Coalgebraic Analysis of Probabilistic Systems

Ana Sokolova

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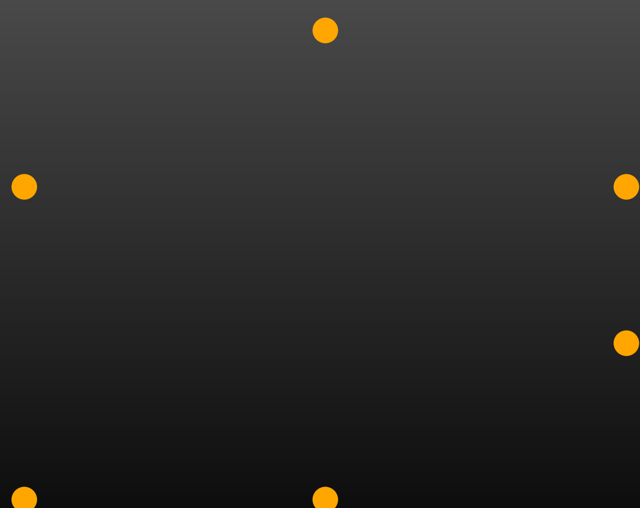

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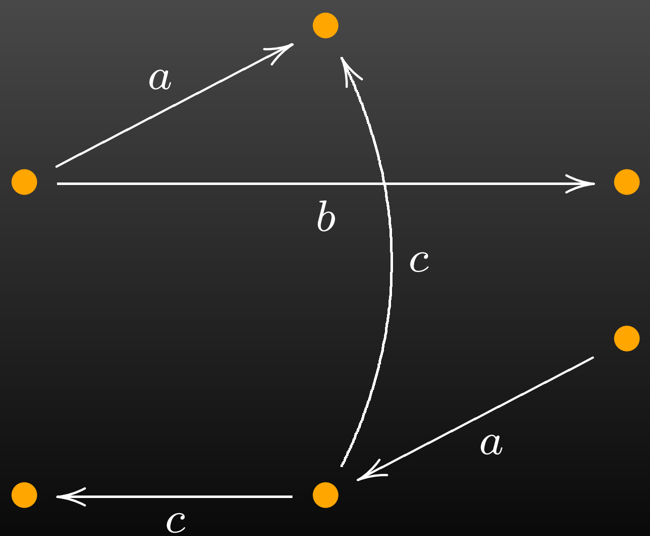

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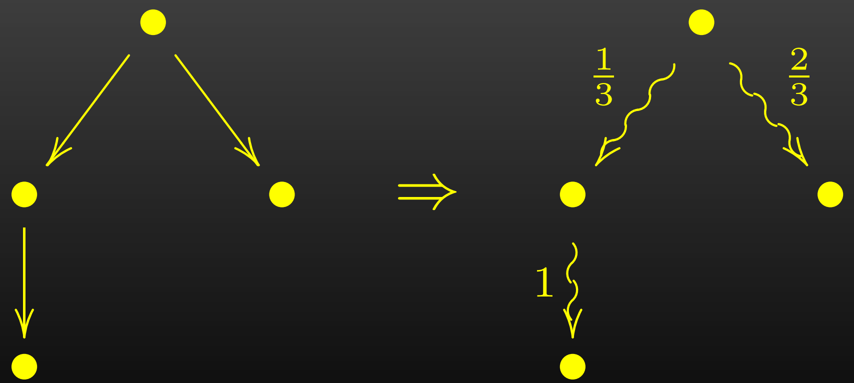

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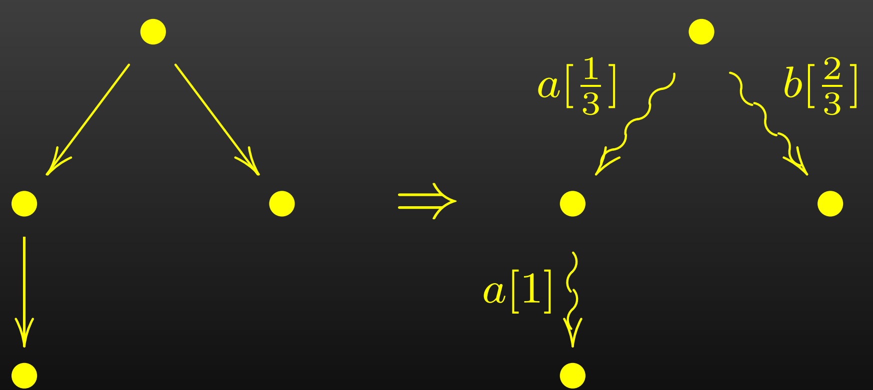

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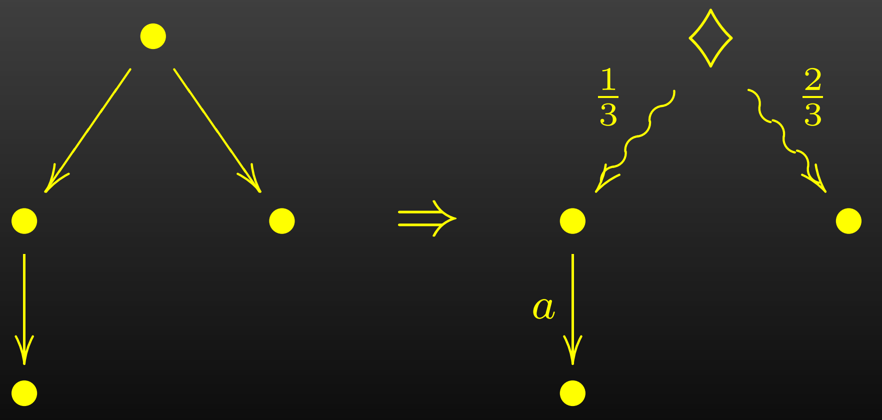

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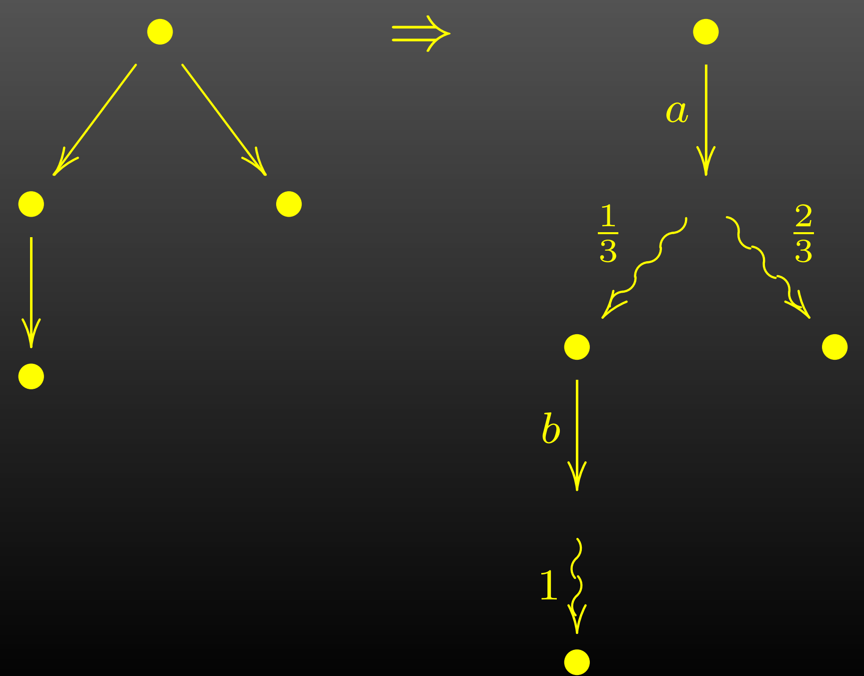

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probabilistic systems in the thesis includes:

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- comparing expressiveness of the types resulting in an expressiveness hierarchy
- study of weak bisimulation
- investigations of notions of linear behavior

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- based on category theory
- provide a uniform way of treating transition systems
- provide general notions and results e.g. a generic notion of bisimulation

Coalgebraic analysis...

MC	$\mathcal{D}$
DLTS	$(\mathcal{I}+1)^A$
LTS	$\mathcal{P}(A\times\mathcal{I})\cong\mathcal{P}^A$
React	$(\mathcal{D}+1)^A$
Gen	$\mathcal{D}(A\times\mathcal{I})+1$
Str	$\mathcal{D}+(A\times\mathcal{I})+1$
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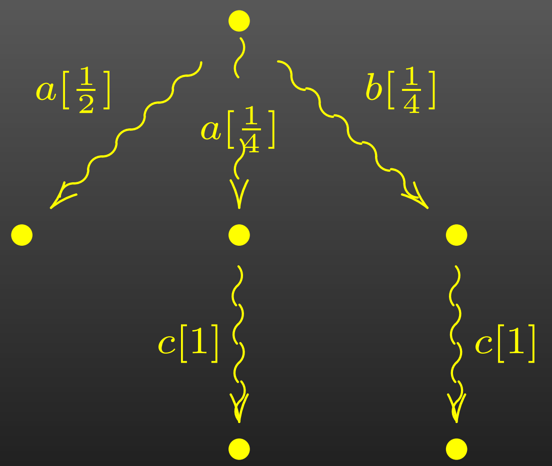

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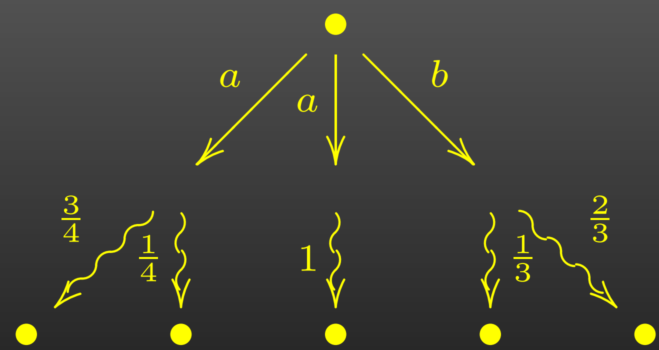

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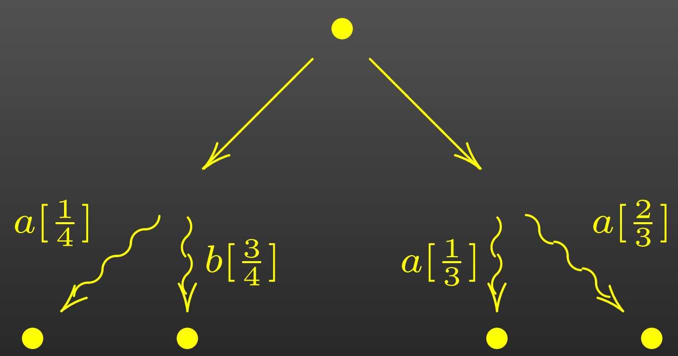
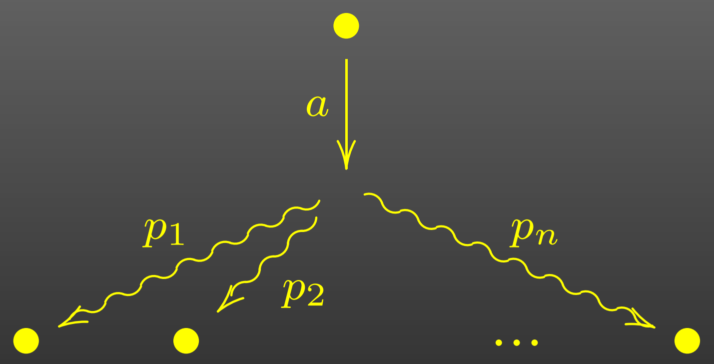


Expressiveness

simple Segala system $\longrightarrow$ Segala system



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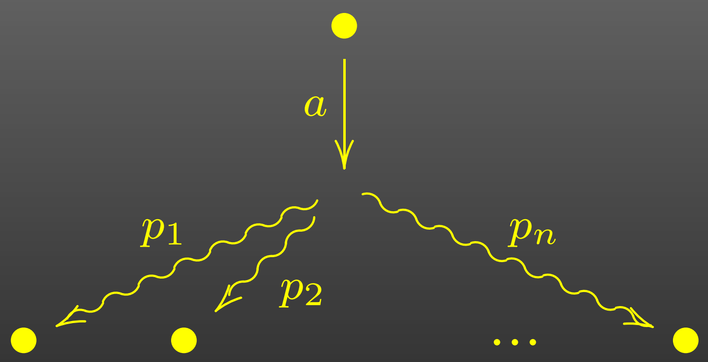

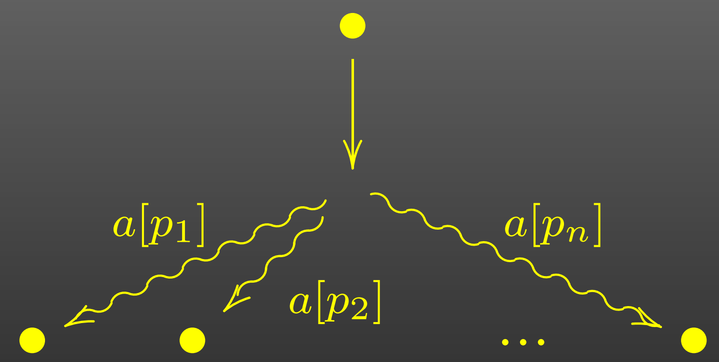

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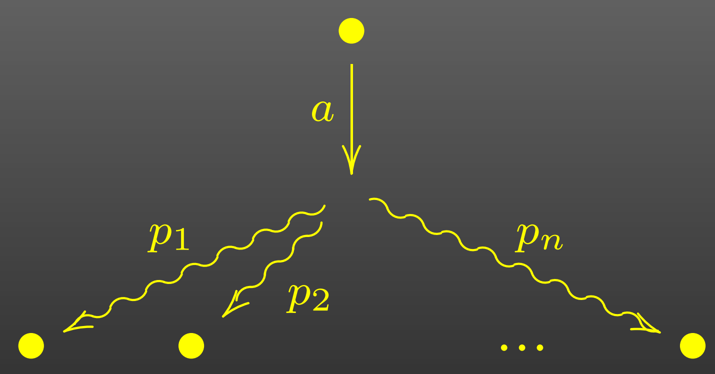
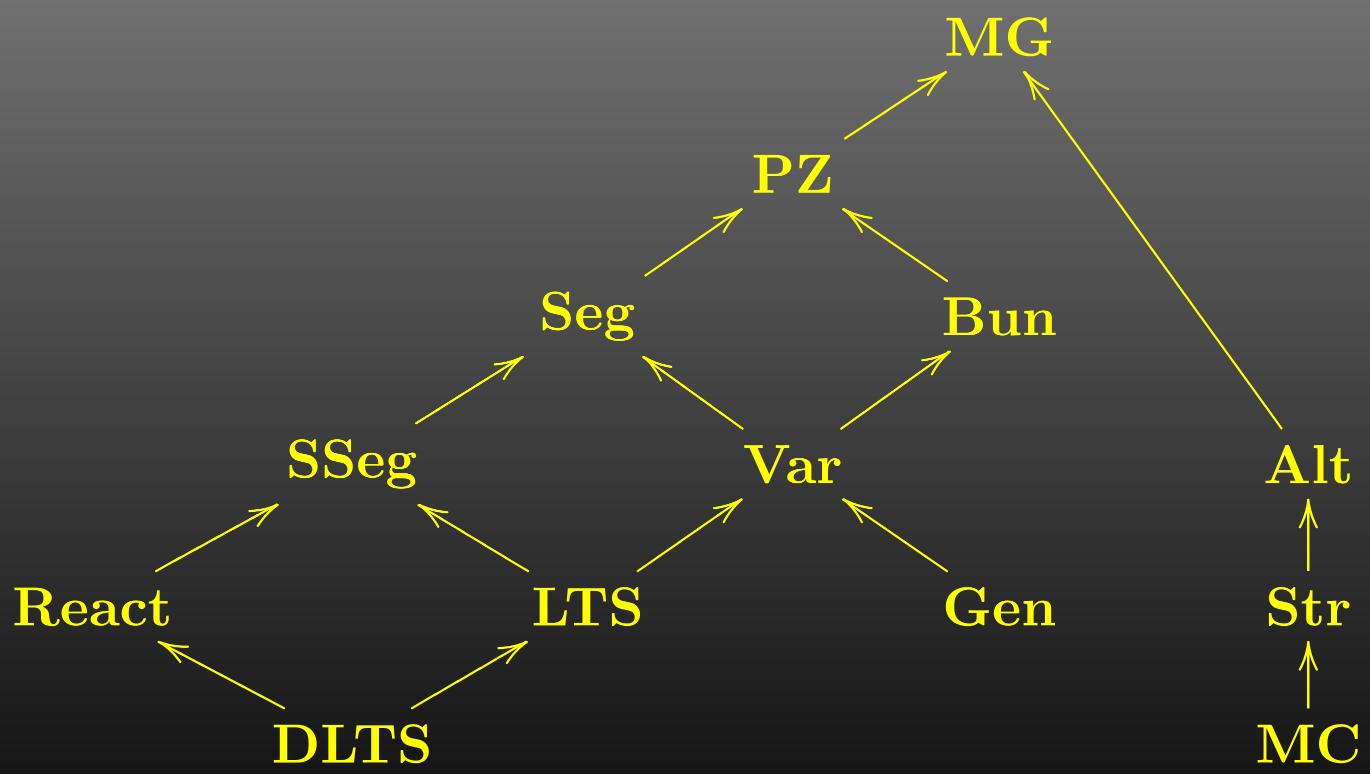


When do we consider one type of systems more expressive than another?

The hierarchy...



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two stages approach:

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weak bisimulation = bisimulation for the ”double-arrow"

Weak bisimulation

$$\begin{tikzcd} \langle S, A, \alpha \rangle \arrow[r, squiggly, "\Phi"] \arrow[d] & \langle S, A^*, \alpha' \rangle \arrow[r, squiggly, "\Psi_\tau"] \arrow[d] & \langle S, (A \setminus \tau)^*, \alpha'' \rangle \arrow[d] \\ \mathcal{F}_A\text{- coalgebra} & \mathcal{G}_{A^*}\text{- coalgebra} & \mathcal{G}_{(A \setminus \tau)^*}\text{- coalgebra} \end{tikzcd}$$

Other semantic relations..

for coalgebras and probabilistic systems:

- simulations
- colored transition relation
- towards linear semantic relations ...
  - composing systems (sequentially)
  - paths in systems

Stellingen

... instead of conclusions

- Coalgebras are a perfect abstract tool for modelling dynamic systems of various kinds including probabilistic ones.
- The expressiveness hierarchy brings order and clarifies the inter-relationships between the models.
- A weak bisimulation on a given system must be strong bisimulation on a suitably transformed system obtained from the original one.
- Sometimes it is good to stop inventing new things but focus on the existing instead.
- The process of learning deserves full respect and wonder.