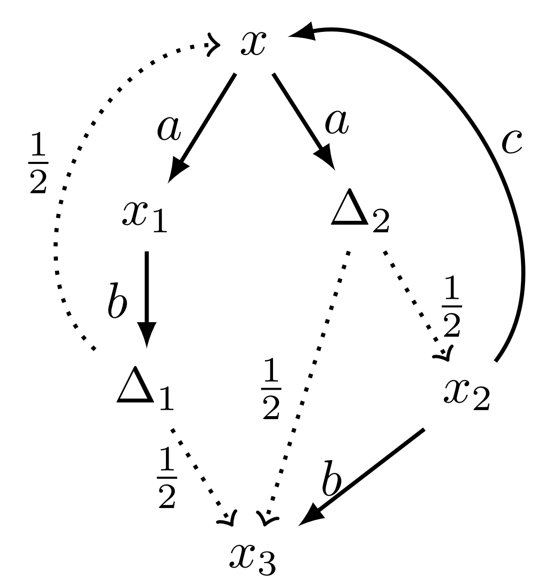
The Theory of Traces for Nondeterminism and Probability

Ana Sokolova
UNIVERSITY of SALZBURG



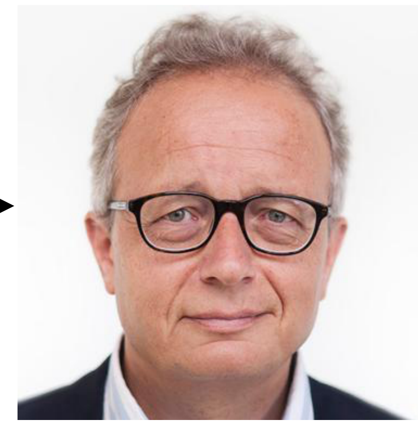
Coalgebra Day @ NII, Tokyo 28.10.19

It’s all about leaving a trace…

Joint work with

Ichiro Hasuo

NII 国立情報学研究所 National Institute of Informatics



Bart Jacobs

Radboud University

Alexandra Silva

UCL

Harald Woracek

TU WIEN

Filippo Bonchi

Valeria Vignudelli

ENS DE LYON

I will talk about:

1. The absolute basics of coalgebra

2. Trace semantics via determinisation

3. …enabled by algebraic structure

Mathematical framework based on category theory for state-based systems semantics

I will talk about:

1. The absolute basics of coalgebra

2. Trace semantics via determinisation

3. …enabled by algebraic structure

I will talk about:

Mathematical framework based on category theory for state-based systems semantics

1. The absolute basics of coalgebra

for nondeterministic/probabilistic systems

2. Trace semantics via determinisation

3. …enabled by algebraic structure

I will talk about:

Mathematical framework based on category theory for state-based systems semantics

1. The absolute basics of coalgebra

for nondeterministic/ probabilistic systems

systems with algebraic effects

2. Trace semantics via determinisation

3. …enabled by algebraic structure

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

$$X \xrightarrow{c} FX$$

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

$$X \xrightarrow{c} FX$$

states

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

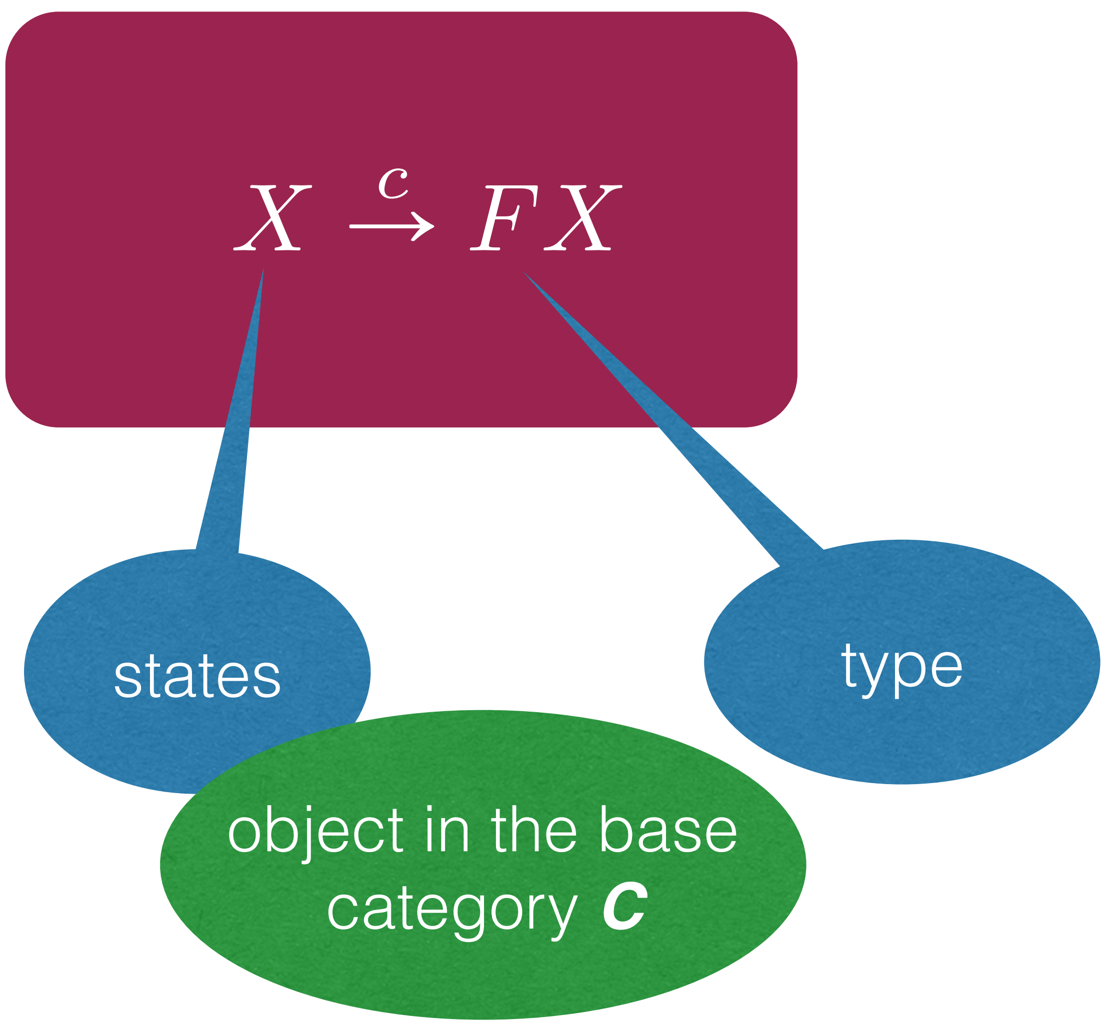
$$X \xrightarrow{c} FX$$

states

object in the base category ***C***

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.



Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

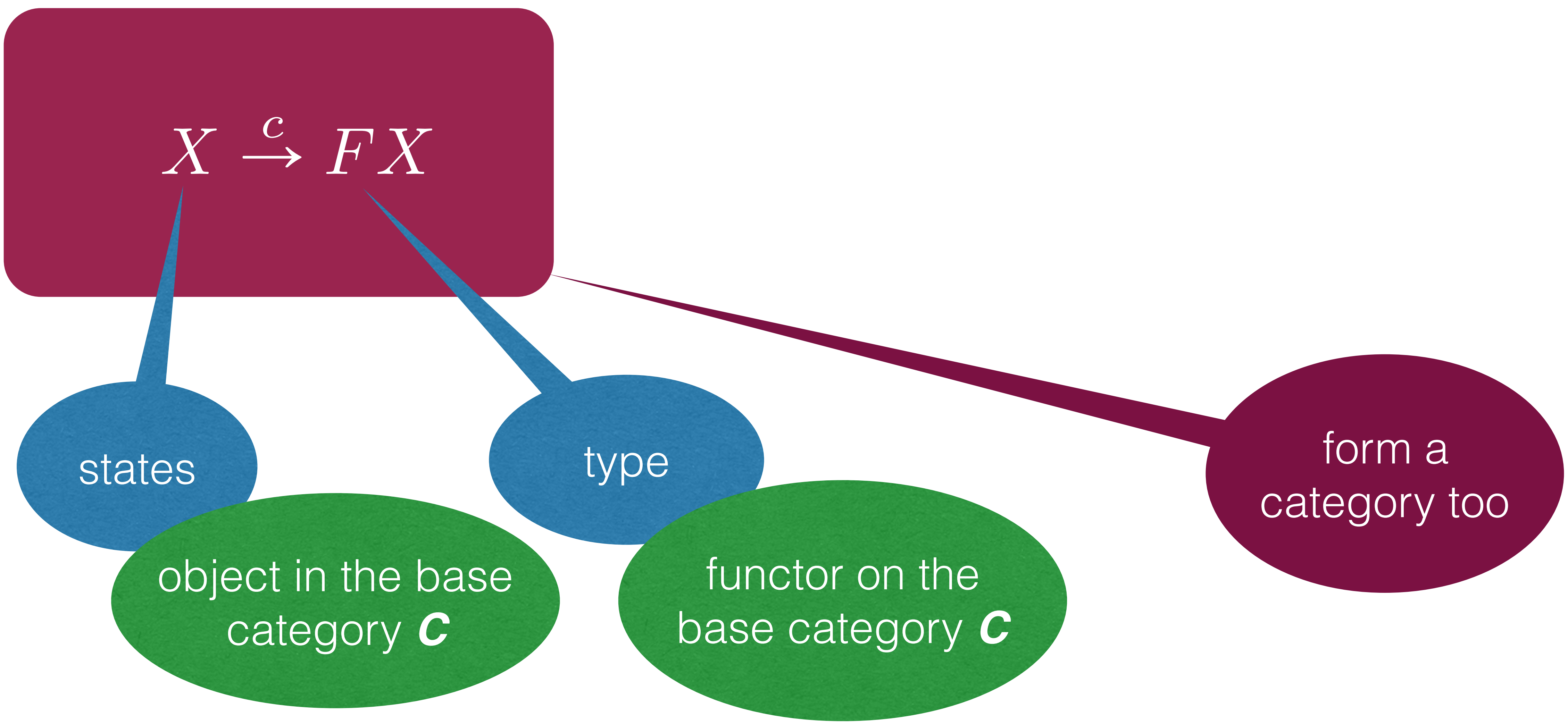
$$X \xrightarrow{c} FX$$

states — object in the base category ***C***

type — functor on the base category ***C***

Coalgebras

Uniform framework for dynamic transition systems, based on category theory.



Coalgebras

Uniform framework for dynamic transition systems, based on category theory.

$$X \xrightarrow{c} FX$$

generic notion of behavioural equivalence (bisimilarity)

$\approx$

states

type

form a category too

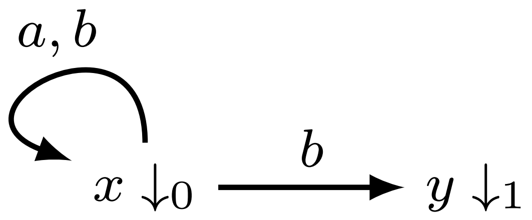
object in the base category ***C***

functor on the base category ***C***

Examples

NFA

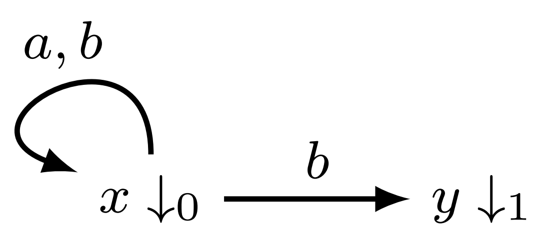
X → 2 x ($\mathcal{P}$X)^A



Examples

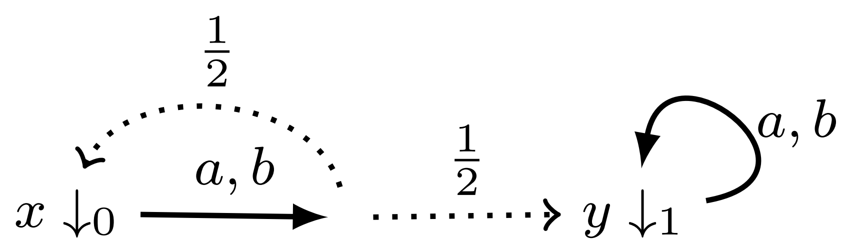
NFA

$X \to 2 \times (\mathcal{P}X)^A$



Rabin PA

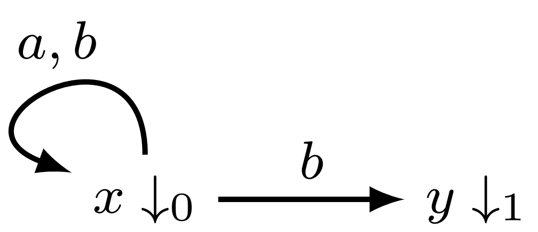
$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



Examples

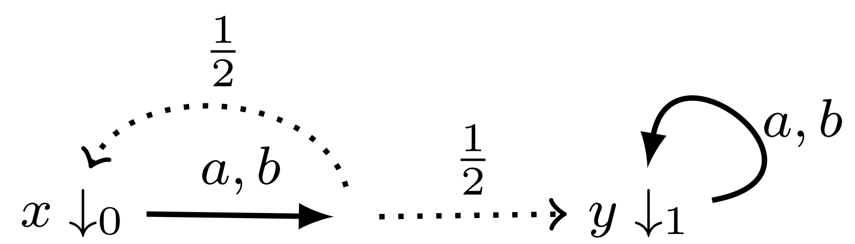
NFA

$X \to 2 \times (\mathcal{P}X)^A$



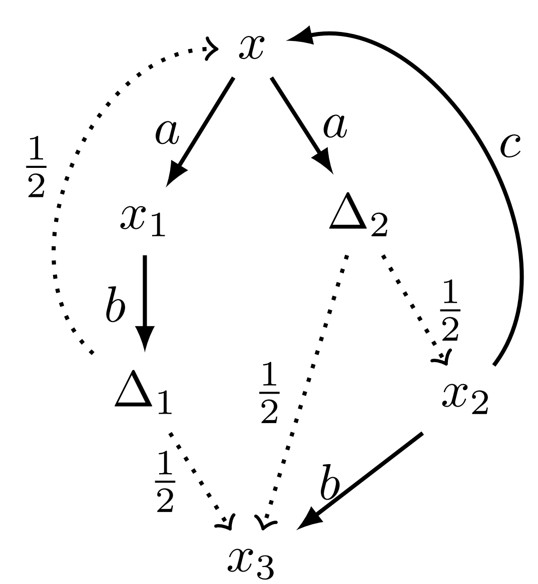
Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



Simple NPA

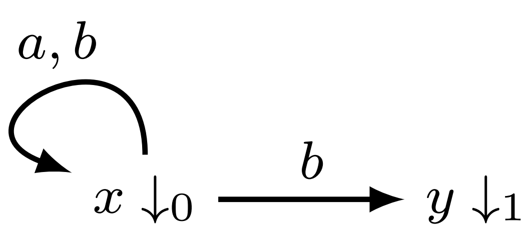
$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$



Examples

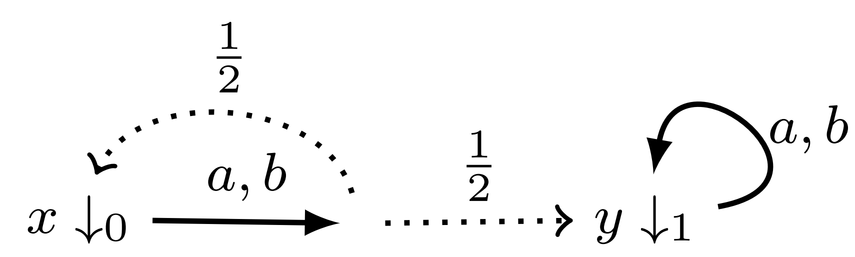
NFA

$X \to 2 \times (\mathcal{P}X)^A$



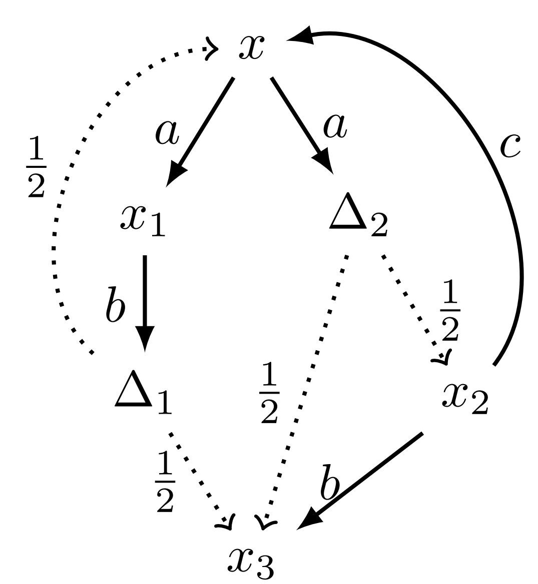
Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



Simple NPA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$



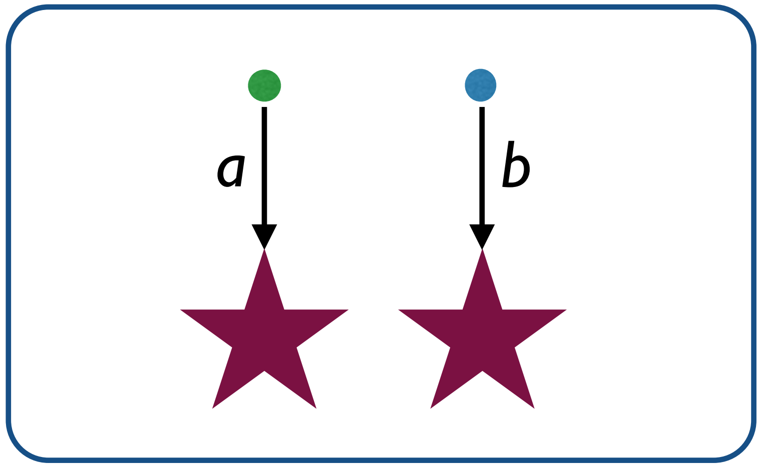
systems with nondeterminism and probability

In general

In general

Automata

$X \to O \times (MX)^A$

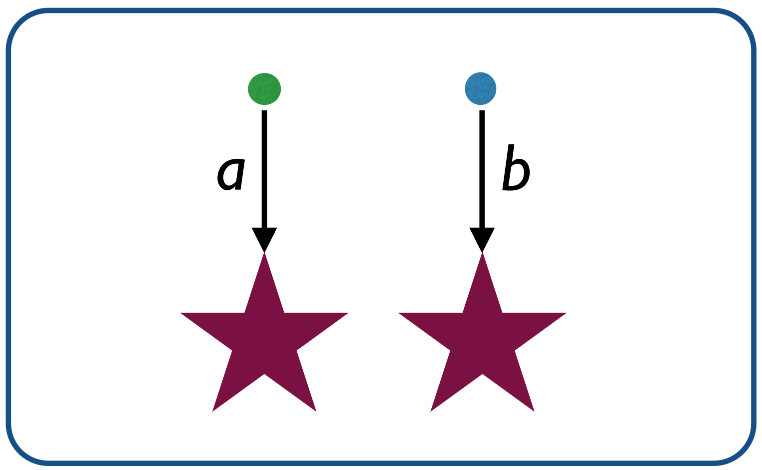


In general

Automata

with observations in O

$$X \to O \times (MX)^A$$

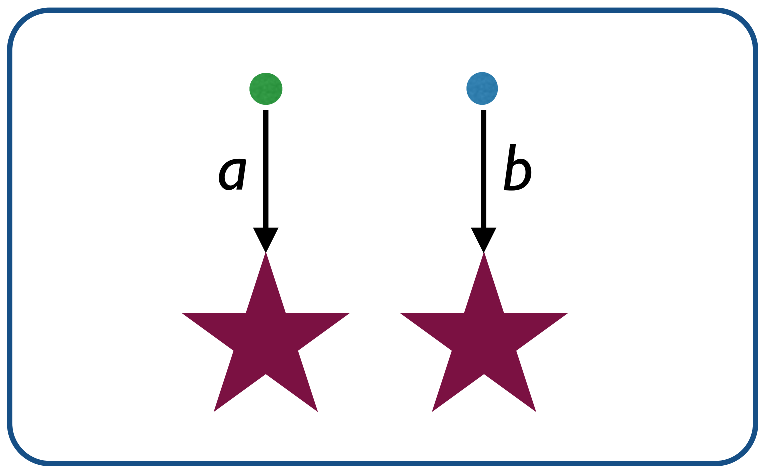


In general

Automata

with observations in O

$X \to O \times (MX)^A$



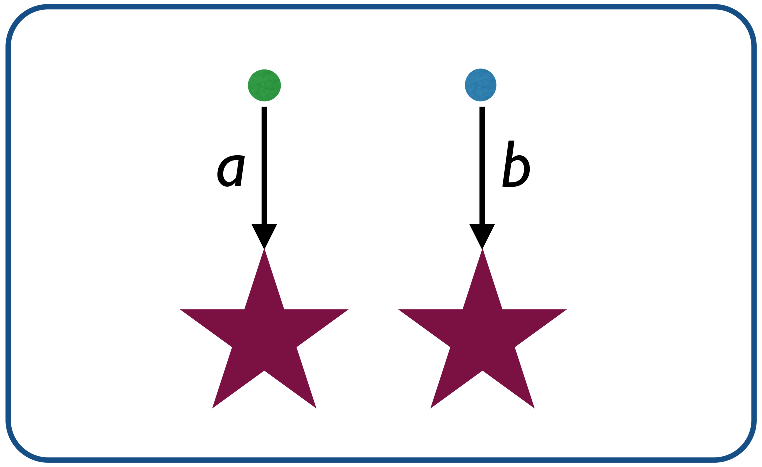
and M-effects

In general

Automata

with observations in O

$X \to O \times (MX)^A$



and M-effects

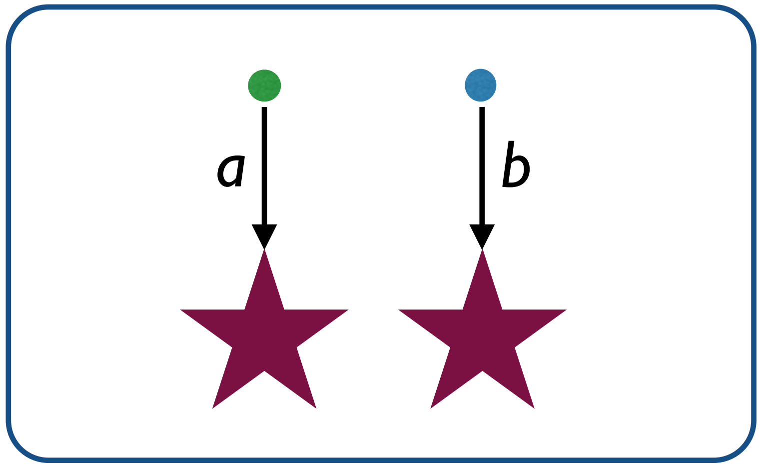
for a monad M

In general

Automata

with observations in O

$X \to O \times (MX)^A$



and M-effects

for a monad M

$\mu \colon MM \Rightarrow M$

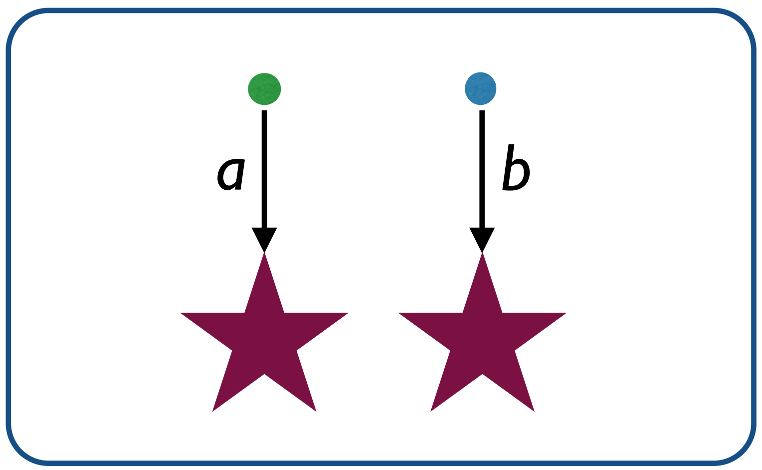
$\eta \colon Id \Rightarrow M$

In general

Automata

with observations in O

$X \to O \times (MX)^A$



and M-effects

for a monad M

$\mu\colon MM \Rightarrow M$

$\eta\colon Id \Rightarrow M$

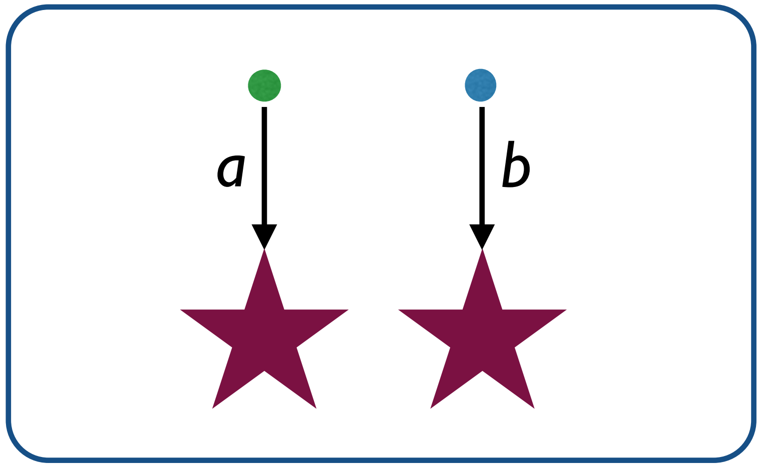
providing algebraic effects

In general

Automata

with observations in O

$X \to O \times (MX)^A$



and M-effects

for a monad M

$\mu \colon MM \Rightarrow M$

$\eta \colon Id \Rightarrow M$

providing algebraic effects

we write $x{\downarrow}o, \;\; x \xrightarrow{a} t_x$

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

Simple PA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

M = $\mathcal{P}$
for nondeterminism

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

Simple PA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

M = $\mathcal{P}$
for nondeterminism

Powerset, subsets

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

Simple PA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

$M = \mathcal{P}$
for nondeterminism

Powerset, subsets

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

$M = \mathcal{D}_{\leq 1}$
for probability

Simple PA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

$M = \mathcal{P}$
for nondeterminism

Powerset, subsets

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

$M = \mathcal{D}_{\leq 1}$
for probability

Subdistributions

Simple PA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

$M = \mathcal{P}$
for nondeterminism

Powerset, subsets

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

$M = \mathcal{D}_{\leq 1}$
for probability

Subdistributions

Simple PA

$X \to ? \times (\mathcal{P}\mathcal{D}X)^A$

$M = \mathcal{P}\mathcal{D}$???
for nondeterminism
and probability

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

$M = \mathcal{P}$
for nondeterminism

Powerset, subsets

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

$M = \mathcal{D}_{\leq 1}$
for probability

Subdistributions

Simple PA

$X \to ? \times (\mathcal{C}X)^A$

$M = \mathcal{C}$
for nondeterminism
and probability !

In our examples

NFA

$X \to 2 \times (\mathcal{P}X)^A$

$M = \mathcal{P}$
for nondeterminism

Powerset, subsets

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

$M = \mathcal{D}_{\leq 1}$
for probability

Subdistributions

Simple PA

$X \to ? \times (\mathcal{C}X)^A$

$M = \mathcal{C}$
for nondeterminism
and probability !

Nonempty f.g. convex
subsets of
distributions

Trace Semantics

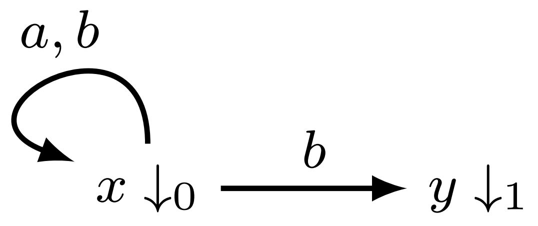
Trace Semantics

language semantics

Trace Semantics

NFA = LTS + termination

$X \to 2 \times (\mathcal{P}X)^A$



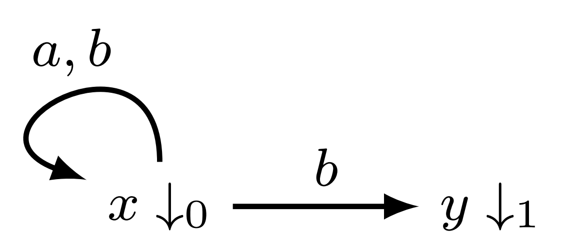
language semantics

Trace Semantics

language semantics

NFA = LTS + termination

$X \to 2 \times (\mathcal{P}X)^A$



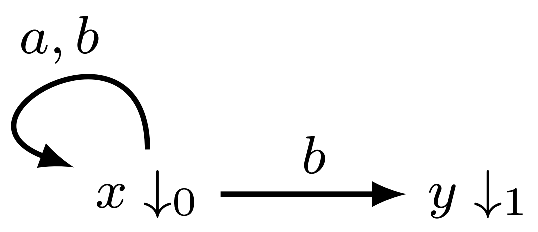
$\mathrm{tr}\colon X \to 2^{A^*}$

Trace Semantics

language semantics

NFA = LTS + termination

$X \to 2 \times (\mathcal{P}X)^A$



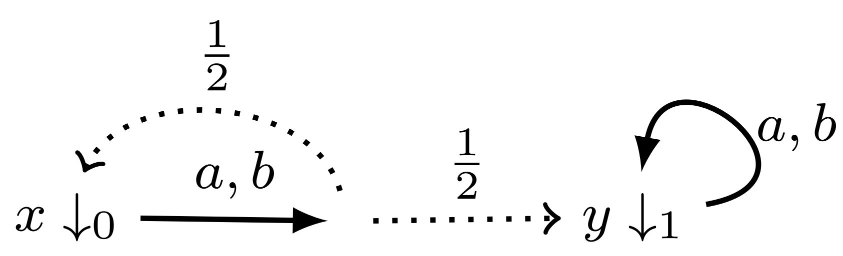
$\mathrm{tr}\colon X \to 2^{A^*}$

$\mathrm{tr}(x) = (a \cup b)^* b = \{w \in \{a,b\}^* \mid w \text{ ends with a } b\}$

Trace Semantics

Rabin PA = RPTS + termination

$X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



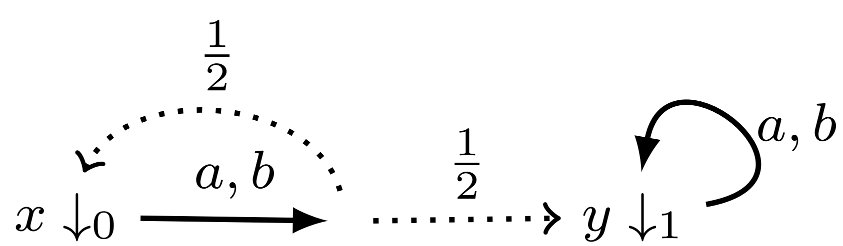
$$\mathrm{tr}(x) = \left(a \mapsto \frac{1}{2}, aa \mapsto \frac{3}{4}, \dots\right)$$

$$\mathrm{tr}\colon X \to [0,1]^{A^*}$$

Trace Semantics

Rabin PA = RPTS + termination

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



probabilistic language semantics

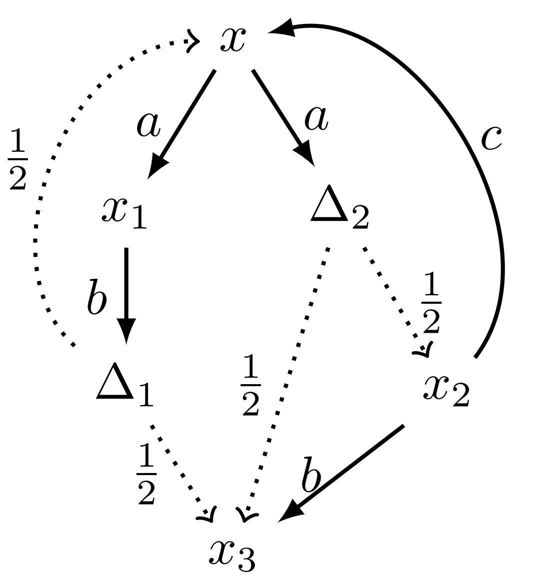
$$\mathrm{tr}(x) = \left(a \mapsto \frac{1}{2}, aa \mapsto \frac{3}{4}, \dots\right)$$

$$\mathrm{tr}\colon X \to [0,1]^{A^*}$$

Trace Semantics

Simple NPA

$X \to ? \times (\mathcal{PD}X)^A$



$\mathrm{tr}(x) = ???$

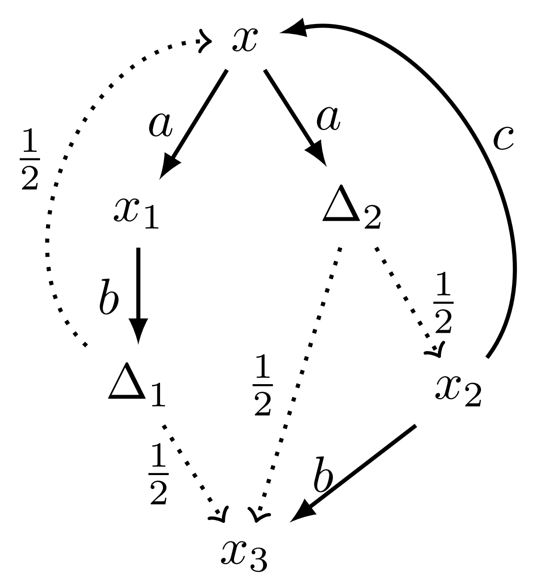
$\mathrm{tr}\colon X \to ?^{A^*}$

Trace Semantics

nondet. probabilistic language semantics ?

Simple NPA

X → ? x (𝒫𝒟X)^A



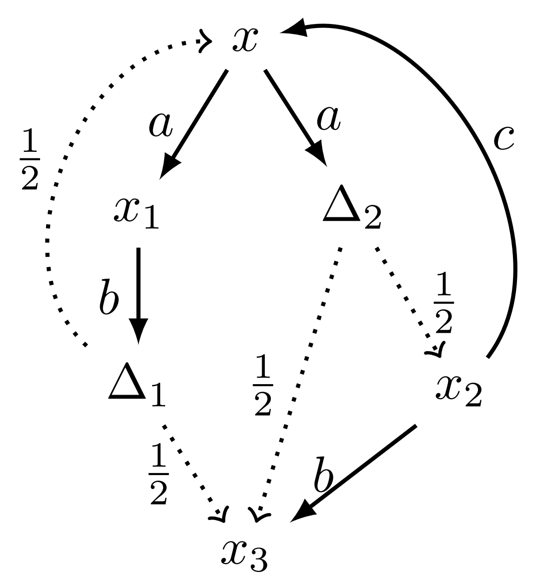
$\mathrm{tr}(x) = ???$

$\mathrm{tr}\colon X \to ?^{A^*}$

Trace Semantics

Simple NPA

$X \to ? \times (\mathcal{PD}X)^A$



nondet. probabilistic language semantics ?

Existing definitions are “local” given in terms of schedulers

$\mathrm{tr}(x) = ???$

$\mathrm{tr}\colon X \to ?^{A^*}$

Trace semantics coalgebraically?

Two ideas: NFA / LTS

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation

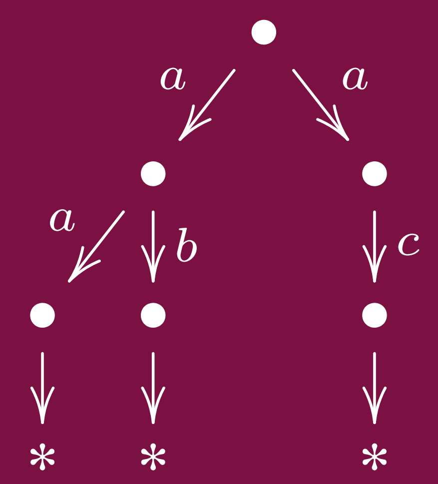
Trace semantics coalgebraically?

NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation



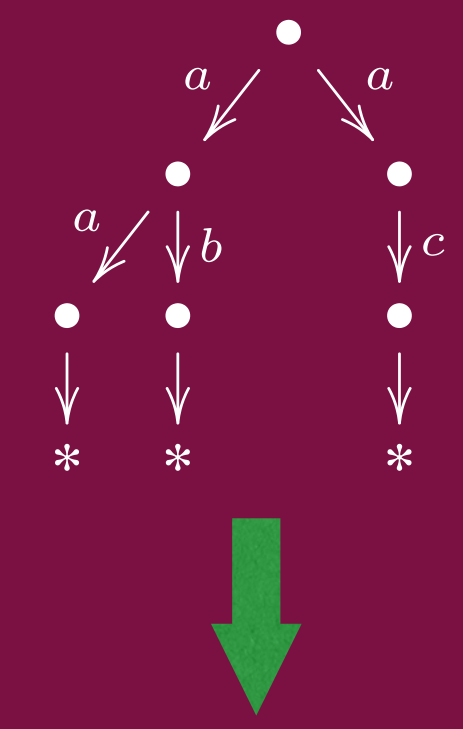
Trace semantics coalgebraically?

NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation



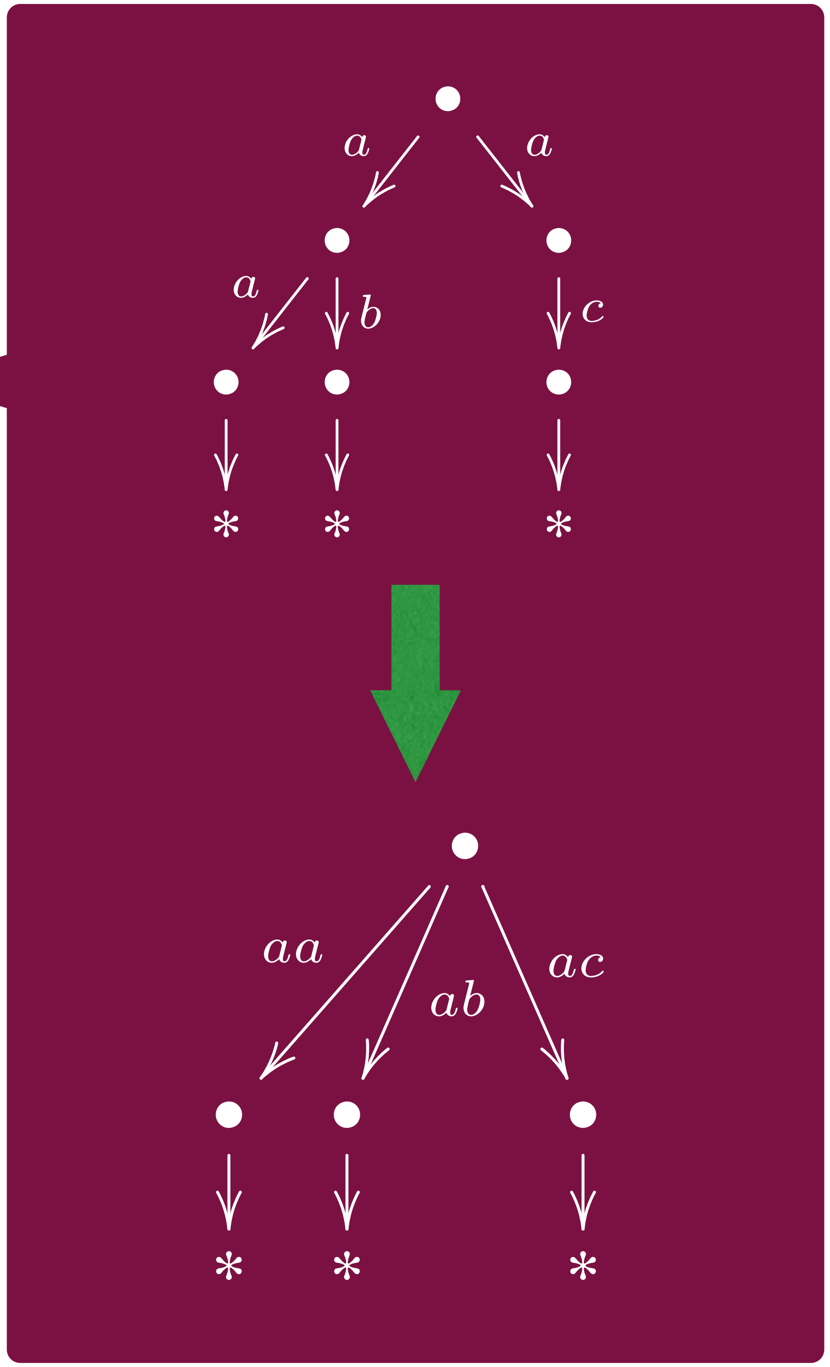
Trace semantics coalgebraically?

NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation



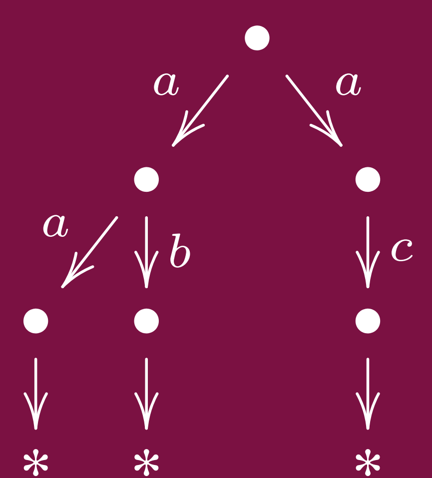
Trace semantics coalgebraically?

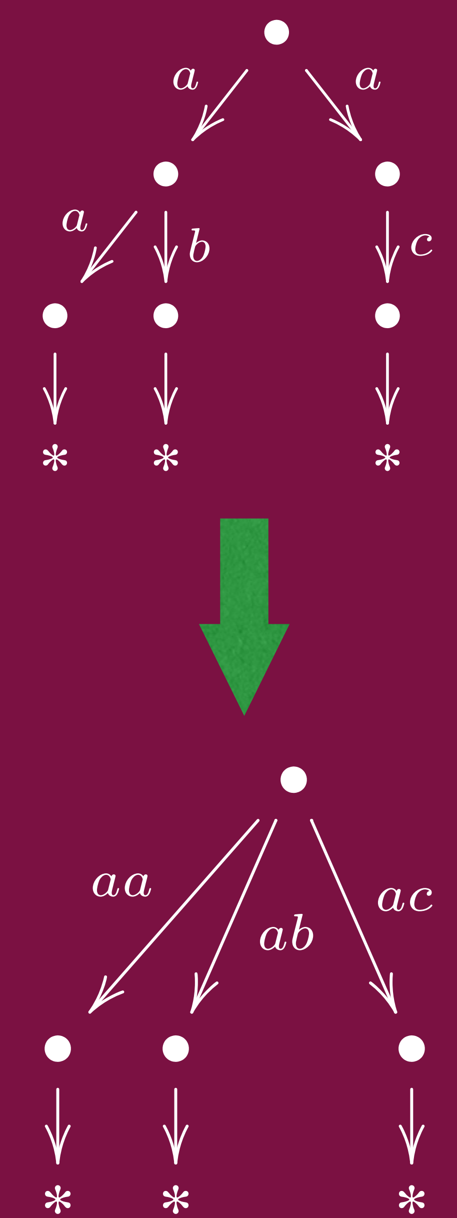
NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation





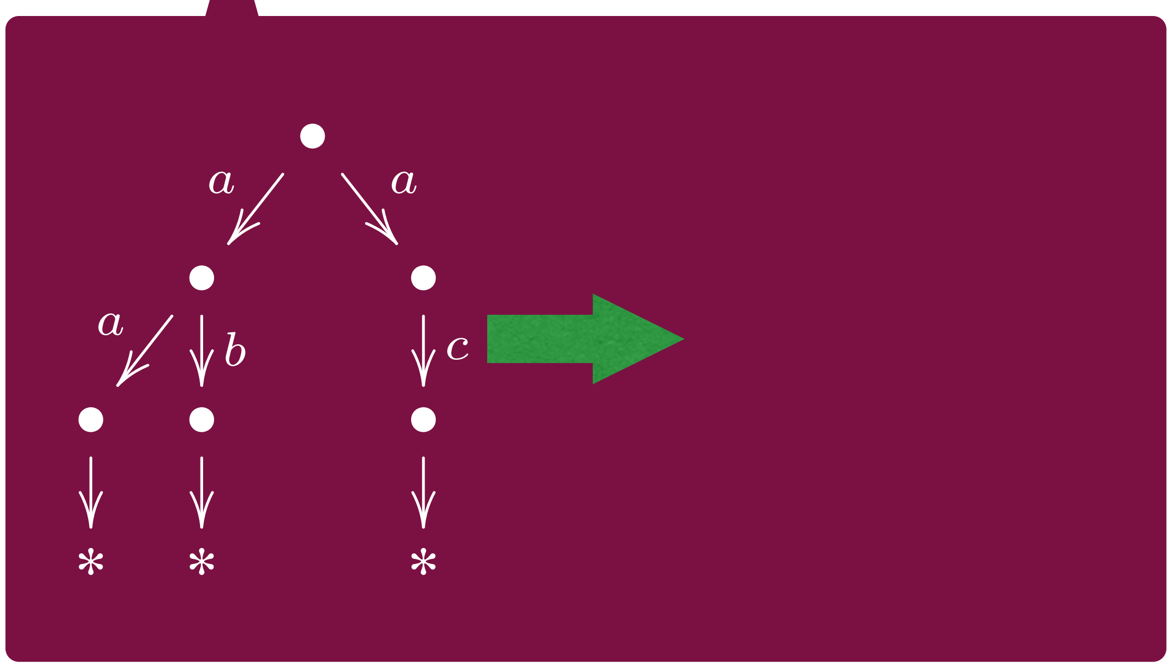
Trace semantics coalgebraically?

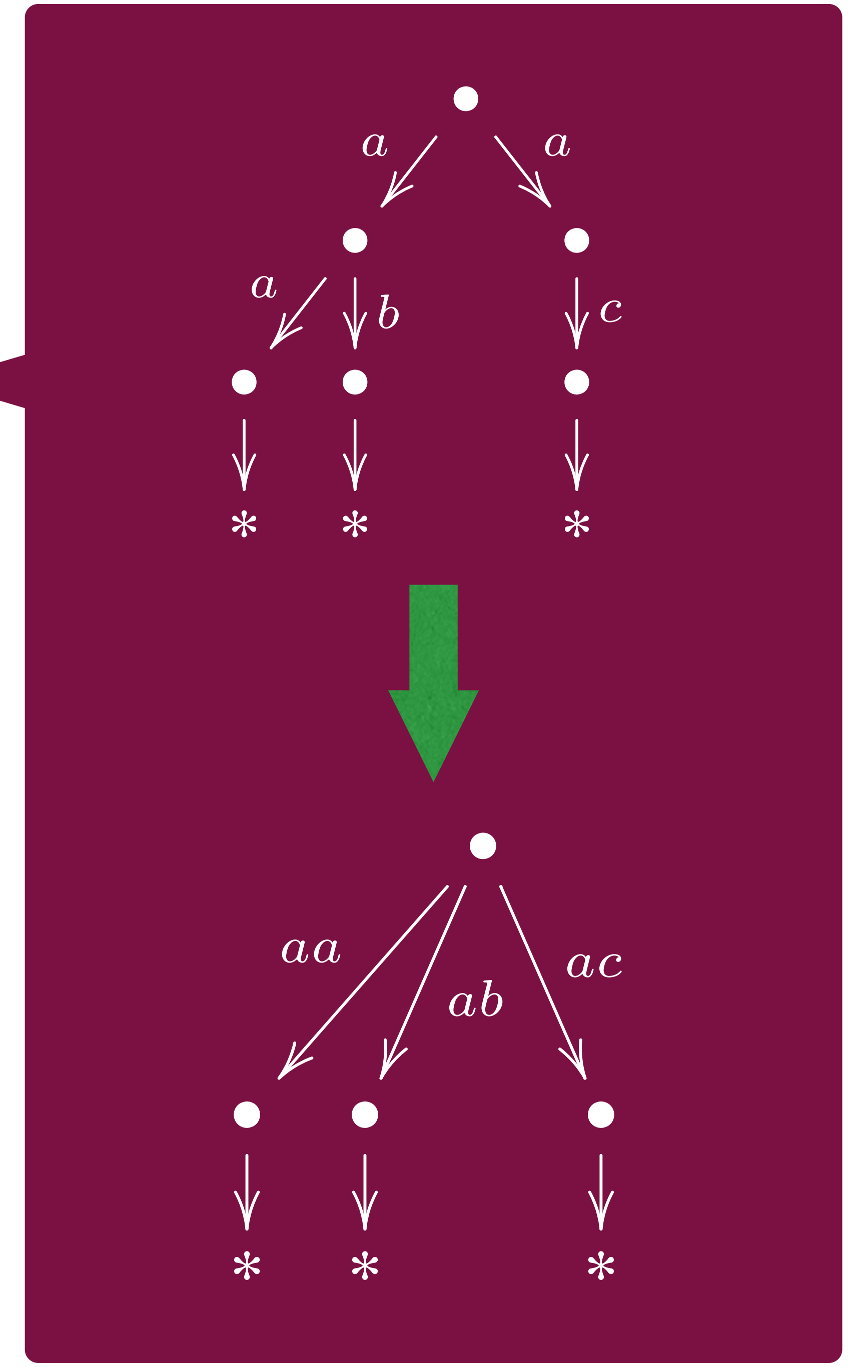
NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation





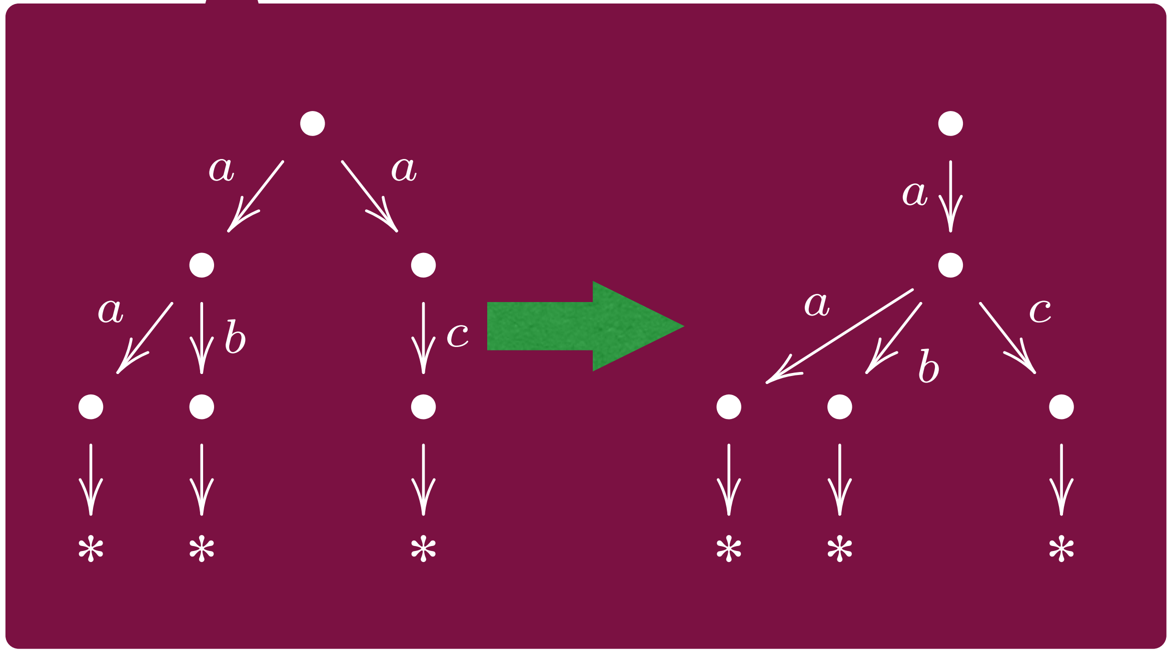
Trace semantics coalgebraically?

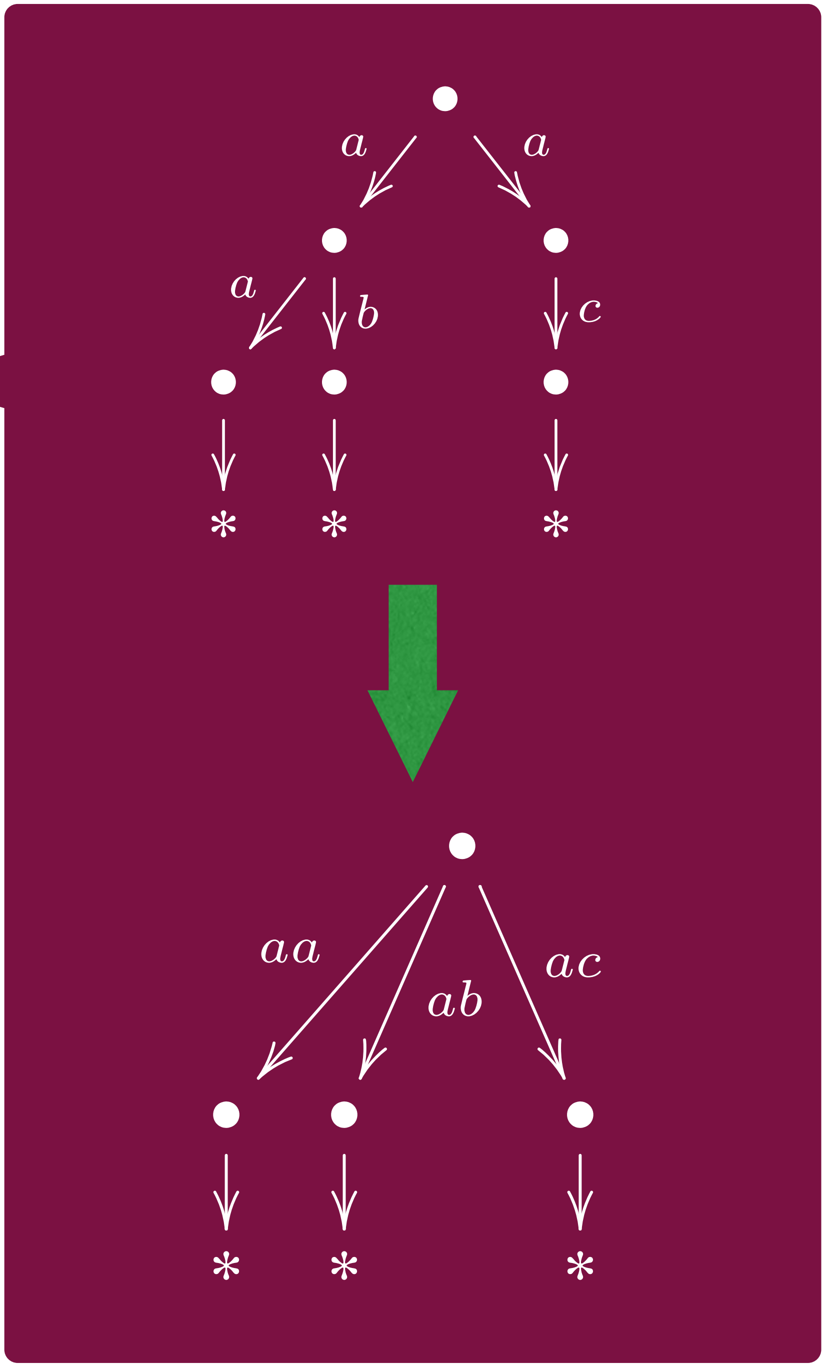
NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation





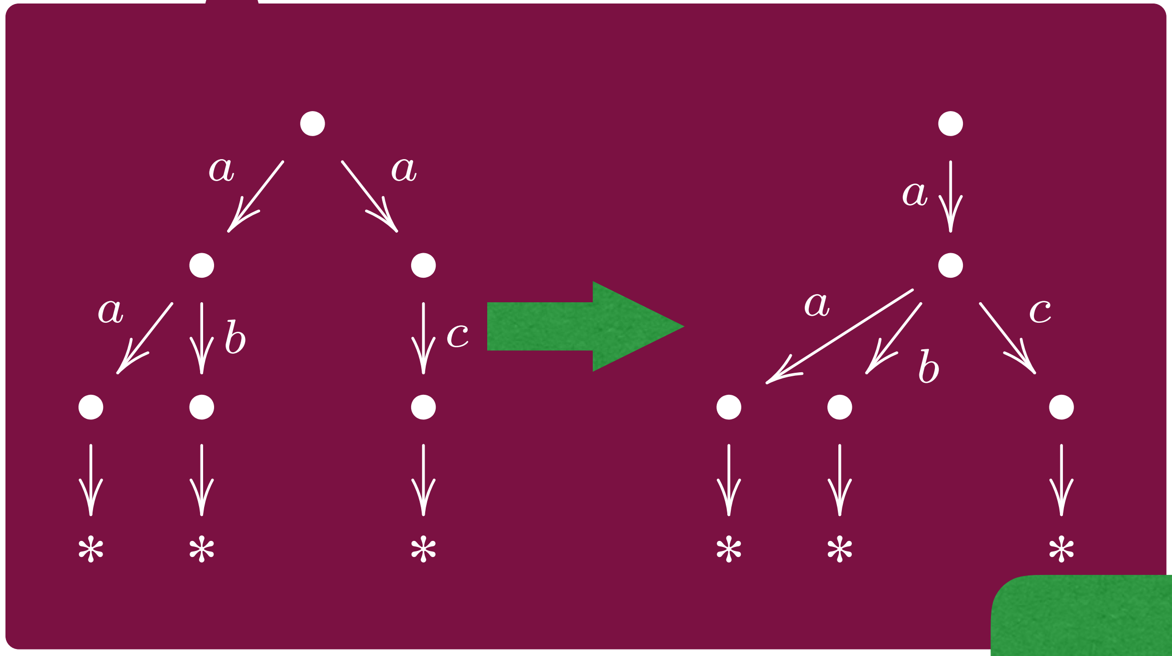
Trace semantics coalgebraically?

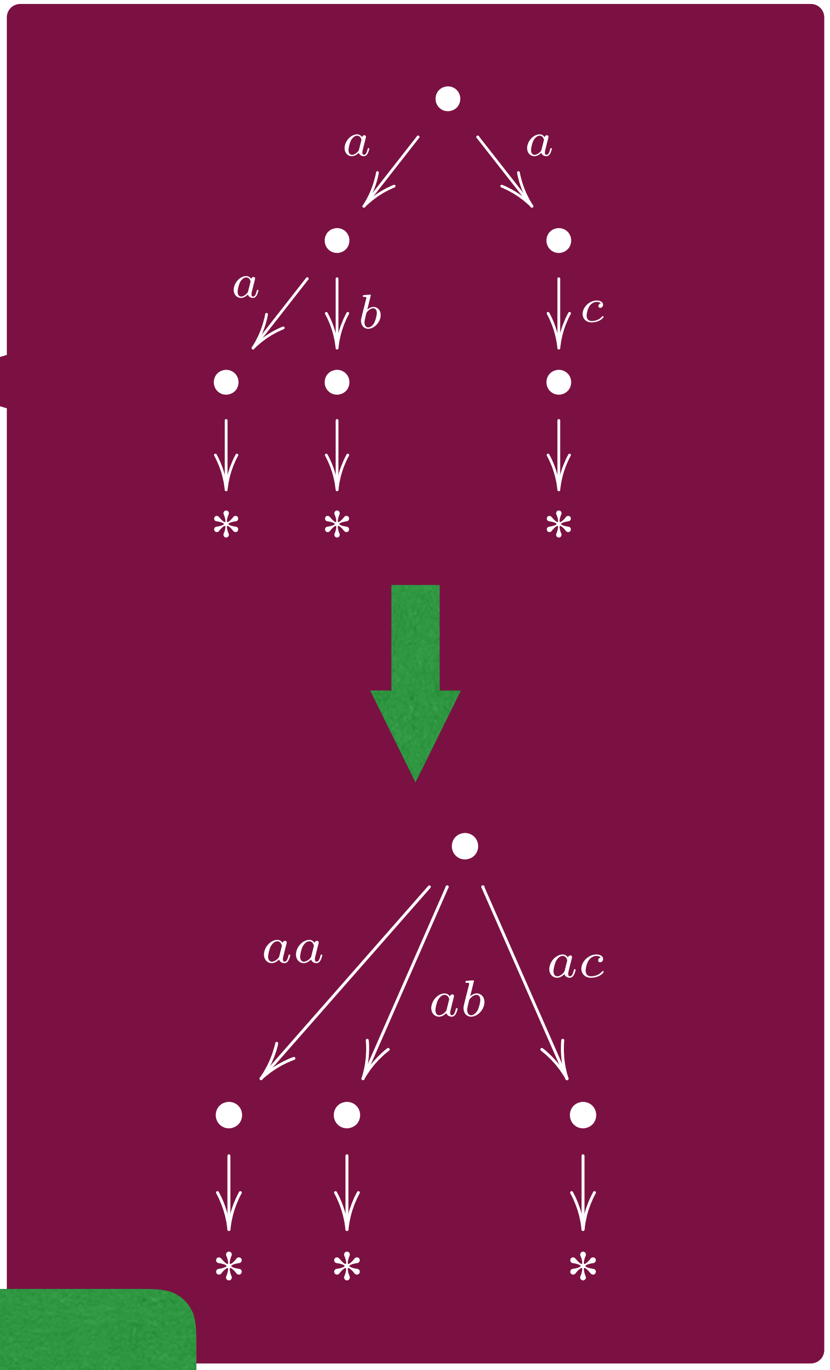
NFA / LTS

Two ideas:

(1) unfold branching + transitions on words

(2) trace = bisimilarity after determinisation





monads !

Trace semantics coalgebraically

Trace semantics coalgebraically

Two approaches:

(1) modelling in a Kleisli category

(2) modelling in an Eilenberg-Moore category

Trace semantics coalgebraically

Two approaches:

(1) modelling in a Kleisli category

(2) modelling in an Eilenberg-Moore category

algebras of a monad M

Trace semantics coalgebraically

Two approaches:

(1) modelling in a Kleisli category

Hasuo, Jacobs, S. LMCS '07

(2) modelling in an Eilenberg-Moore category

algebras of a monad M

Trace semantics coalgebraically

Hasuo, Jacobs, S. LMCS '07

Silva, Bonchi, Bonsangue, Rutten FSTTCS'10

Two approaches:

(1) modelling in a Kleisli category

(2) modelling in an Eilenberg-Moore category

algebras of a monad M

Trace semantics coalgebraically

Two approaches:

(1) modelling in a Kleisli category

(2) modelling in an Eilenberg-Moore category

Hasuo, Jacobs, S. LMCS '07

Silva, Bonchi, Bonsangue, Rutten FSTTCS'10

algebras of a monad M

(1) and (2) are related

Trace semantics coalgebraically

Two approaches:

(1) modelling in a Kleisli category

(2) modelling in an Eilenberg-Moore category

Hasuo, Jacobs, S. LMCS '07

Silva, Bonchi, Bonsangue, Rutten FSTTCS'10

algebras of a monad M

Jacobs, Silva, S. JCSS'15

(1) and (2) are related

Traces via determinisation

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$ →

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

→

Determinisation

$MX \to O \times (MX)^A$

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

Determinisation

$MX \to O \times (MX)^A$

trace = bisimilarity after determinisation

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

Determinisation

$MX \to O \times (MX)^A$

trace = bisimilarity after determinisation

Algebras for M

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

Determinisation

$MX \to O \times (MX)^A$

trace = bisimilarity after determinisation

Algebras for M

ideally we have a presentation

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

O has to be an M-algebra !

Determinisation

$MX \to O \times (MX)^A$

trace = bisimilarity after determinisation

Algebras for M

ideally we have a presentation

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

O has to be an M-algebra !

Determinisation

$MX \to O \times (MX)^A$

MX carries the free M-algebra

trace = bisimilarity after determinisation

Algebras for M

ideally we have a presentation

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

O has to be an M-algebra !

Determinisation

$MX \to O \times (MX)^A$

MX carries the free M-algebra

trace = bisimilarity after determinisation

Algebras for M

ideally we have a presentation

Eilenberg-Moore algebras

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

O has to be an M-algebra !

Determinisation

$MX \to O \times (MX)^A$

MX carries the free M-algebra

trace = bisimilarity after determinisation

$\mathrm{tr}\colon X \to O^{A^*}$

Algebras for M

ideally we have a presentation

Eilenberg-Moore algebras

Traces via determinisation

Automaton with M-effects

$X \to O \times (MX)^A$

O has to be an M-algebra !

Determinisation

$MX \to O \times (MX)^A$

MX carries the free M-algebra

trace = bisimilarity after determinisation

$\mathrm{tr}\colon X \to O^{A^*}$

Algebras for M

ideally we have a presentation

Eilenberg-Moore algebras

$$\mathrm{tr}(x)(a_1 a_2 \dots a_n) = o \;\Leftrightarrow\; x \xrightarrow{a_1} t_1 \xrightarrow{a_2} t_2 \cdots t_{n-1} \xrightarrow{a_n} t_n \;\wedge\; t_n{\downarrow}o$$

Eilenberg-Moore Algebras

abstractly

$\mathcal{EM}(M)$

- objects

$$\begin{array}{c} MA \\ \downarrow a \\ A \end{array}$$

satisfying

$$\begin{array}{ccc} A & \xrightarrow{\eta} & MA \\ & \overset{a}{\searrow\!\!\!\searrow} & \downarrow a \\ & & A \end{array} \qquad \begin{array}{ccc} MMA & \xrightarrow{\mu} & MA \\ Ma \downarrow & & \downarrow a \\ MA & \xrightarrow{a} & A \end{array}$$

- morphisms

$$\begin{array}{c} MA \\ \downarrow a \\ A \end{array} \xrightarrow{h} \begin{array}{c} MB \\ \downarrow b \\ B \end{array}$$

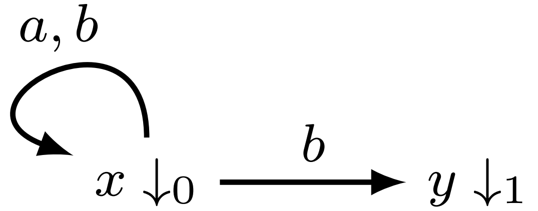
$$\begin{array}{ccc} MA & \xrightarrow{Mh} & MB \\ a \downarrow & & \downarrow b \\ A & \xrightarrow{h} & B \end{array}$$

Traces via determinisation

Traces via determinisation

NFA

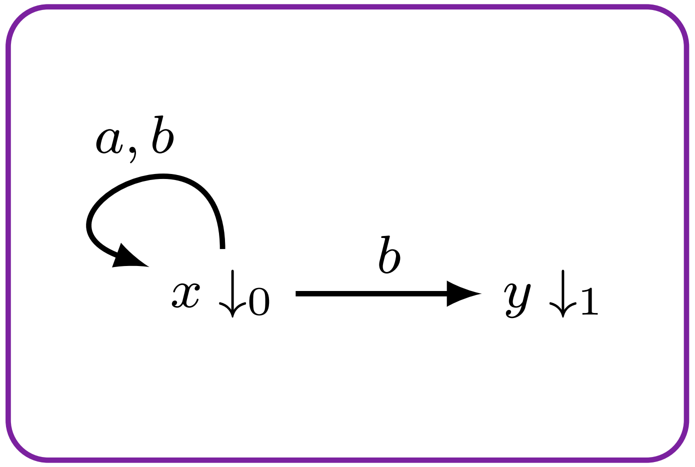
$X \to 2 \times (\mathcal{P}X)^A$



Traces via determinisation

NFA

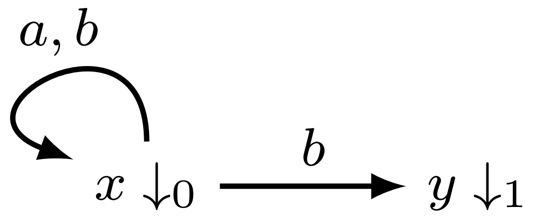
$X \to 2 \times (\mathcal{P}X)^A$



Traces via determinisation

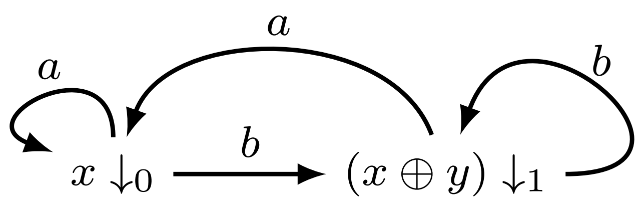
NFA

$X \to 2 \times (\mathcal{P}X)^A$



DFA

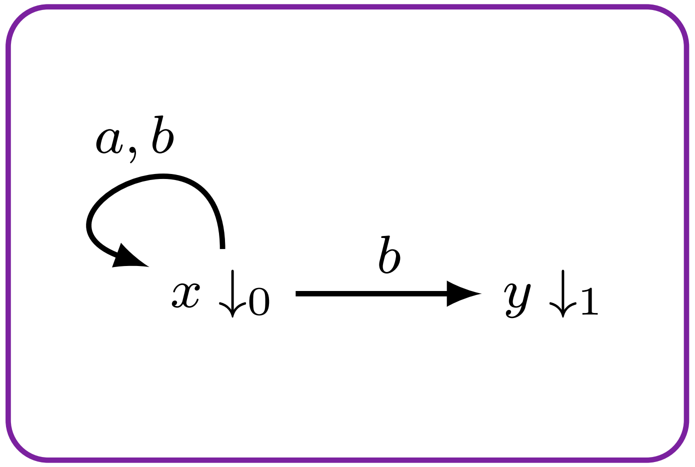
$\mathcal{P}X \to 2 \times (\mathcal{P}X)^A$



Traces via determinisation

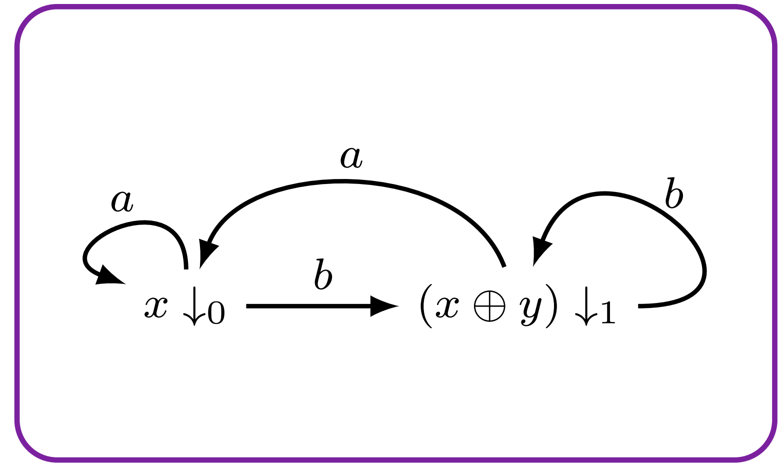
NFA

$X \to 2 \times (\mathcal{P}X)^A$



DFA

$\mathcal{P}X \to 2 \times (\mathcal{P}X)^A$



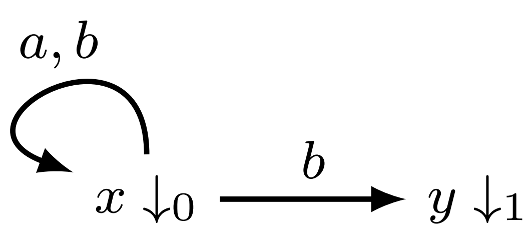
$$\frac{x \xrightarrow{a} t_x,\ y \xrightarrow{a} t_y}{x \oplus y \xrightarrow{a} t_x \oplus t_y}$$

$$\frac{x{\downarrow}o_x,\ y{\downarrow}o_y}{x \oplus y \downarrow o_x \oplus o_y}$$

Traces via determinisation

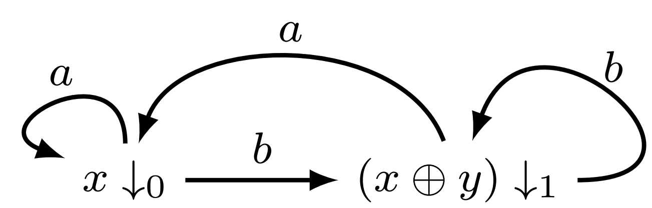
NFA

X → 2 x ($\mathcal{P}$X)^A



DFA

$\mathcal{P}$X → 2 x ($\mathcal{P}$X)^A



$$\frac{x \xrightarrow{a} t_x,\ y \xrightarrow{a} t_y}{x \oplus y \xrightarrow{a} t_x \oplus t_y}$$

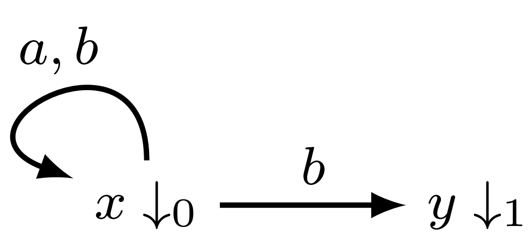
$$\frac{x{\downarrow}o_x,\ y{\downarrow}o_y}{x \oplus y \downarrow o_x \oplus o_y}$$

Algebras for $\mathcal{P}$

Traces via determinisation

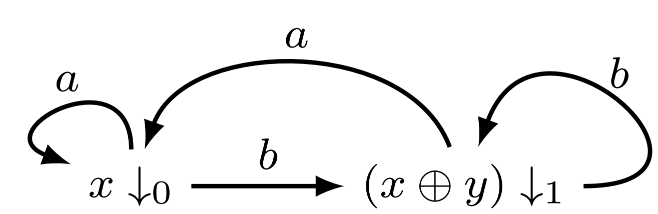
NFA

$X \to 2 \times (\mathcal{P}X)^A$



DFA

$\mathcal{P}X \to 2 \times (\mathcal{P}X)^A$



$$\frac{x \xrightarrow{a} t_x,\ y \xrightarrow{a} t_y}{x \oplus y \xrightarrow{a} t_x \oplus t_y}$$

$$\frac{x{\downarrow}o_x,\ y{\downarrow}o_y}{x \oplus y \downarrow o_x \oplus o_y}$$

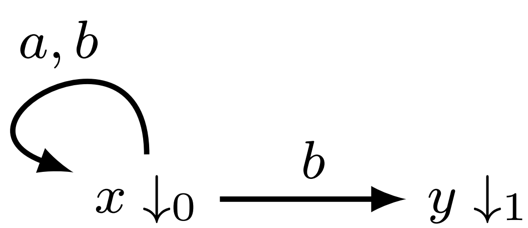
Algebras for $\mathcal{P}$

join semilattices with bottom

Traces via determinisation

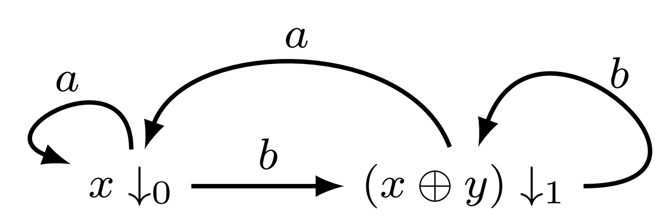
NFA

$X \to 2 \times (\mathcal{P}X)^A$



DFA

$\mathcal{P}X \to 2 \times (\mathcal{P}X)^A$



$$\frac{x \xrightarrow{a} t_x,\ y \xrightarrow{a} t_y}{x \oplus y \xrightarrow{a} t_x \oplus t_y}$$

$$\frac{x{\downarrow}o_x,\ y{\downarrow}o_y}{x \oplus y \downarrow o_x \oplus o_y}$$

Algebras for $\mathcal{P}$

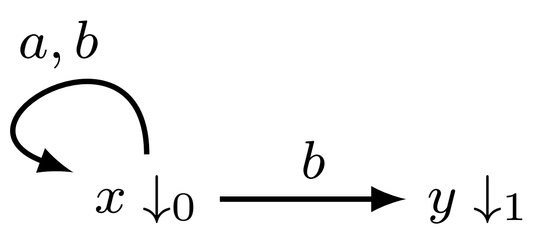
join semilattices with bottom

finite powerset !

Traces via determinisation

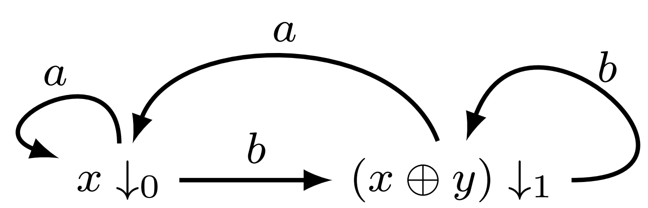
NFA

$X \to 2 \times (\mathcal{P}X)^A$



DFA

$\mathcal{P}X \to 2 \times (\mathcal{P}X)^A$



$$\frac{x \xrightarrow{a} t_x,\ y \xrightarrow{a} t_y}{x \oplus y \xrightarrow{a} t_x \oplus t_y}$$

$$\frac{x{\downarrow}o_x,\ y{\downarrow}o_y}{x \oplus y \downarrow o_x \oplus o_y}$$

Algebras for $\mathcal{P}$

join semilattices with bottom

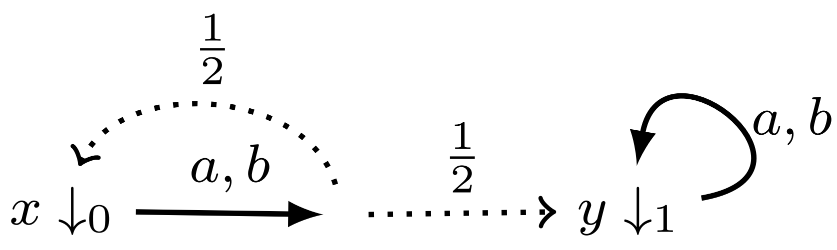
finite powerset !

$2 = \mathcal{P}1$

Traces via determinisation

Rabin PA

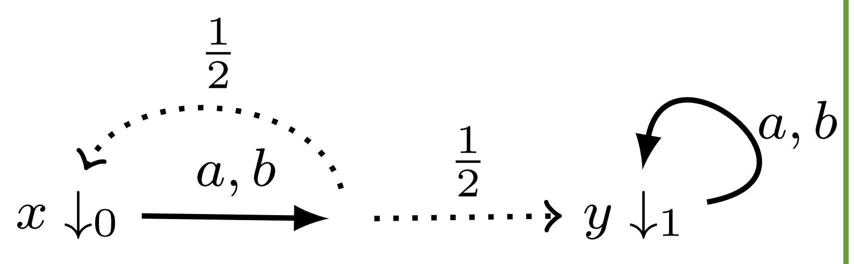
$X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



Traces via determinisation

Rabin PA

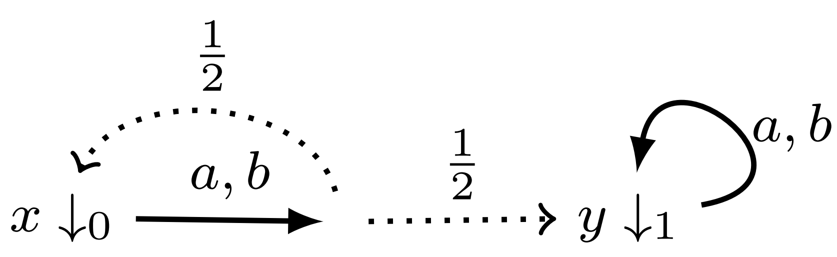
$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



Traces via determinisation

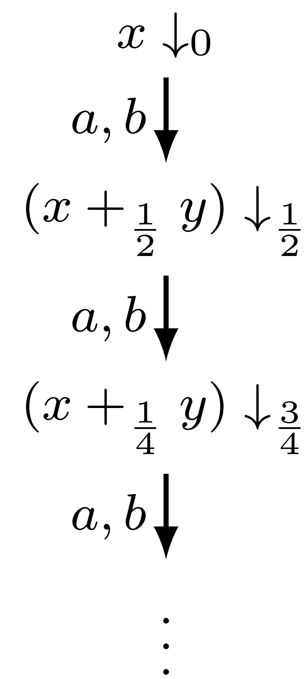
Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



DPA

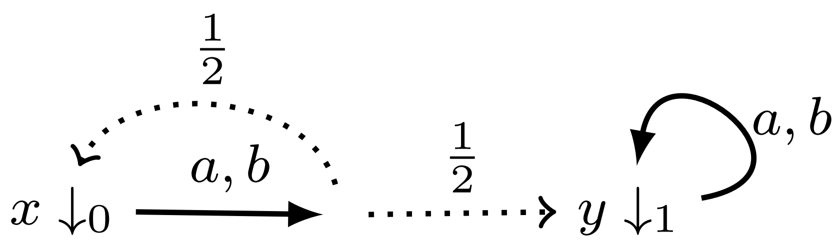
$\mathcal{D}_{\leq 1}X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



Traces via determinisation

Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



DPA

$\mathcal{D}_{\leq 1}X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$

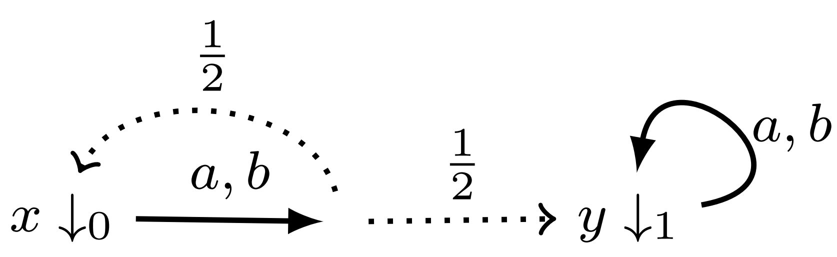
$$x \downarrow_0$$
$$\xrightarrow{a,b}$$
$$(x +_{\frac{1}{2}} y) \downarrow_{\frac{1}{2}}$$
$$\xrightarrow{a,b}$$
$$(x +_{\frac{1}{4}} y) \downarrow_{\frac{3}{4}}$$
$$\xrightarrow{a,b}$$
$$\vdots$$

Algebras for $\mathcal{D}_{\leq 1}$

Traces via determinisation

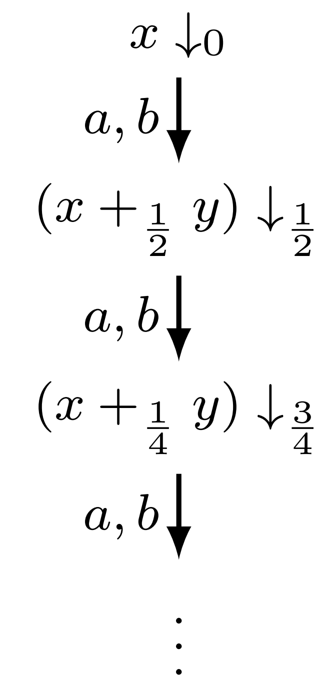
Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



DPA

$\mathcal{D}_{\leq 1} X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



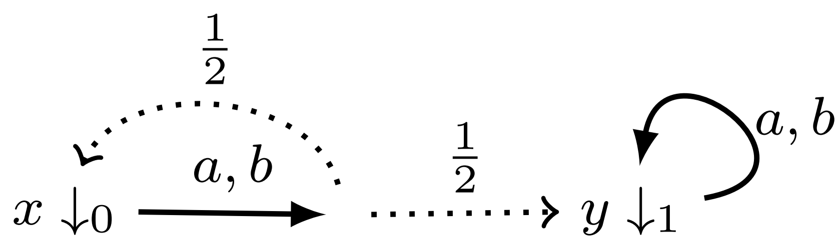
Algebras for $\mathcal{D}_{\leq 1}$

positive convex algebras

Traces via determinisation

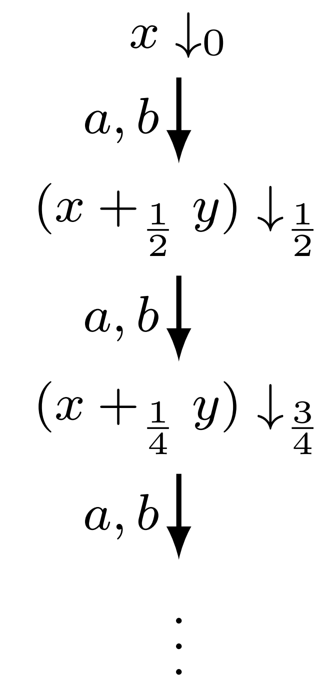
Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



DPA

$\mathcal{D}_{\leq 1}X \to [0,1] \times (\mathcal{D}_{\leq 1}X)^A$



Algebras for $\mathcal{D}_{\leq 1}$

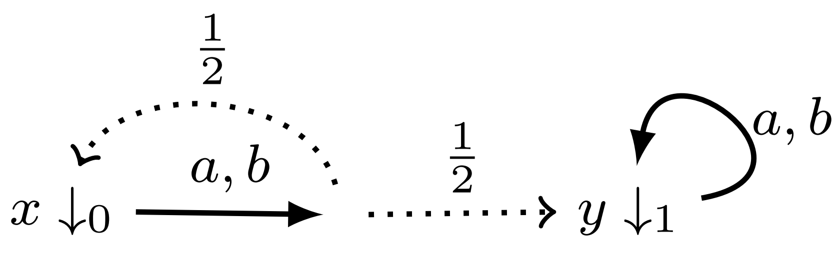
positive convex algebras

finitely supported subdistributions!

Traces via determinisation

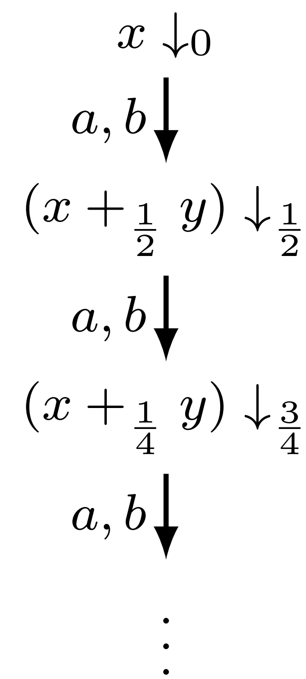
Rabin PA

$X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



DPA

$\mathcal{D}_{\leq 1} X \to [0,1] \times (\mathcal{D}_{\leq 1} X)^A$



Algebras for $\mathcal{D}_{\leq 1}$

positive convex algebras

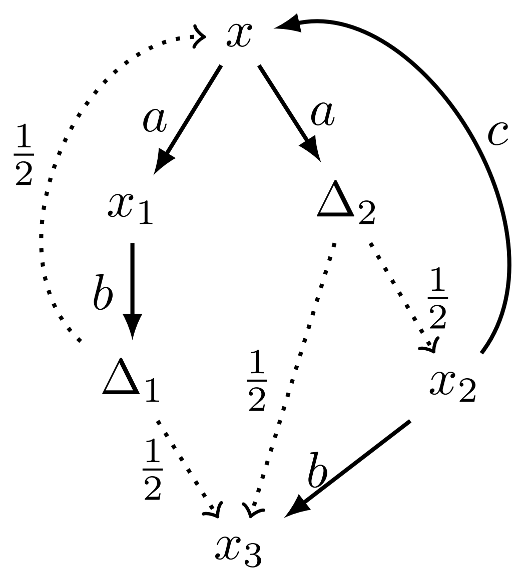
finitely supported subdistributions!

$[0,1] = \mathcal{D}_{\leq 1} 1$

Traces via determinisation

Simple NPA

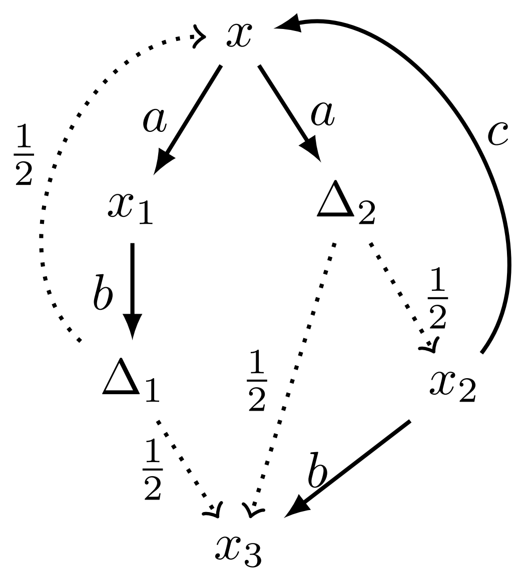
$X \to ? \times (\mathcal{C}X)^A$

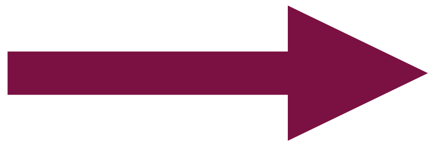


Traces via determinisation

Simple NPA

X → ? x ($\mathcal{C}$X)^A

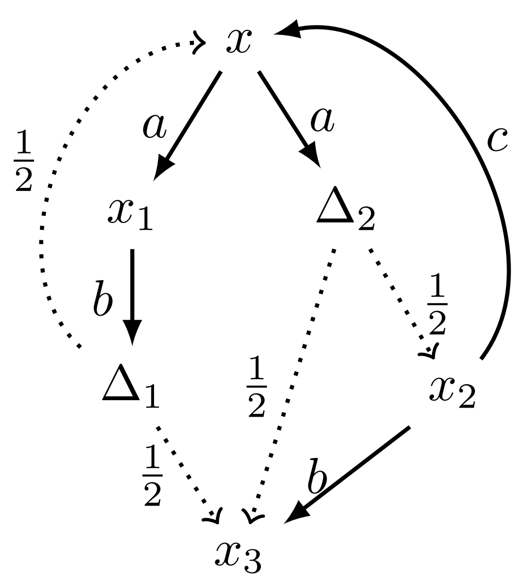




Traces via determinisation

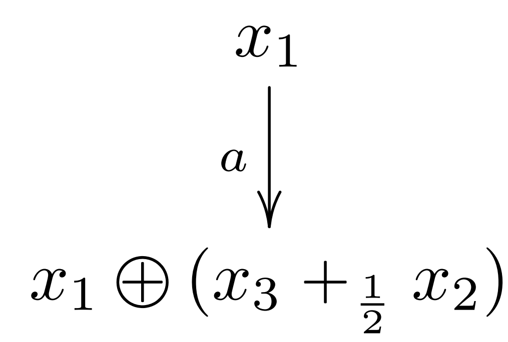
Simple NPA

$X \to ? \times (\mathcal{C}X)^A$



DNPA

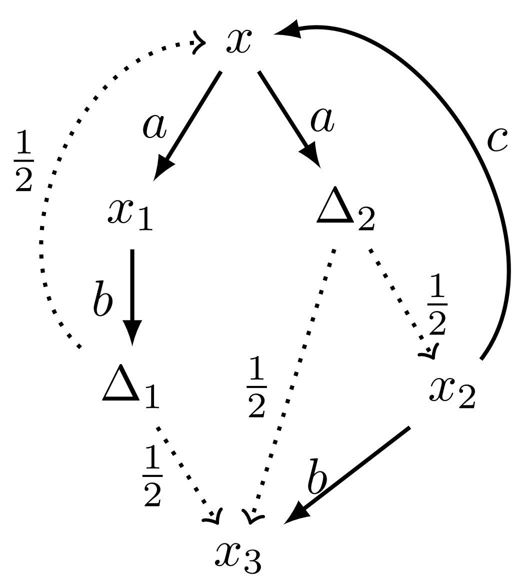
$\mathcal{C}X \to ? \times (\mathcal{C}X)^A$



Traces via determinisation

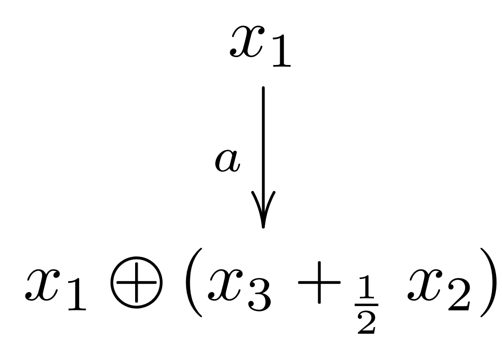
Simple NPA

$X \to ? \times (\mathcal{C}X)^A$



DNPA

$\mathcal{C}X \to ? \times (\mathcal{C}X)^A$

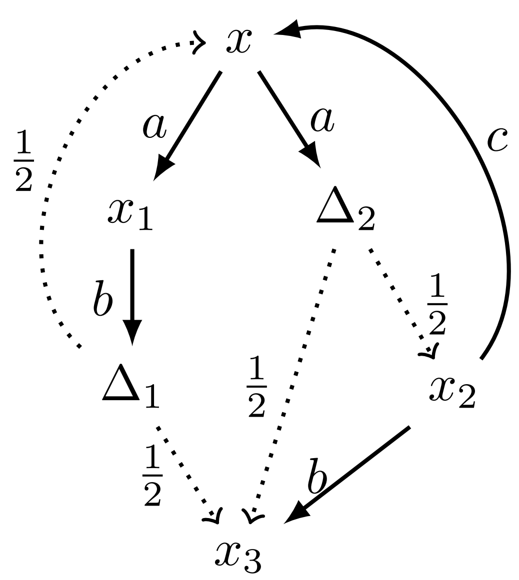


Algebras for C

Traces via determinisation

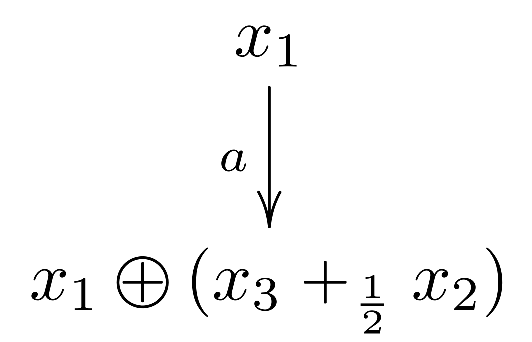
Simple NPA

$X \to ? \times (\mathcal{C}X)^A$



DNPA

$\mathcal{C}X \to ? \times (\mathcal{C}X)^A$



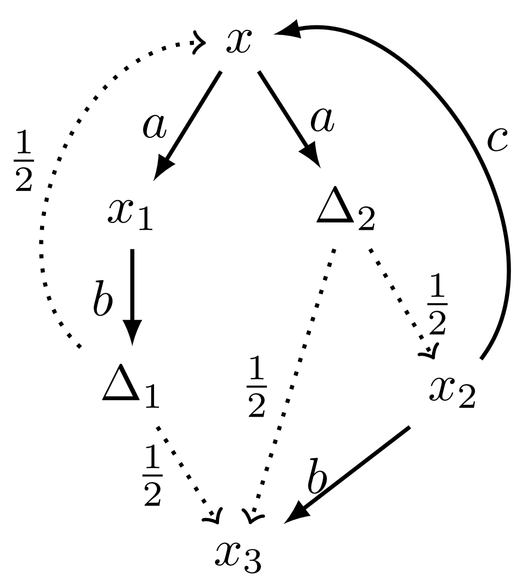
Algebras for C

convex semilattices

Traces via determinisation

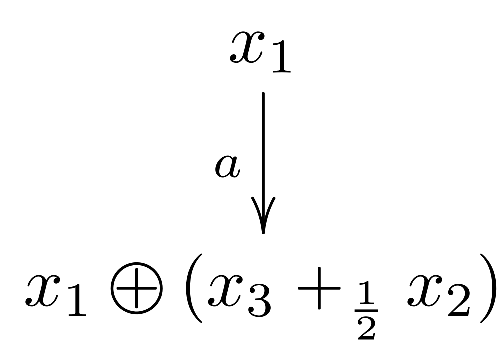
Simple NPA

X → ? x ($\mathcal{C}$X)^A



DNPA

$\mathcal{C}$X → ? x ($\mathcal{C}$X)^A



Algebras for C

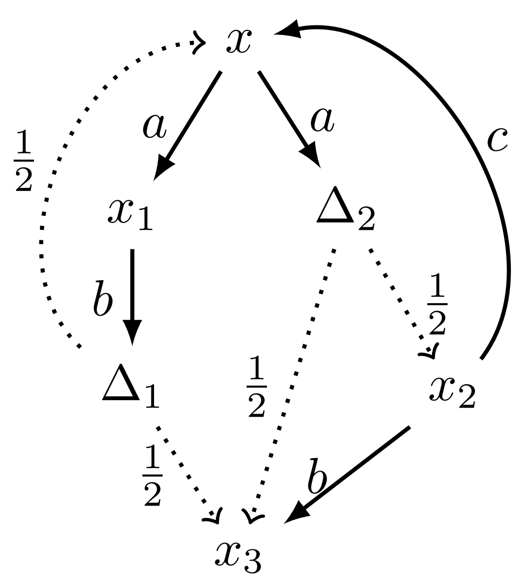
convex semilattices

finitely generated convex sets of distr…

Traces via determinisation

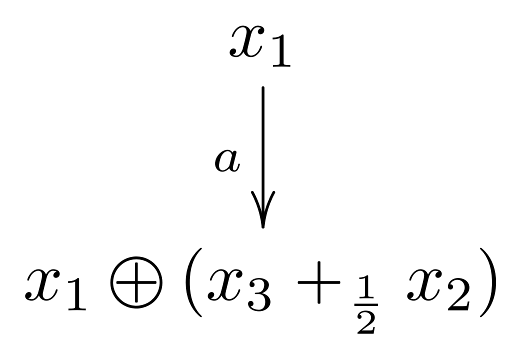
Simple NPA

$X \to ? \times (\mathcal{C}X)^A$



DNPA

$\mathcal{C}X \to ? \times (\mathcal{C}X)^A$



$? = \mathcal{C}1$

Algebras for C

convex semilattices

finitely generated convex sets of distr…

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

finitely generated convex sets of distr…

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

$$\mathbb{A} = (A, \oplus, +_p)$$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

Bonchi, S., Vignudelli '19

finitely generated convex sets of distr…

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

$$(x \oplus y) \oplus z \stackrel{(A)}{=} x \oplus (y \oplus z)$$

$$x \oplus y \stackrel{(C)}{=} y \oplus x$$

$$x \oplus x \stackrel{(I)}{=} x$$

$$(x +_q y) +_p z \stackrel{(A_p)}{=} x +_{pq} (y +_{\frac{p(1-q)}{1-pq}} z)$$

$$x +_p y \stackrel{(C_p)}{=} y +_{1-p} x$$

$$x +_p x \stackrel{(I_p)}{=} x$$

$$(x \oplus y) +_p z \stackrel{(D)}{=} (x +_p z) \oplus (y +_p z)$$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

semilattice

$$(x \oplus y) \oplus z \overset{(A)}{=} x \oplus (y \oplus z)$$

$$x \oplus y \overset{(C)}{=} y \oplus x$$

$$x \oplus x \overset{(I)}{=} x$$

$$(x +_q y) +_p z \overset{(A_p)}{=} x +_{pq} (y +_{\frac{p(1-q)}{1-pq}} z)$$

$$x +_p y \overset{(C_p)}{=} y +_{1-p} x$$

$$x +_p x \overset{(I_p)}{=} x$$

$$(x \oplus y) +_p z \overset{(D)}{=} (x +_p z) \oplus (y +_p z)$$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

semilattice

$$(x \oplus y) \oplus z \stackrel{(A)}{=} x \oplus (y \oplus z)$$
$$x \oplus y \stackrel{(C)}{=} y \oplus x$$
$$x \oplus x \stackrel{(I)}{=} x$$

convex algebra

$$(x +_q y) +_p z \stackrel{(A_p)}{=} x +_{pq} (y +_{\frac{p(1-q)}{1-pq}} z)$$
$$x +_p y \stackrel{(C_p)}{=} y +_{1-p} x$$
$$x +_p x \stackrel{(I_p)}{=} x$$

$$(x \oplus y) +_p z \stackrel{(D)}{=} (x +_p z) \oplus (y +_p z)$$

Presentation for $\mathscr{C}$

Algebras for $\mathscr{C}$

convex semilattices

finitely generated convex sets of distr…

Bonchi, S., Vignudelli '19

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

$$(x \oplus y) \oplus z \stackrel{(A)}{=} x \oplus (y \oplus z)$$
$$x \oplus y \stackrel{(C)}{=} y \oplus x$$
$$x \oplus x \stackrel{(I)}{=} x$$

semilattice

S., Woracek '15, '17, '18

$$(x +_q y) +_p z \stackrel{(A_p)}{=} x +_{pq} (y +_{\frac{p(1-q)}{1-pq}} z)$$
$$x +_p y \stackrel{(C_p)}{=} y +_{1-p} x$$
$$x +_p x \stackrel{(I_p)}{=} x$$

convex algebra

$$(x \oplus y) +_p z \stackrel{(D)}{=} (x +_p z) \oplus (y +_p z)$$

Presentation for $\mathcal{C}$

Algebras for $\mathcal{C}$

convex semilattices

Bonchi, S., Vignudelli '19

finitely generated convex sets of distr…

$$\mathbb{A} = (A, \oplus, +_p)$$

$p \in (0,1)$

$$(x \oplus y) \oplus z \stackrel{(A)}{=} x \oplus (y \oplus z)$$
$$x \oplus y \stackrel{(C)}{=} y \oplus x$$
$$x \oplus x \stackrel{(I)}{=} x$$

semilattice

S., Woracek '15, '17, '18

$$(x +_q y) +_p z \stackrel{(A_p)}{=} x +_{pq} (y +_{\frac{p(1-q)}{1-pq}} z)$$
$$x +_p y \stackrel{(C_p)}{=} y +_{1-p} x$$
$$x +_p x \stackrel{(I_p)}{=} x$$

convex algebra

$$(x \oplus y) +_p z \stackrel{(D)}{=} (x +_p z) \oplus (y +_p z)$$

distributivity

Three variants for "$\mathcal{C}$"

Three variants for “$\mathscr{C}$”

Bonchi, S.,
Vignudelli ‘19

Three variants for "$\mathcal{C}$"

Bonchi, S.,
Vignudelli '19

We explore the whole space
and
prove coincidence with "local"
trace semantics

Three variants for “$\mathscr{C}$”

Algebras for “$\mathscr{C}$”

nonempty f.g. convex subsets of **sub**distr…

Bonchi, S., Vignudelli ‘19

We explore the whole space and prove coincidence with “local” trace semantics

Three variants for "$\mathscr{C}$"

Algebras for "$\mathscr{C}$"

nonempty f.g.
convex subsets of
subdistr…

Bonchi, S.,
Vignudelli '19

We explore the whole space
and
prove coincidence with "local"
trace semantics

l.pointed
convex
semilattices

Intervals in [0,1]
with
min-max
Minkowski
[0,0] = "$\mathscr{C}$"1

Three variants for "$\mathscr{C}$"

Algebras for "$\mathscr{C}$"

nonempty f.g.
convex subsets of
subdistr…

Bonchi, S.,
Vignudelli '19

We explore the whole space
and
prove coincidence with "local"
trace semantics

I.pointed
convex
semilattices

Intervals in [0,1]
with
min-max
Minkowski
[0,0] = "$\mathscr{C}$"1

II.
with bottom

[0,1]
with
max, $+_p$
0 = "$\mathscr{C}$"1

Three variants for "$\mathscr{C}$"

Algebras for "$\mathscr{C}$"

nonempty f.g.
convex subsets of
subdistr…

Bonchi, S.,
Vignudelli '19

We explore the whole space
and
prove coincidence with "local"
trace semantics

I. pointed
convex
semilattices

Intervals in [0,1]
with
min-max
Minkowski
[0,0] = "$\mathscr{C}$"1

II.
with bottom

[0,1]
with
max, $+_p$
0 = "$\mathscr{C}$"1

III.
with top

[0,1]
with
min, $+_p$
0 = "$\mathscr{C}$"1

Three things to take home:

1. Semantics via determinisation
is easy for automata with M-effects

2. Having a presentation for M gives us syntax

3. Having the syntax makes determinisation natural !

Many general properties follow
also a sound
up-to context
proof technique

Three things to take home:

1. Semantics via determinisation
is easy for automata with M-effects

2. Having a presentation for M gives us syntax

3. Having the syntax makes determinisation natural !

Three things to take home:

Many general properties follow
also a sound
up-to context
proof technique

1. Semantics via determinisation
is easy for automata with M-effects

2. Having a presentation for M gives us syntax

3. Having the syntax makes determinisation natural !

combining
nondeterminism
and probability
becomes easy

Many general properties follow
also a sound up-to context proof technique

Three things to take home:

1. Semantics via determinisation
is easy for automata with M-effects

2. Having a presentation for M gives us syntax

3. Having the syntax makes determinisation natural !

combining nondeterminism and probability becomes easy

Thank You !