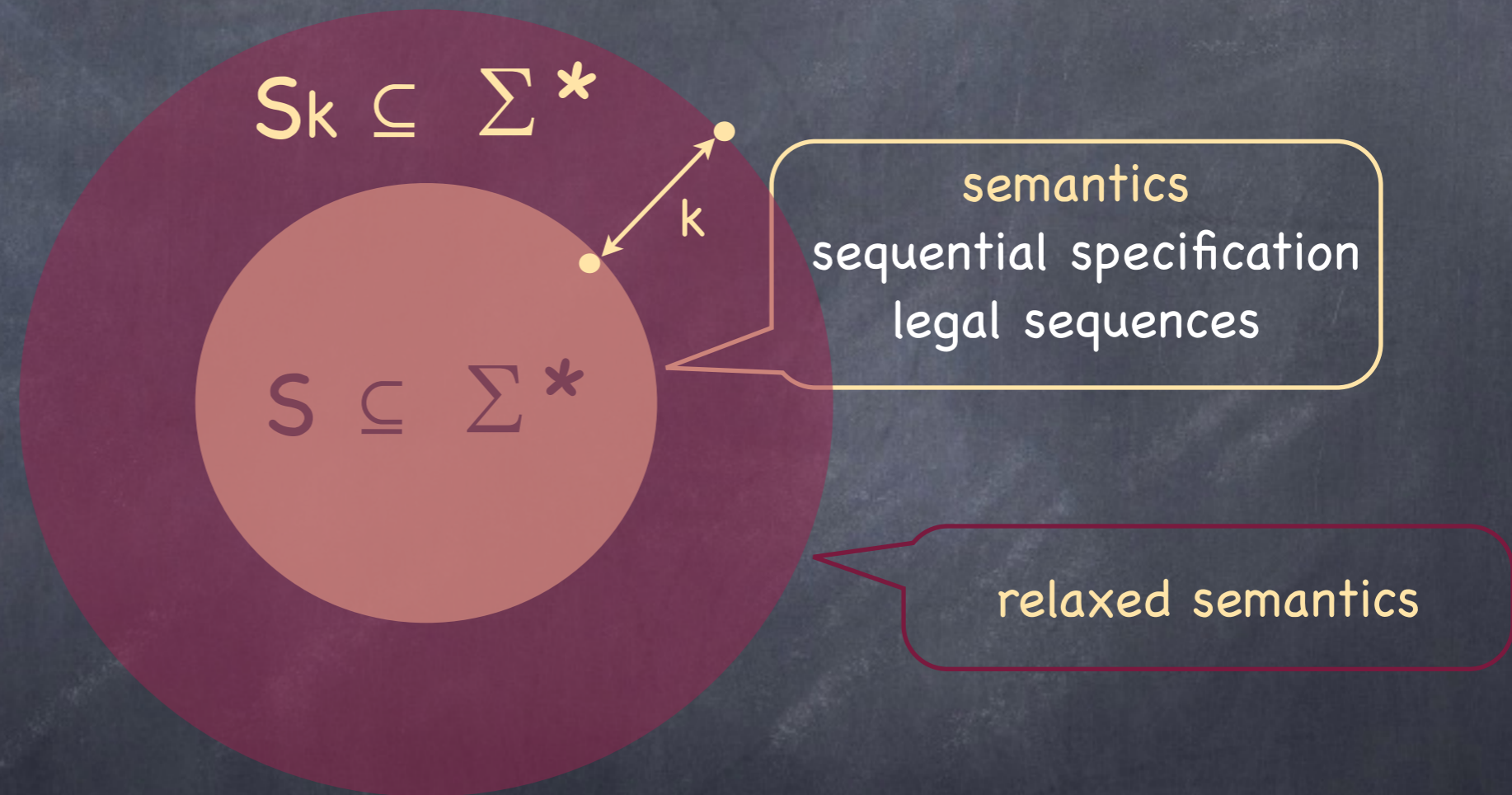
Σ - methods with arguments

The big picture



Σ - methods with arguments

The big picture



Σ - methods with arguments

distance?

Challenge

There are natural concrete relaxations...

Stack

Each **pop** pops one of the $(k+1)$ -youngest elements

Each **push** pushes

k-out-of-order
relaxation

Challenge

There are natural concrete relaxations...

Stack

Each **pop** pops one of the $(k+1)$ -youngest elements

Each **push** pushes

k-out-of-order
relaxation

makes sense also for queues,
priority queues,

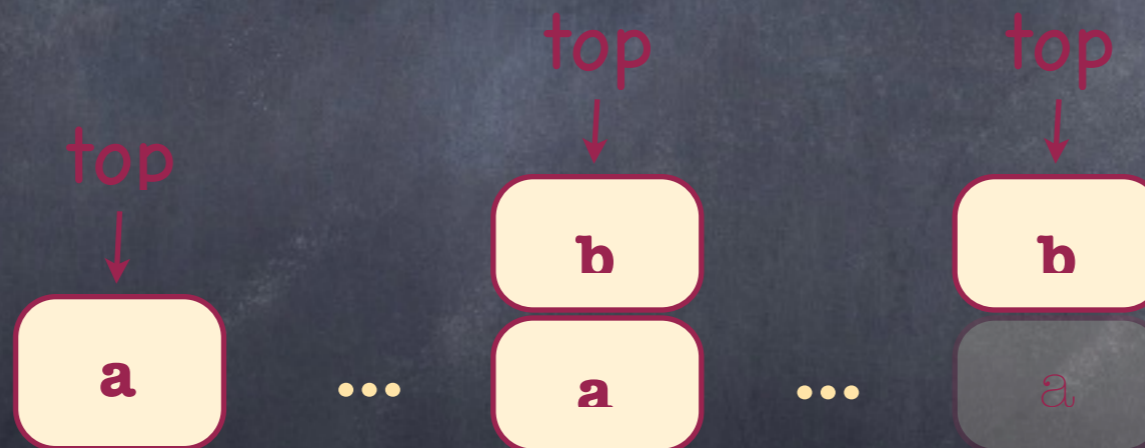
How is it reflected by a distance between sequences?

one distance for all?

Syntactic distances do not help

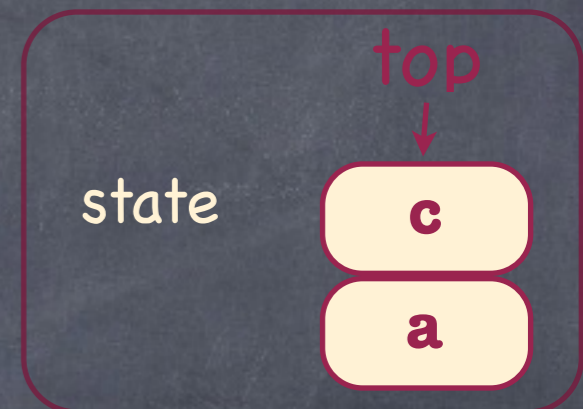
$\text{push}(a) [\text{push}(i)\text{pop}(i)]^n \text{push}(b) [\text{push}(j)\text{pop}(j)]^m \text{pop}(a)$

is a 1-out-of-order stack sequence



its permutation distance is $\min(n,m)$

Semantic distances need a notion of state



- States are equivalence classes of sequences in S

example: for stack

$\text{push}(a)\text{push}(b)\text{pop}(b)\text{push}(c) \equiv \text{push}(a)\text{push}(c)$

- Two sequences in S are equivalent if they have an indistinguishable future

$$\mathbf{x} \equiv \mathbf{y} \iff \forall \mathbf{u} \in \Sigma^*. (\mathbf{xu} \in \mathbf{S} \iff \mathbf{yu} \in \mathbf{S})$$

Semantics goes operational

- $S \subseteq \Sigma^*$ is the sequential specification

states

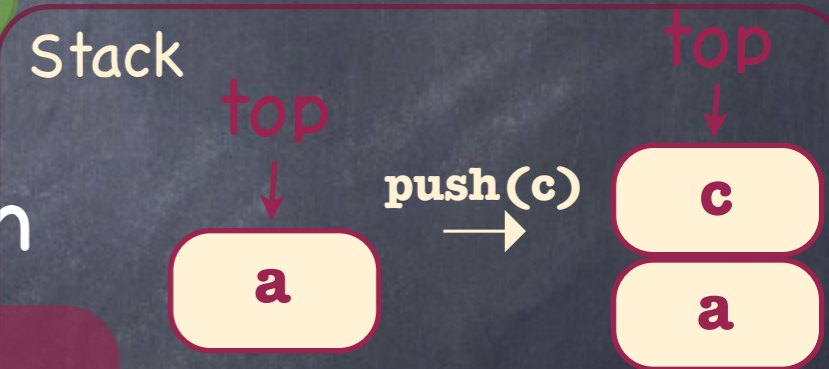
labels

initial state

- $LTS(S) = (S/\equiv, \Sigma, \rightarrow, [\varepsilon]_{\equiv})$ with

transition relation

$$[s]_{\equiv} \xrightarrow{m} [sm]_{\equiv} \iff sm \in S$$



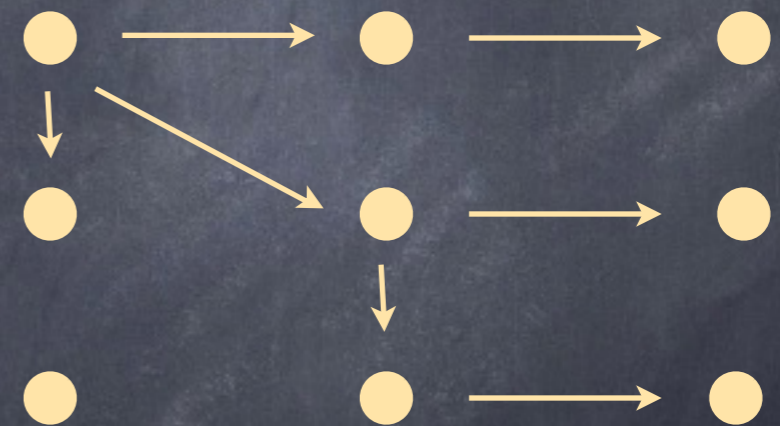
The framework

- Start from LTS(S)
- Add transitions with transition costs
- Fix a path cost function

The framework

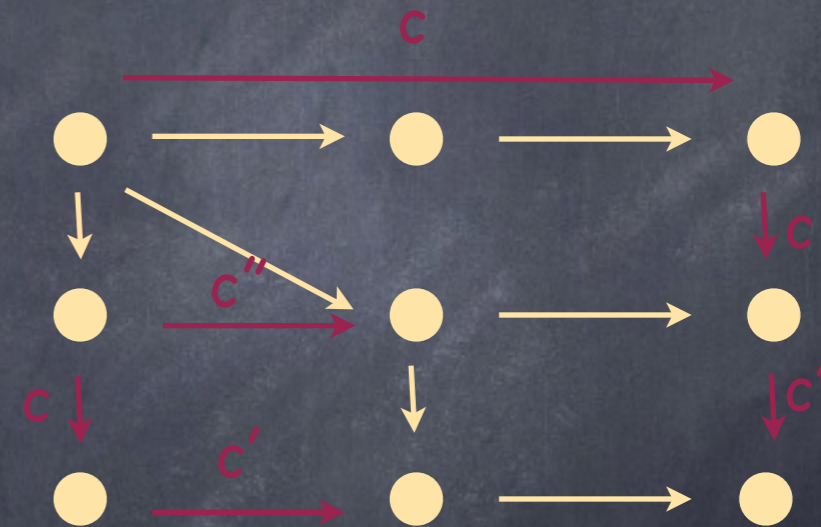
- Start from $LTS(S)$
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Σ - singleton



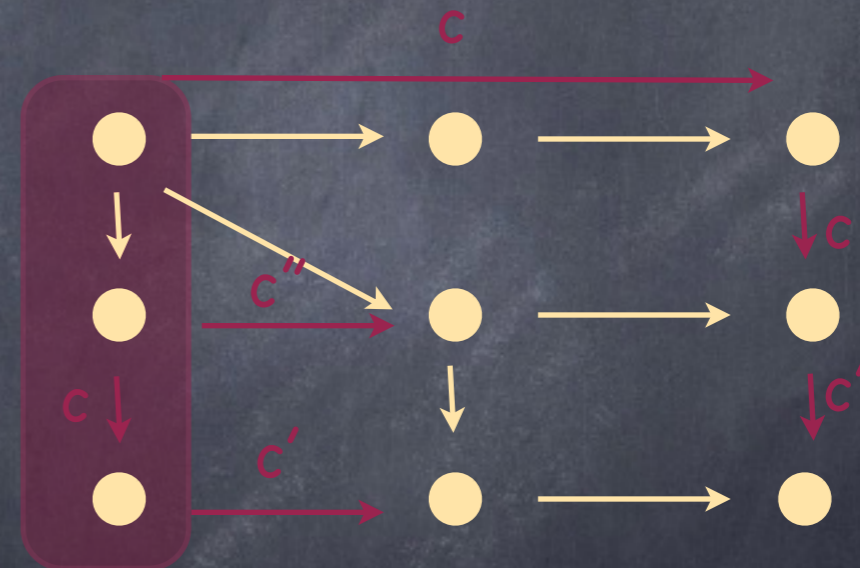
The framework

- Start from LTS(S)
- Add transitions with transition costs
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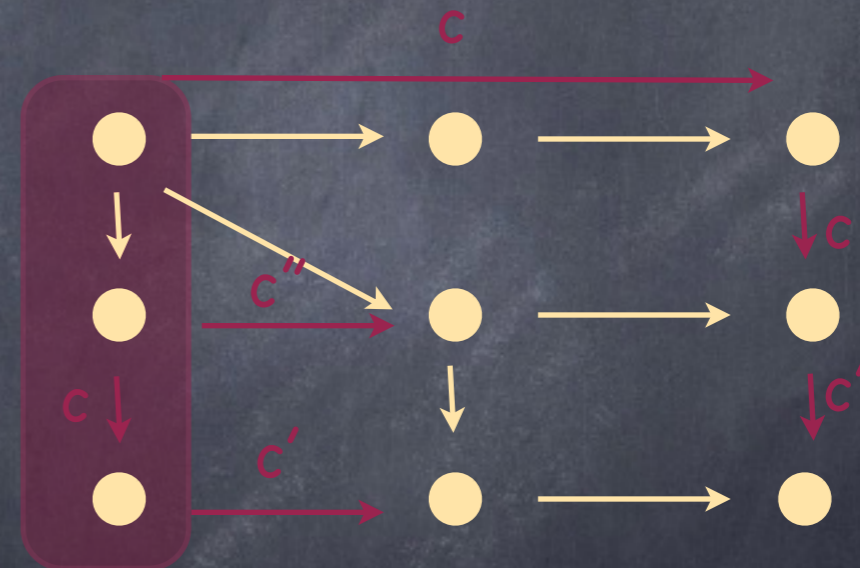
The framework

- Start from LTS(S)
- Add transitions with transition costs
- Fix a path cost function



The framework

- Start from LTS(S)
- Add transitions with transition costs
- Fix a path cost function

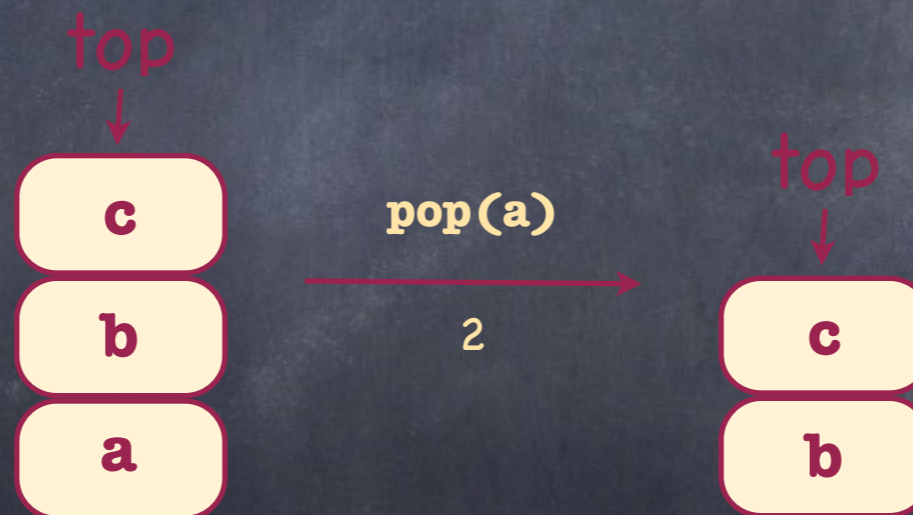


distance - minimal cost on all paths
labelled by the sequence

Out-of-order stack

Sequence of **push**'s with no matching **pop**

- Canonical representative of a state
- Add incorrect transitions with **segment-costs**



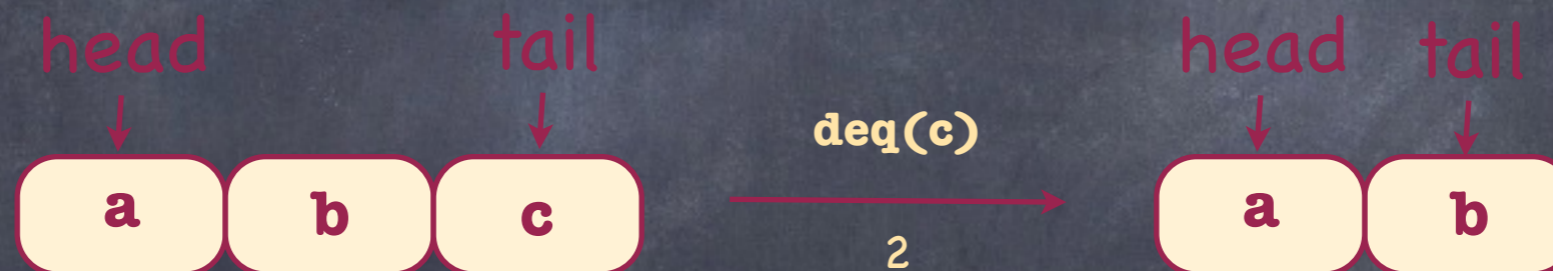
- Possible path cost functions **max**, **sum**,...

also more advanced

Out-of-order queue

Sequence of **enq**'s with no matching **deq**

- Canonical representative of a state
- Add incorrect transitions with **segment-costs**



- Possible path cost functions **max**, **sum**,...

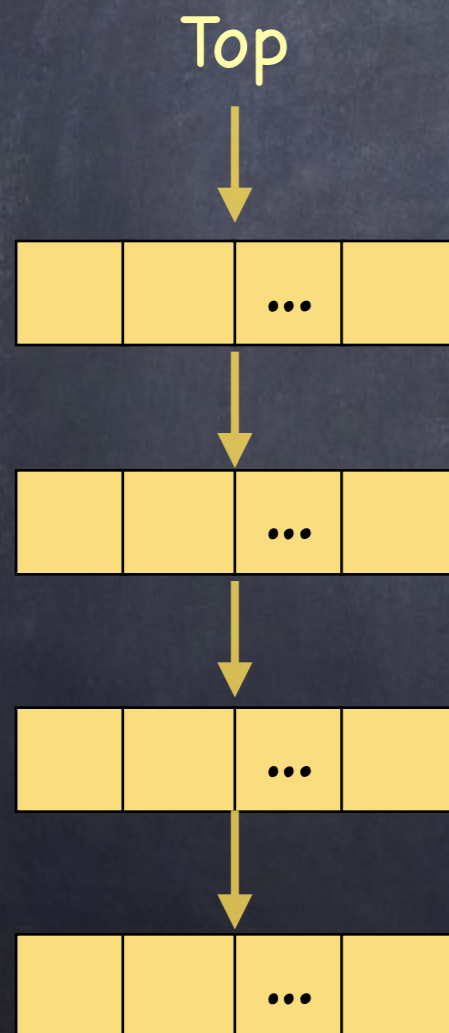
also more advanced

Implementations and Performance

Relaxed implementations

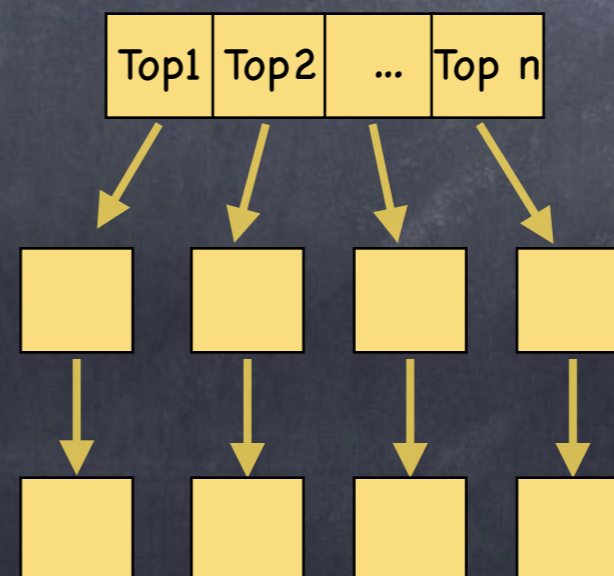
k-Stack

Henzinger, Kirsch, Payer, Sezgin, S.
POPL 2013



Distributed queues / stacks

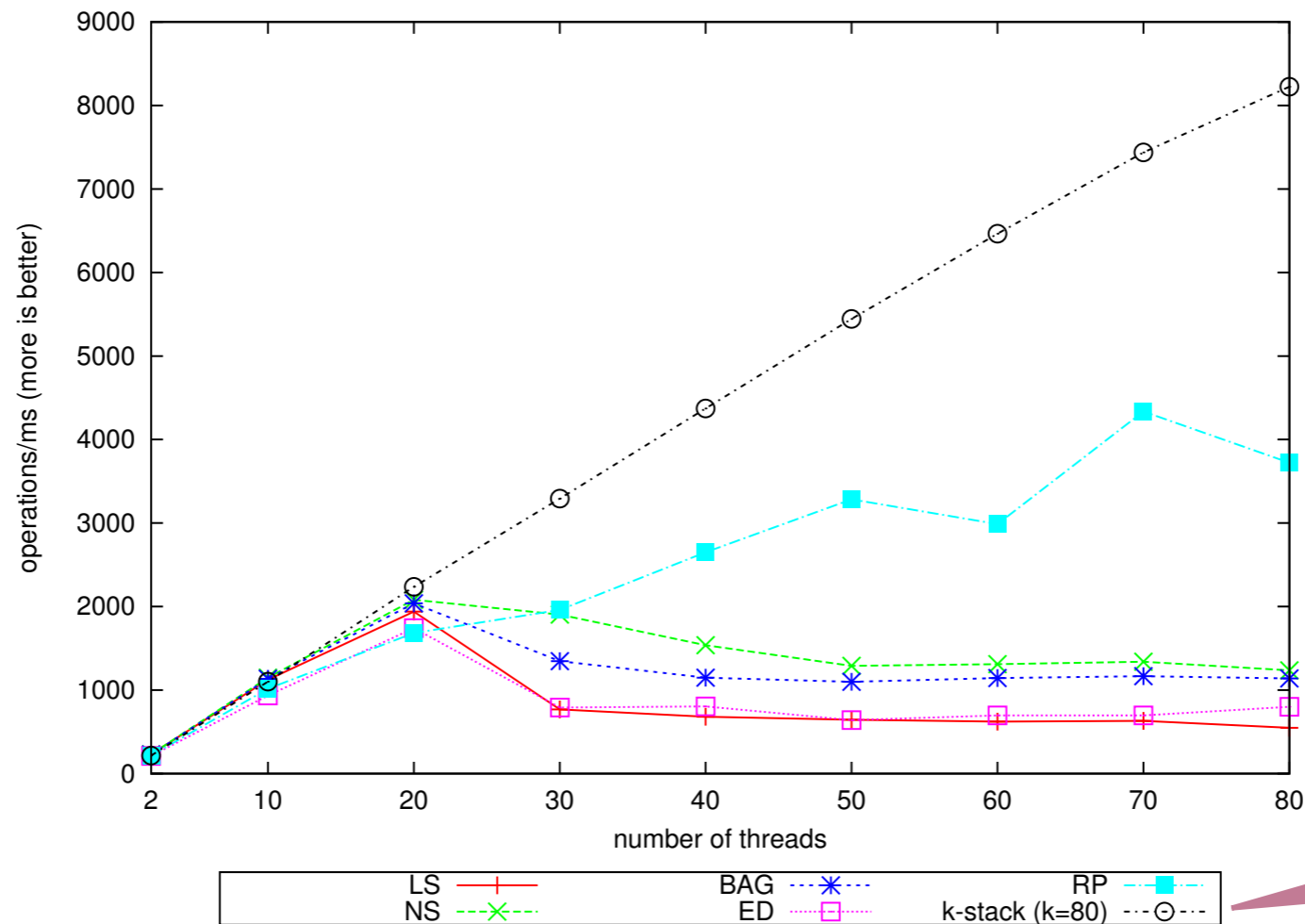
Haas, Henzinger, Kirsch, Lippautz, Payer, Sezgin, S.
CF 2013



k-Stack

Performance and Scalability comparison

"80"-core machine

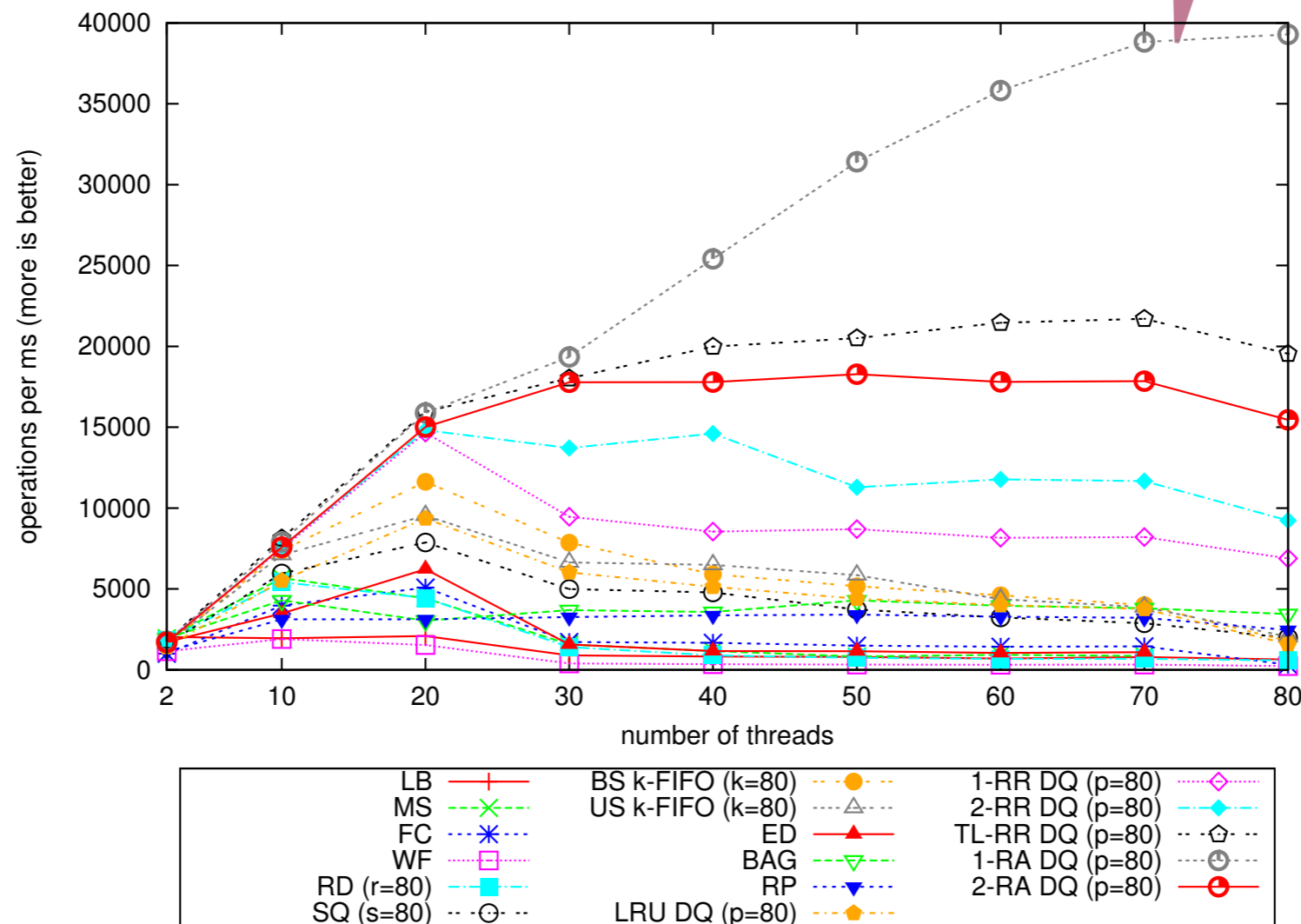


lock-free segment stack

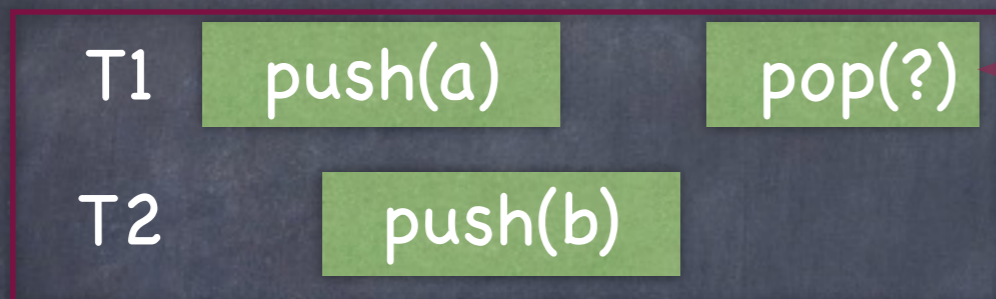
Distributed queues

Performance and Scalability comparison

"80"-core machine

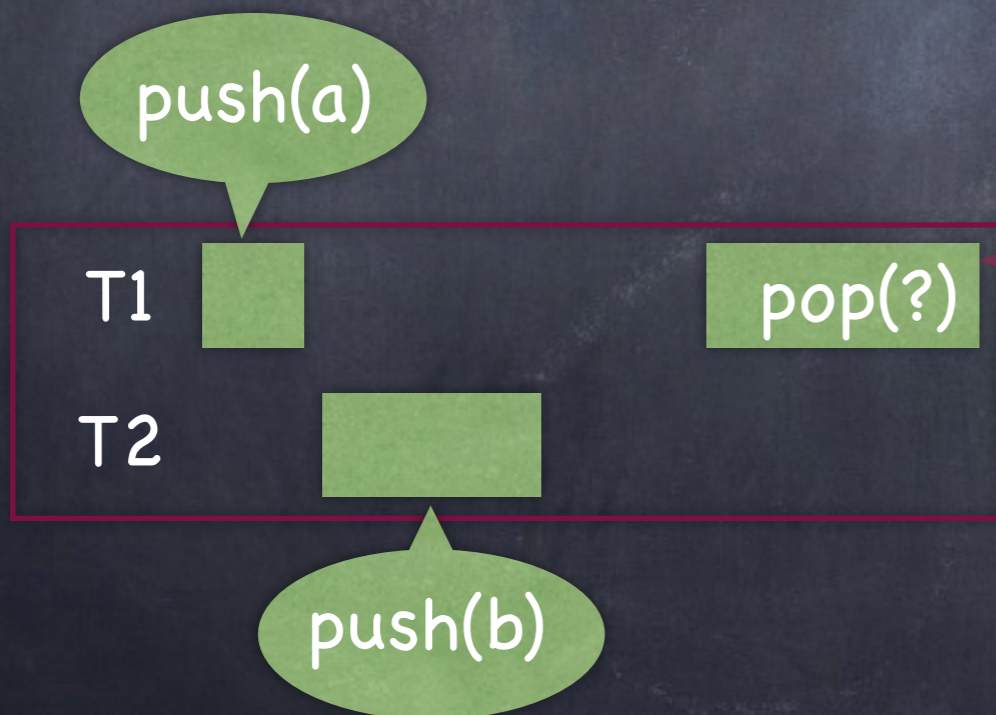


Bad performance also relaxes semantics



may return a or b

The slower the implementation, the more nondeterminism



must return a

Semantics vs. performance comparison (Con²Colic testing)
Haas, Henzinger, Holzer, Kirsch, ... S. work in progress