Local Linearizability

Ana Sokolova UNIVERSITY of SALZBURG

joint work with:

Andreas Haas Google
Andreas Holzer UNIVERSITY OF TORONTO
Michael Lippautz Google
Ali Sezgin UNIVERSITY OF CAMBRIDGE
Tom Henzinger IST AUSTRIA
Christoph Kirsch UNIVERSITY of SALZBURG
Hannes Payer Google
Helmut Veith TU WIEN

Rigorous methods
for
engineering of
and
reasoning about
reactive systems

Rigorous methods
for
engineering of
and
reasoning about
reactive systems

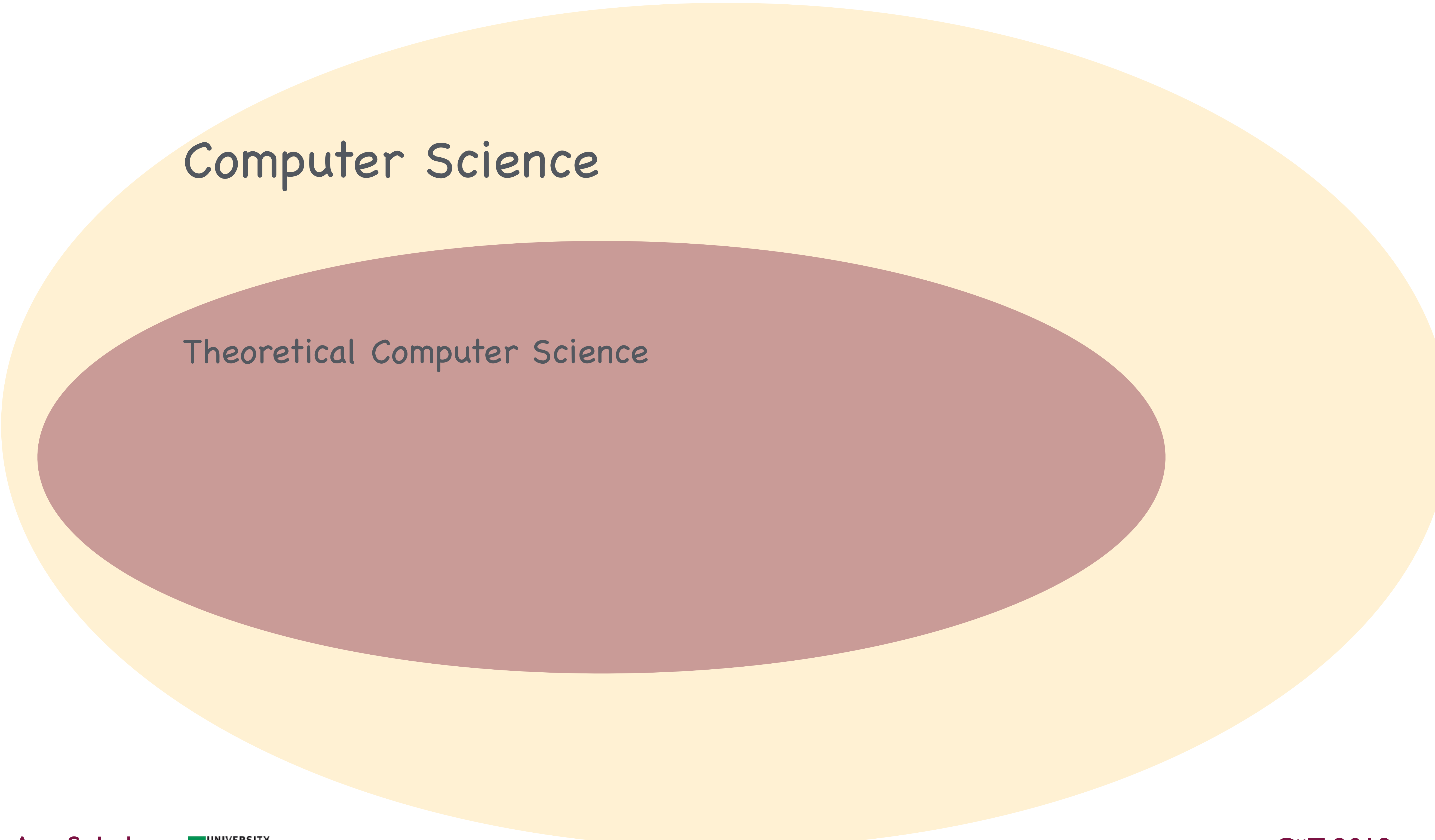
concurrent

Background big picture

Background big picture

Computer Science

Background big picture



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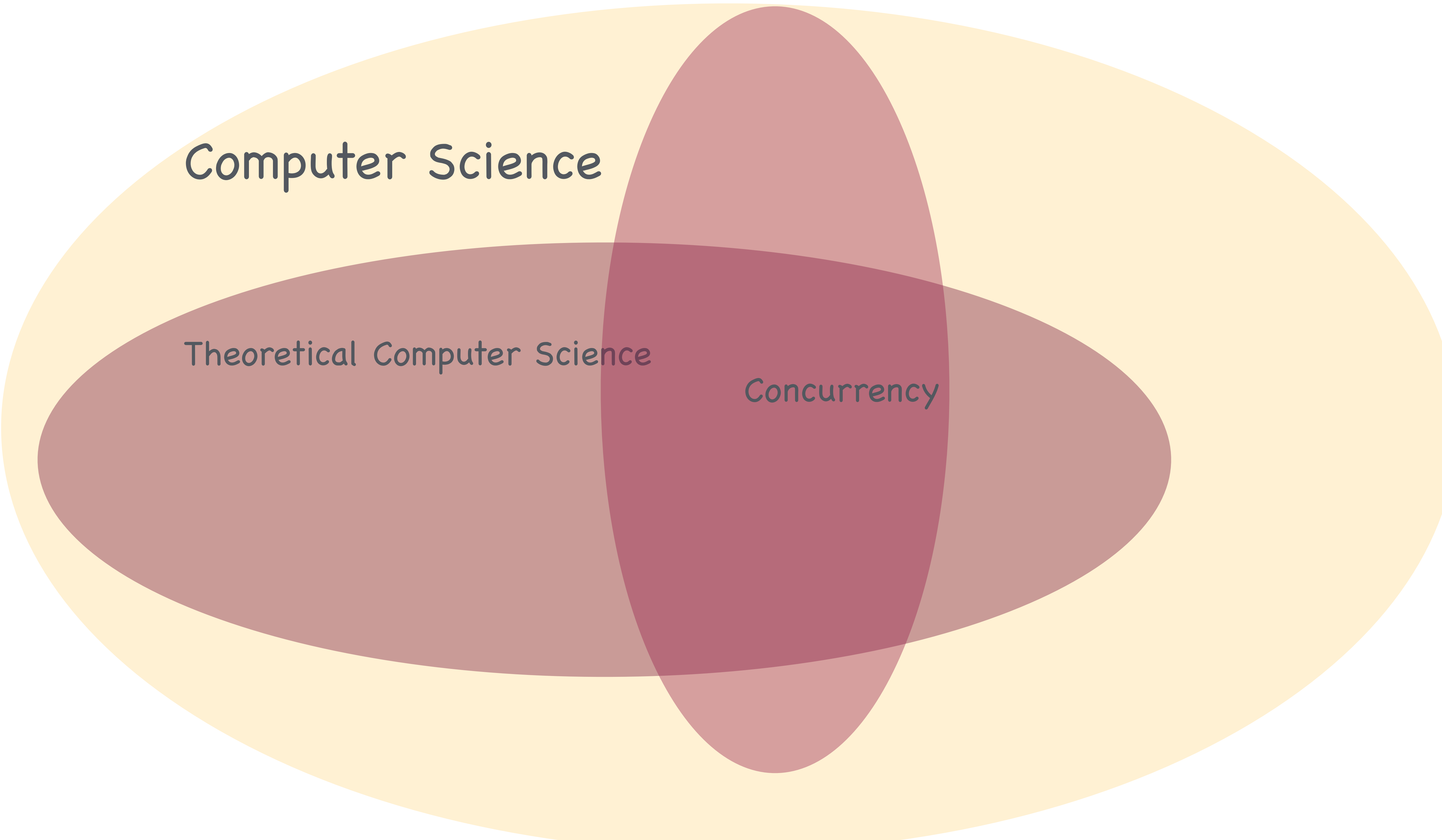

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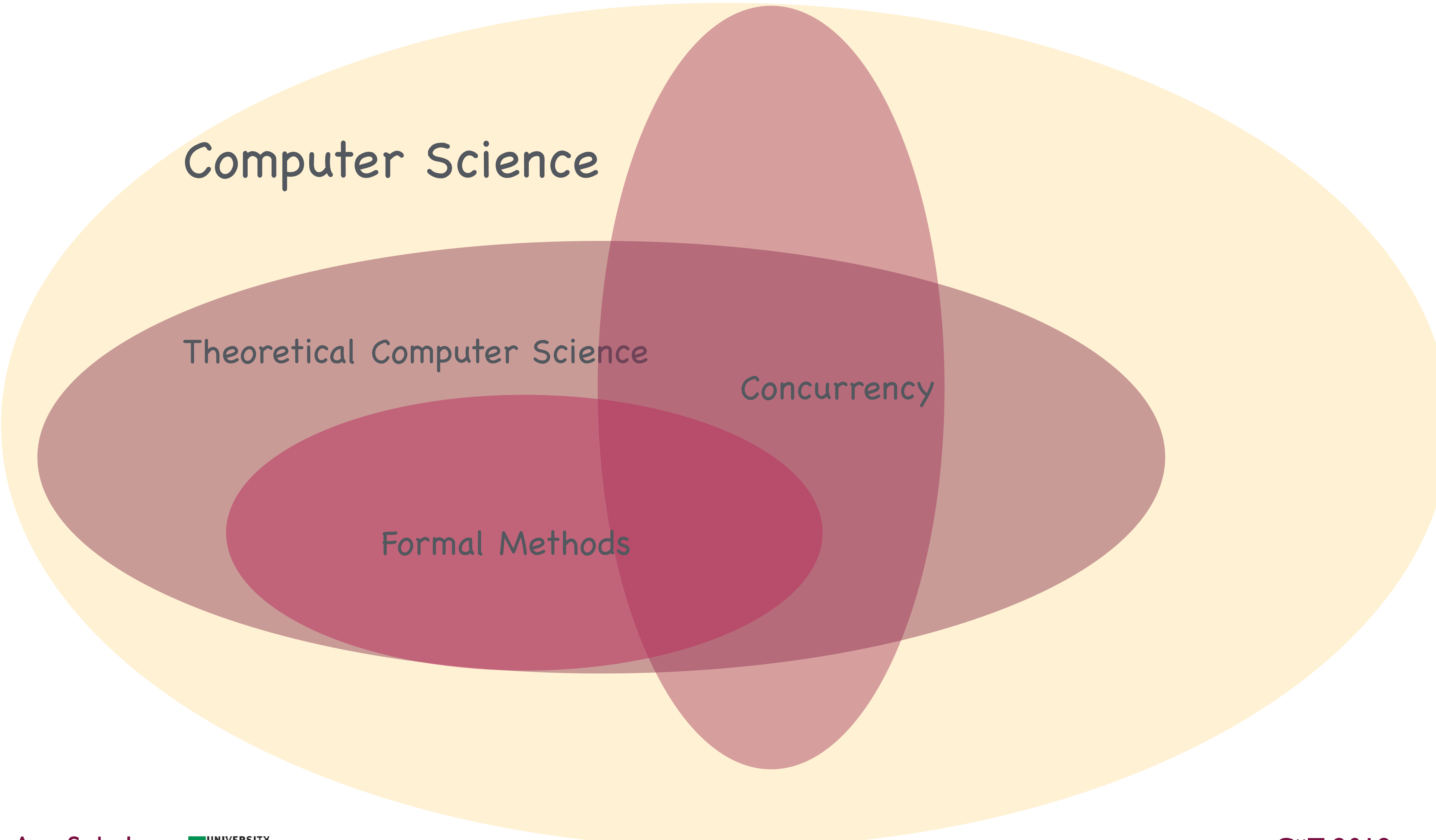

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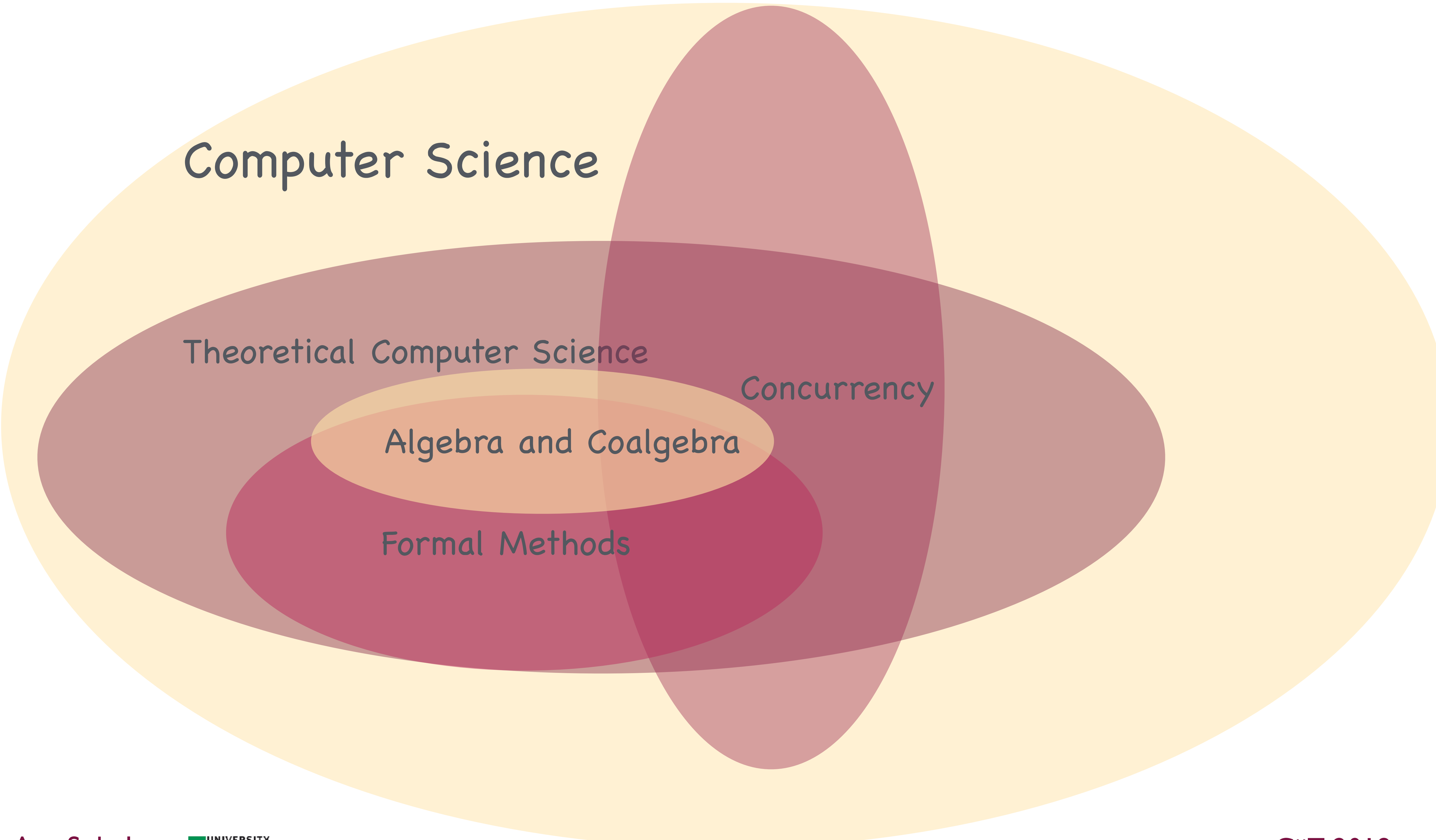

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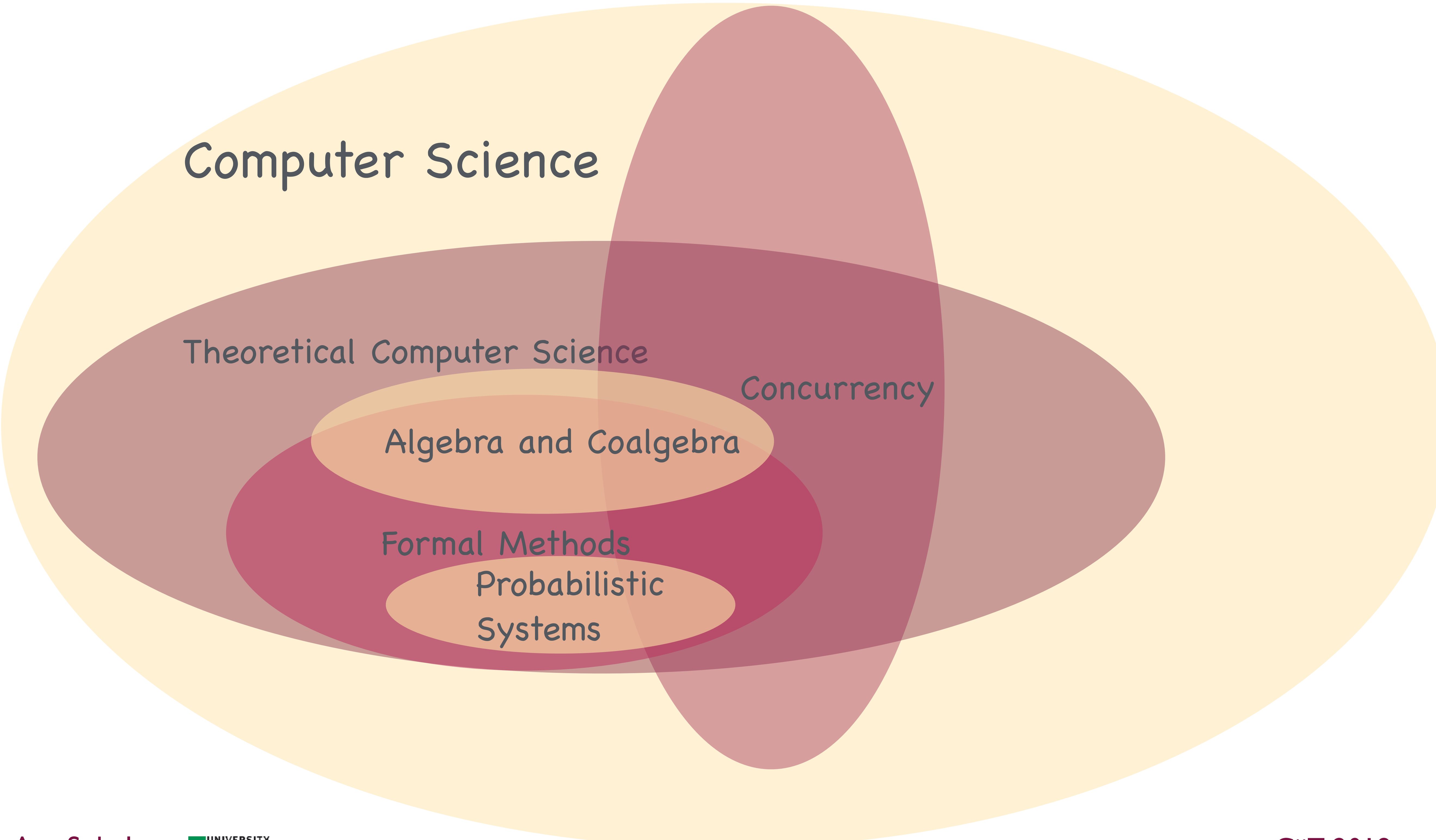

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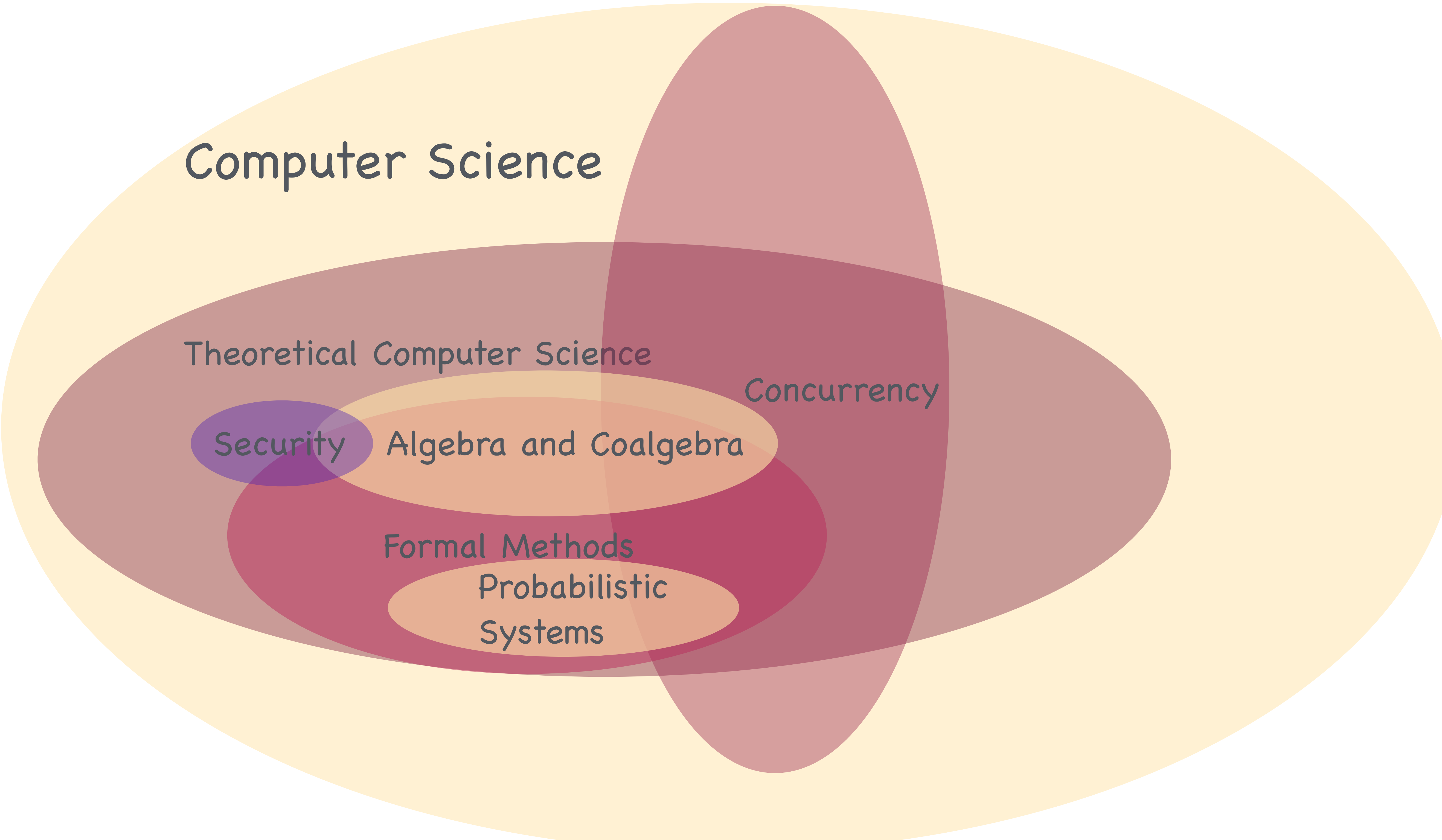

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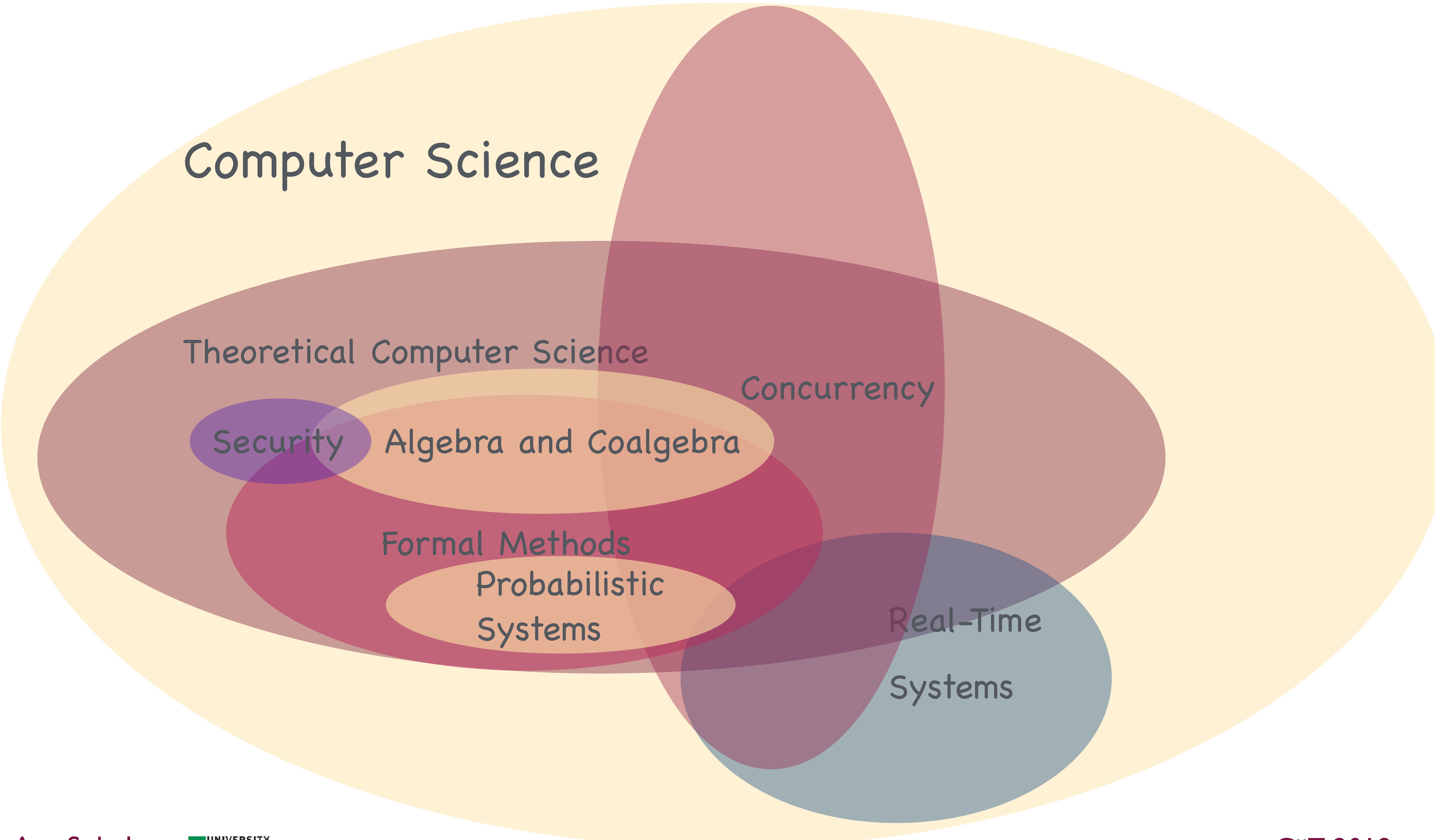

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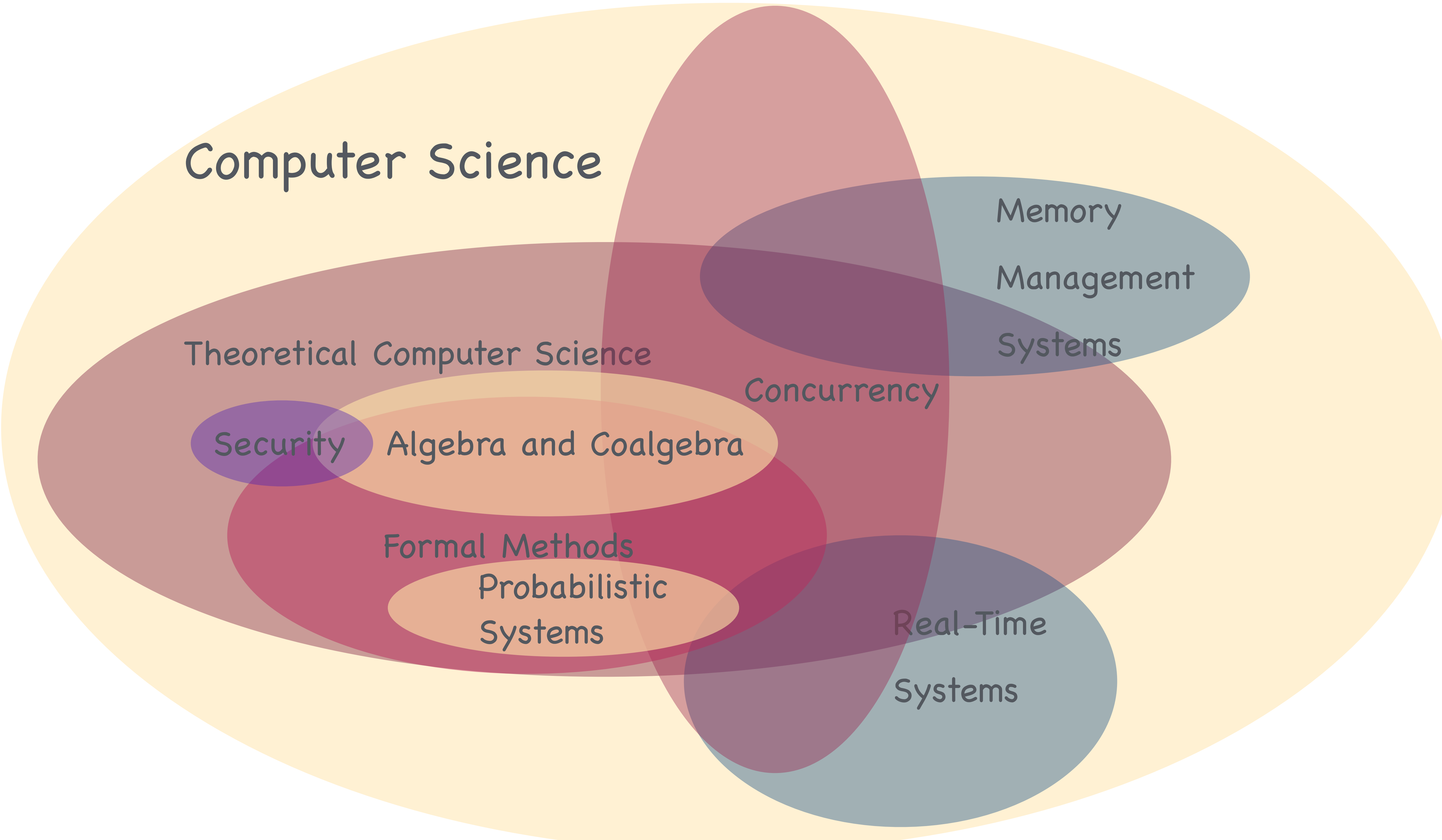

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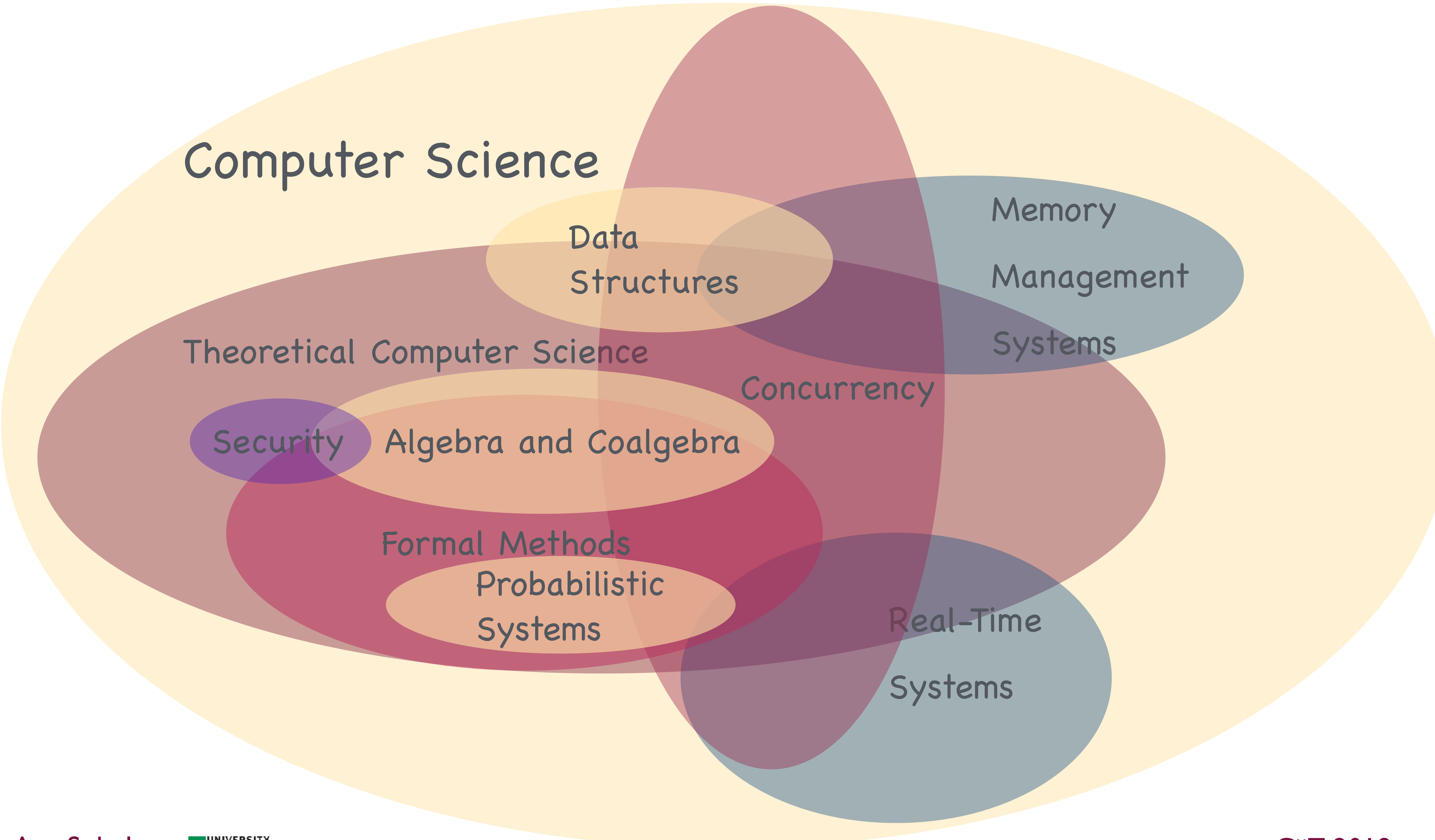

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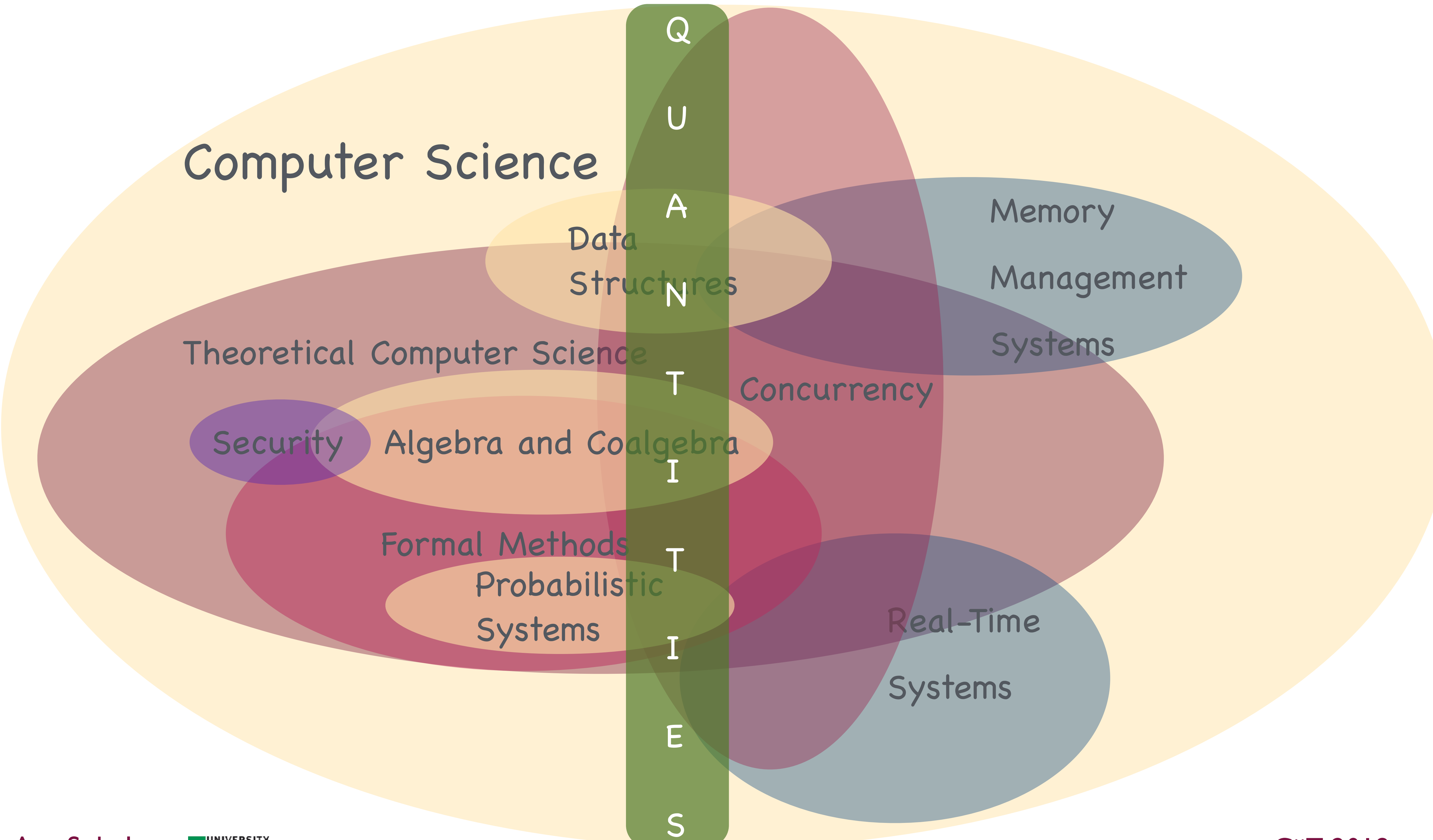
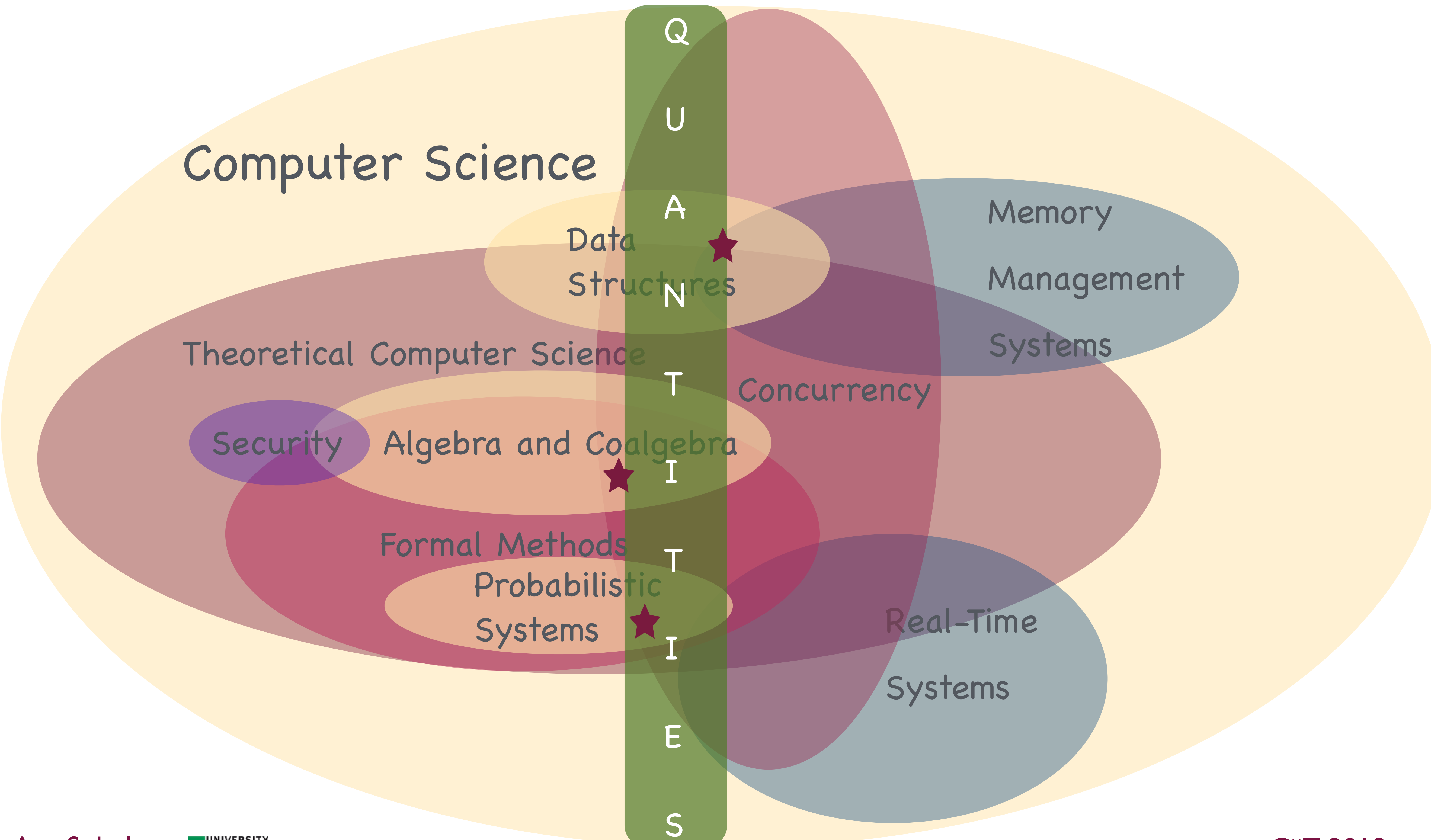


Current favourites



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Concurrent Data Structures: Semantics and Relaxations

Concurrent Data Structures
Correctness and Performance

Concurrent Data Structures
Correctness and Performance

structure and power

Semantics of concurrent data structures

Semantics of concurrent data structures

e.g. pools, queues, stacks

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t1: enq(2) deq(1)

t2: enq(1) deq(2)

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- Consistency condition = e.g. linearizability / sequential consistency

Semantics of concurrent data structures

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e.g. queue legal sequence
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e.g. the concurrent history above is a linearizable queue concurrent history

Consistency conditions

Linearizability [Herlihy,Wing '90]

↓

Sequential Consistency [Lamport'79]

Consistency conditions

there exists a legal sequence that preserves precedence

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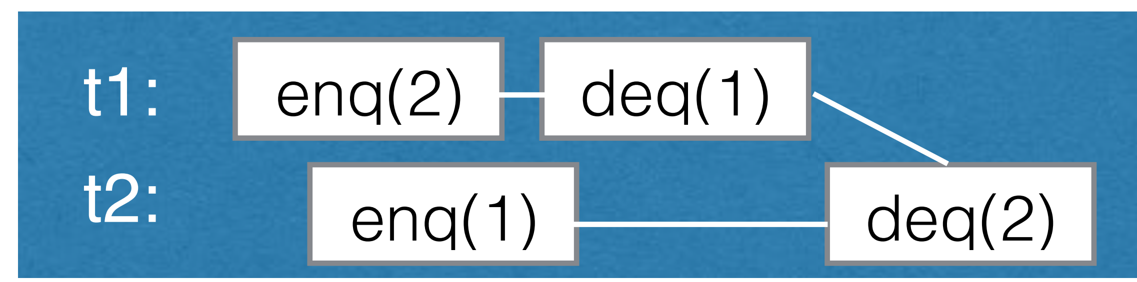

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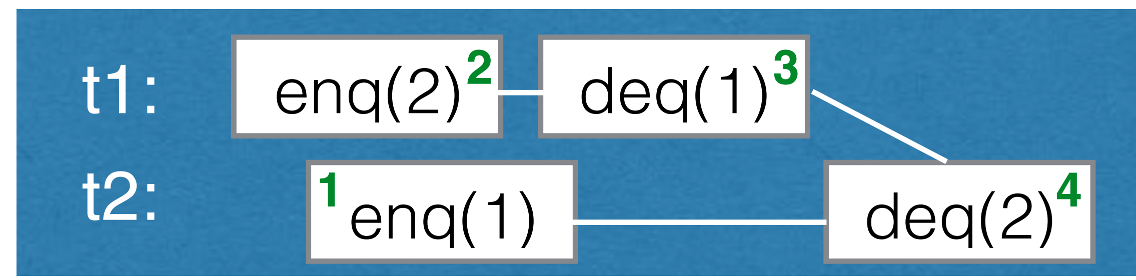

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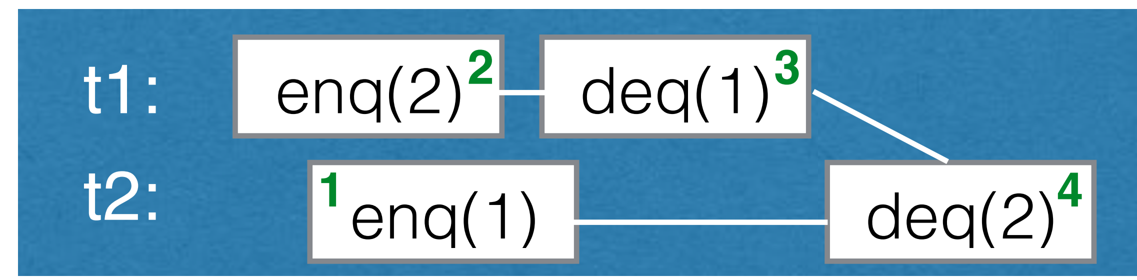

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Sequential Consistency [Lamport'79]

there exists a legal sequence that preserves per-thread precedence (program order)

Consistency conditions

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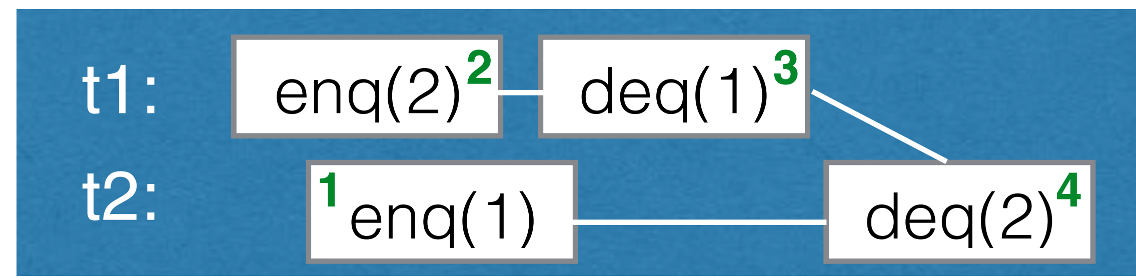

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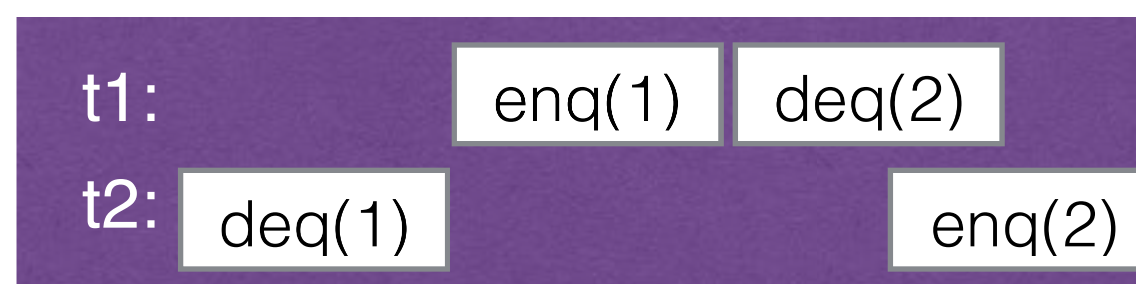

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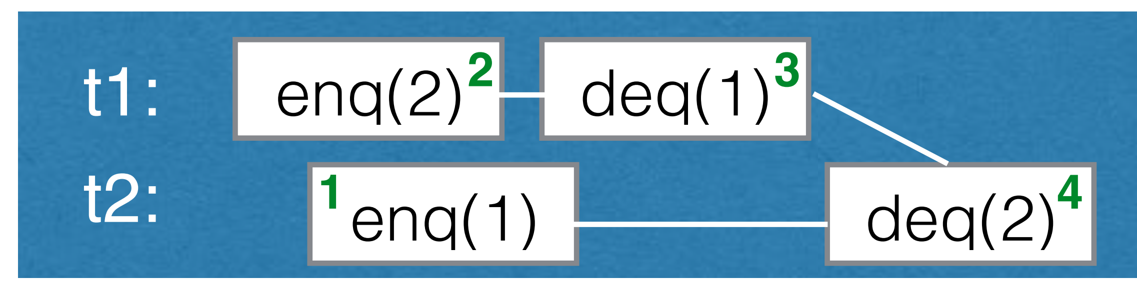

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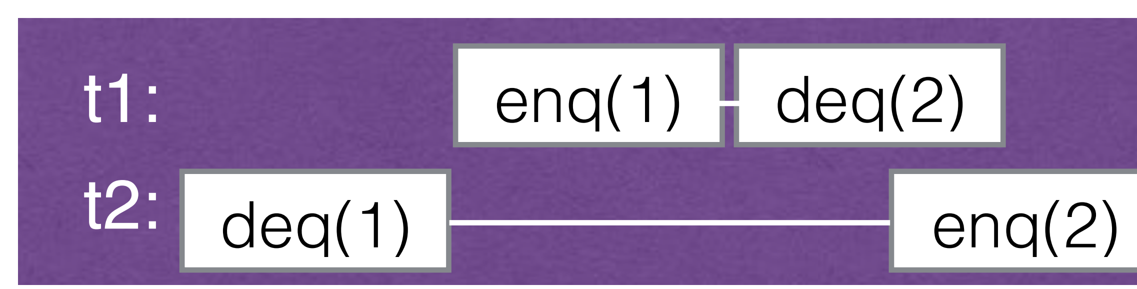

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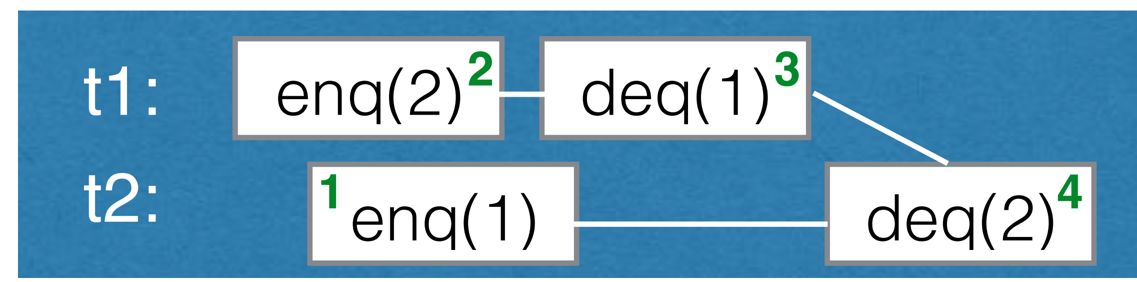

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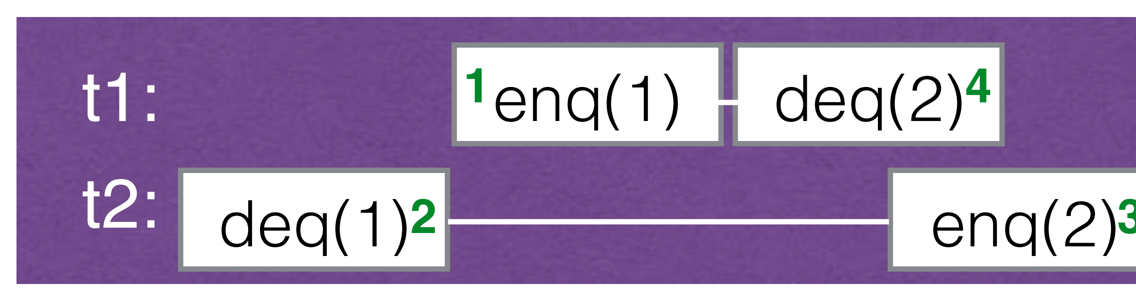
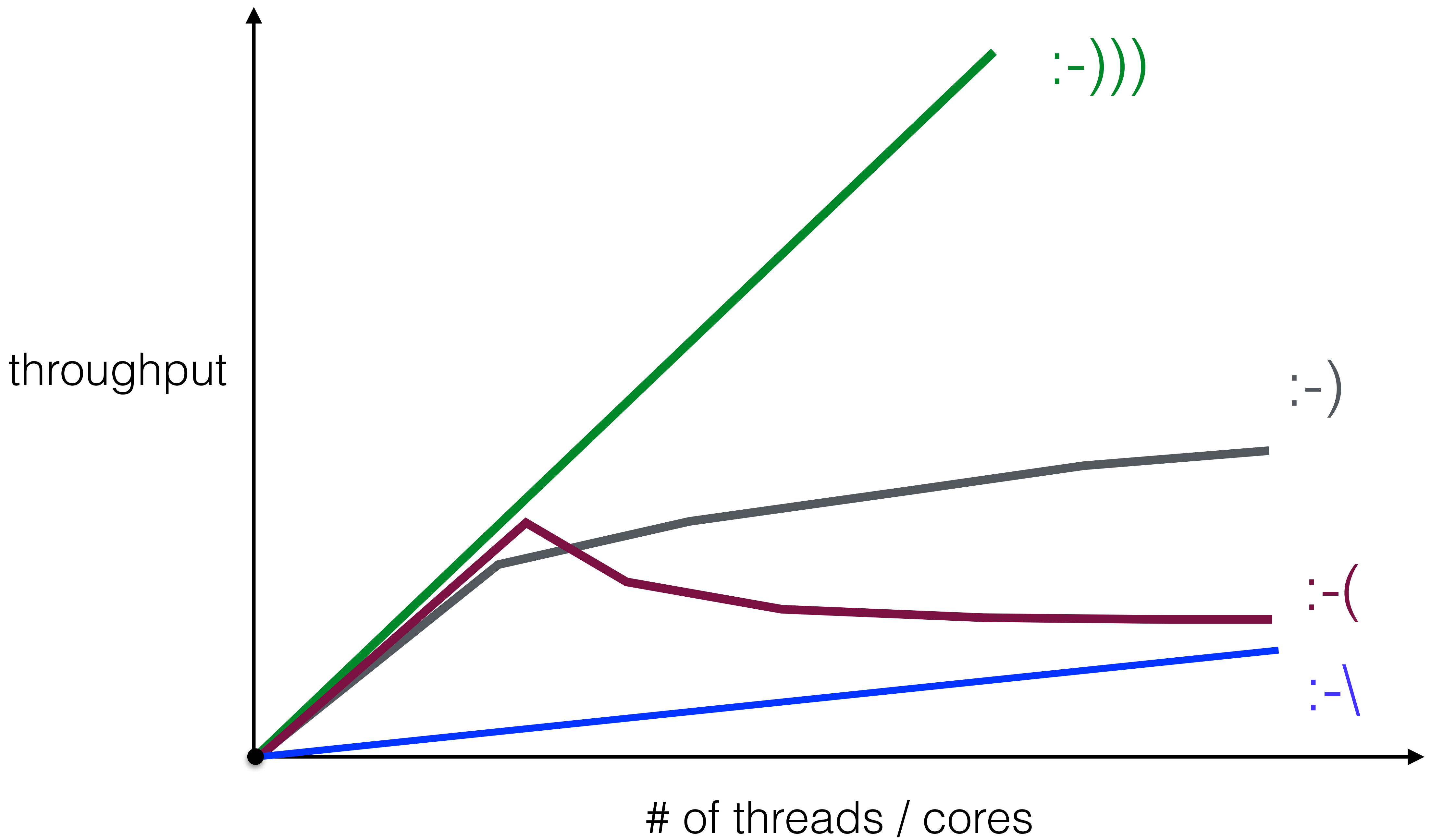


Performance and scalability



Relaxations allow trading

correctness
for
performance

Relaxations allow trading

correctness
for
performance

provide the potential
for better-performing
implementations

Relaxing the Semantics

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- Sequential specification = set of legal sequences
- Consistency condition = e.g. linearizability / sequential consistency

Relaxing the Semantics

Quantitative relaxations
Henzinger, Kirsch, Payer, Sezgin,S. POPL13

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not
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Local linearizability
in this talk

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for queues only
(feel free to ask for more)

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Local linearizability
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too weak

Relaxing the Consistency Condition

Relaxing the Consistency Condition

Local Linearizability
(CONCUR16)

Local Linearizability
main idea

Local Linearizability main idea

- Partition a history into a set of local histories
- Require linearizability per local history

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Already present in some shared-memory consistency conditions
(not in our form of choice)

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Local sequential consistency… is also possible

Local Linearizability main idea

Already present in some shared-memory consistency conditions
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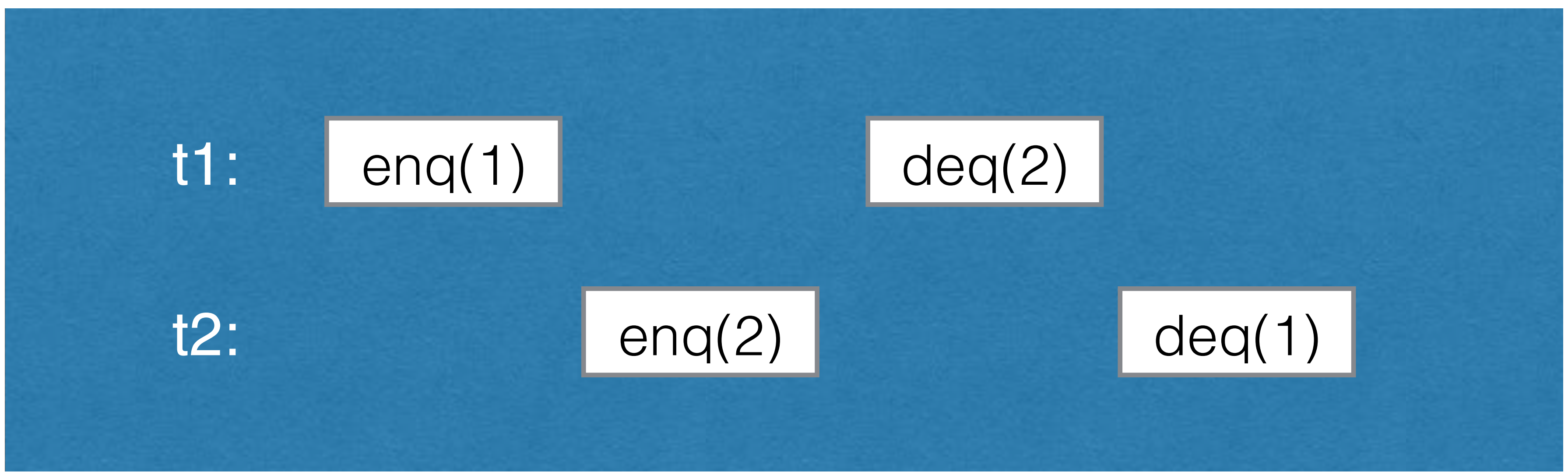
- Partition a history into a set of local histories
- Require linearizability per local history

no global witness

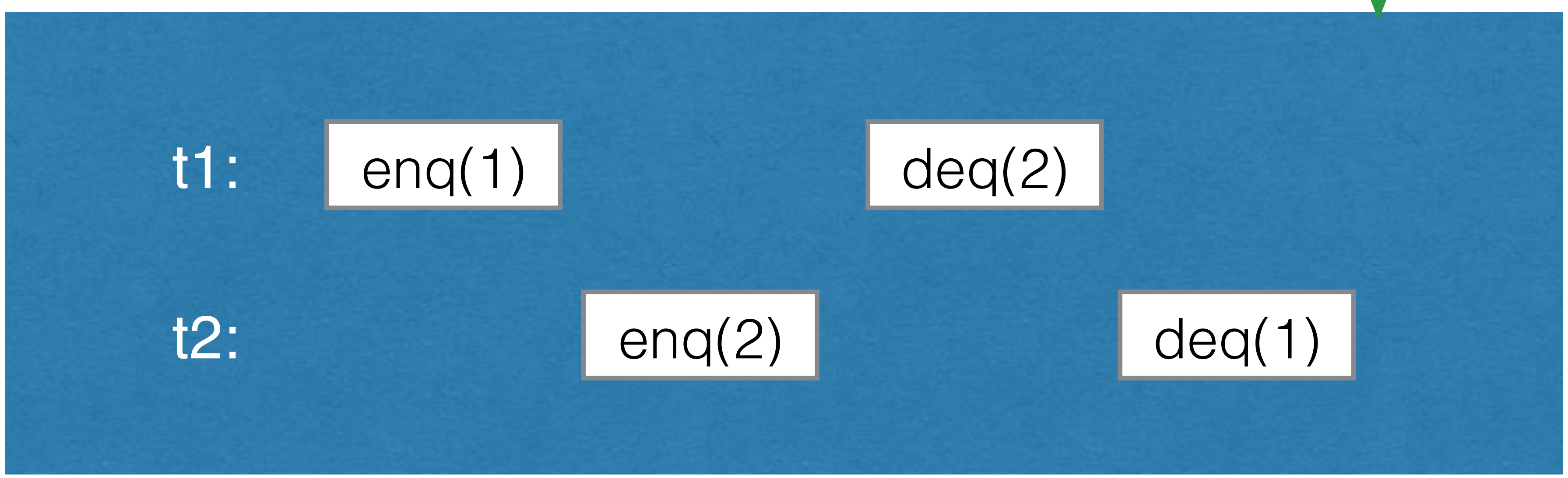
Local sequential consistency… is also possible

Local Linearizability (queue) example



Local Linearizability (queue) example

(sequential) history not linearizable



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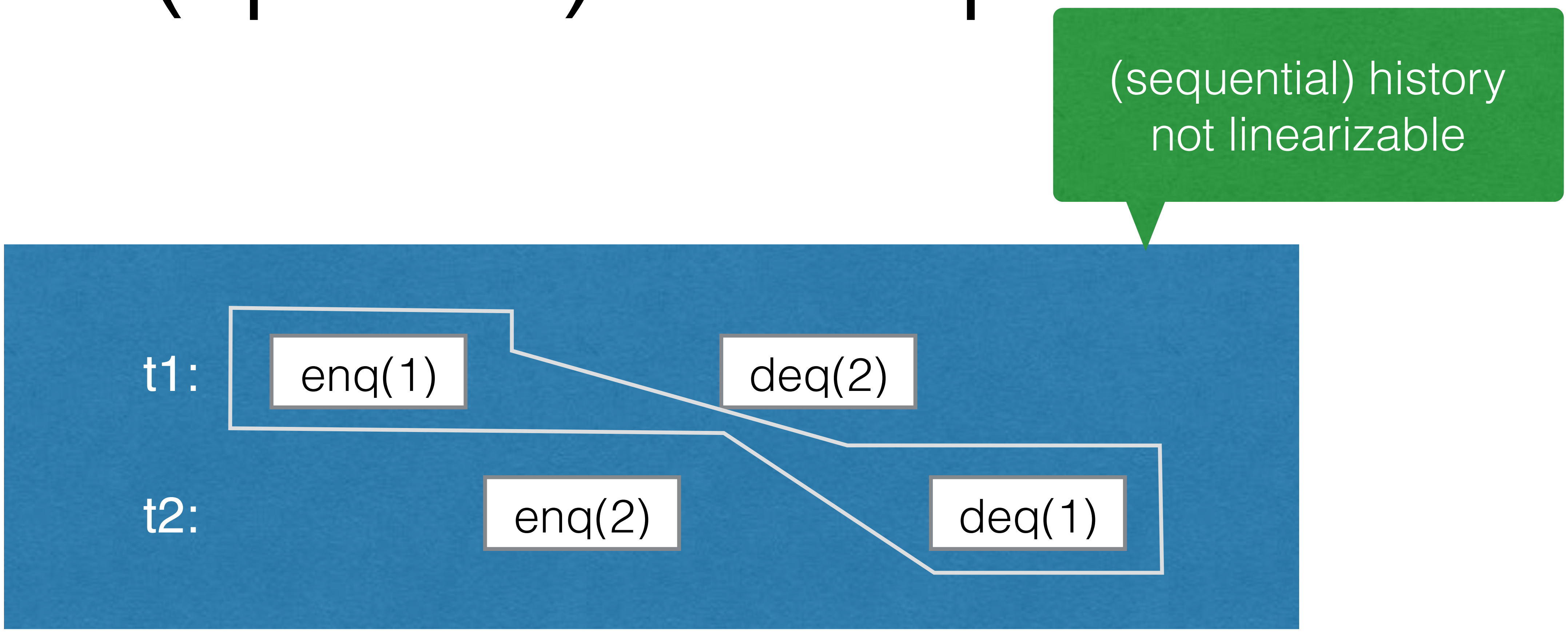

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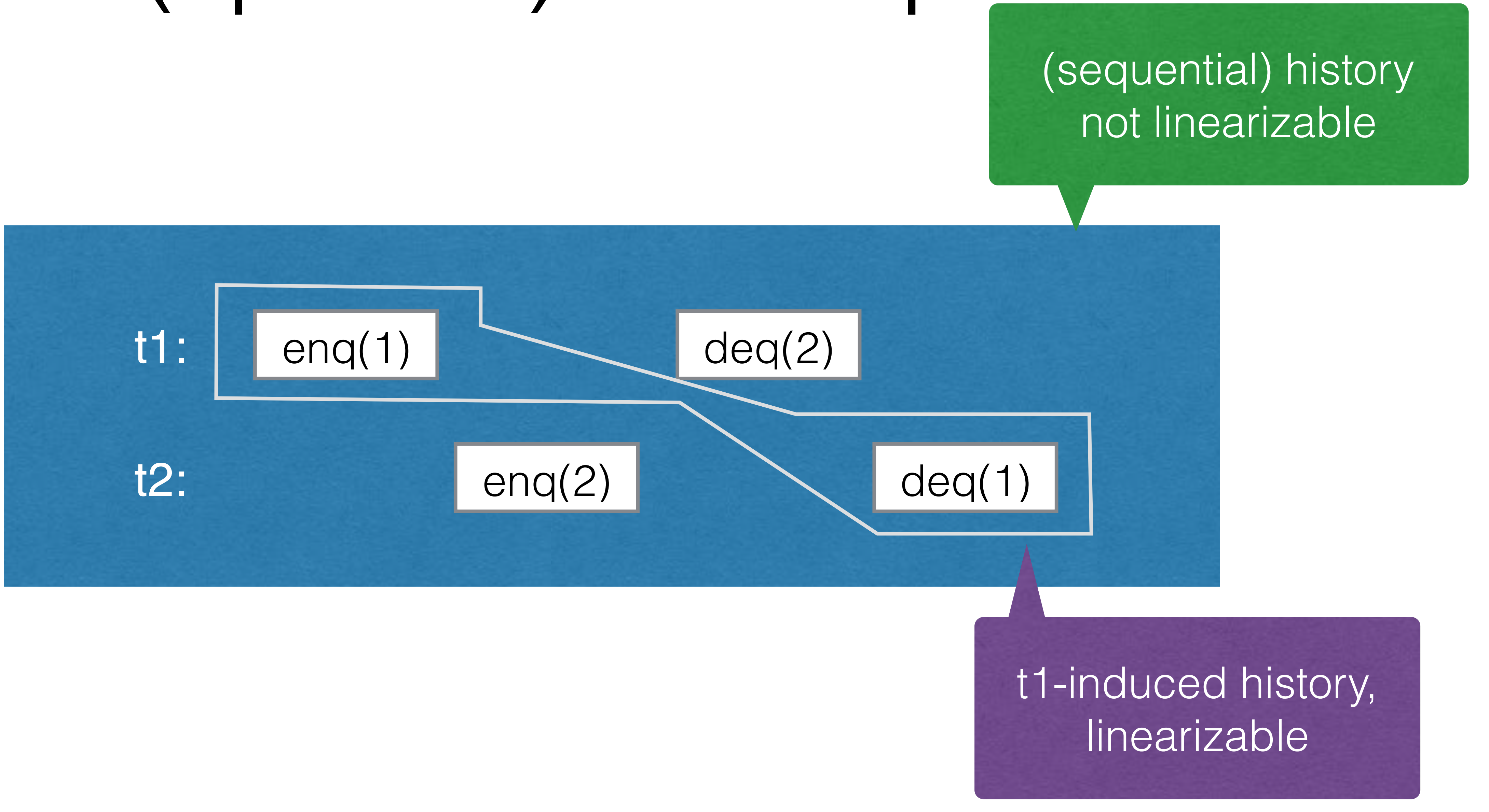

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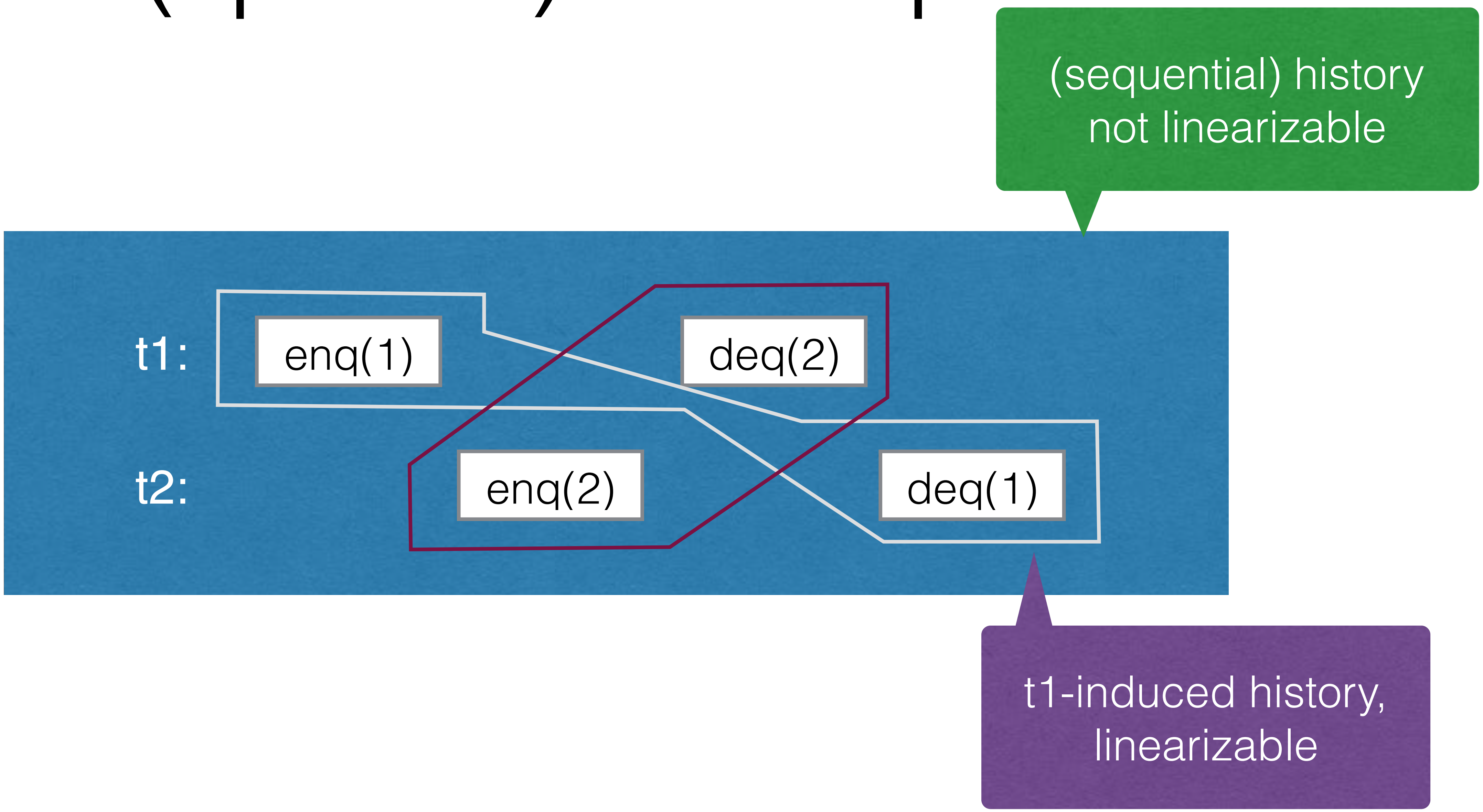

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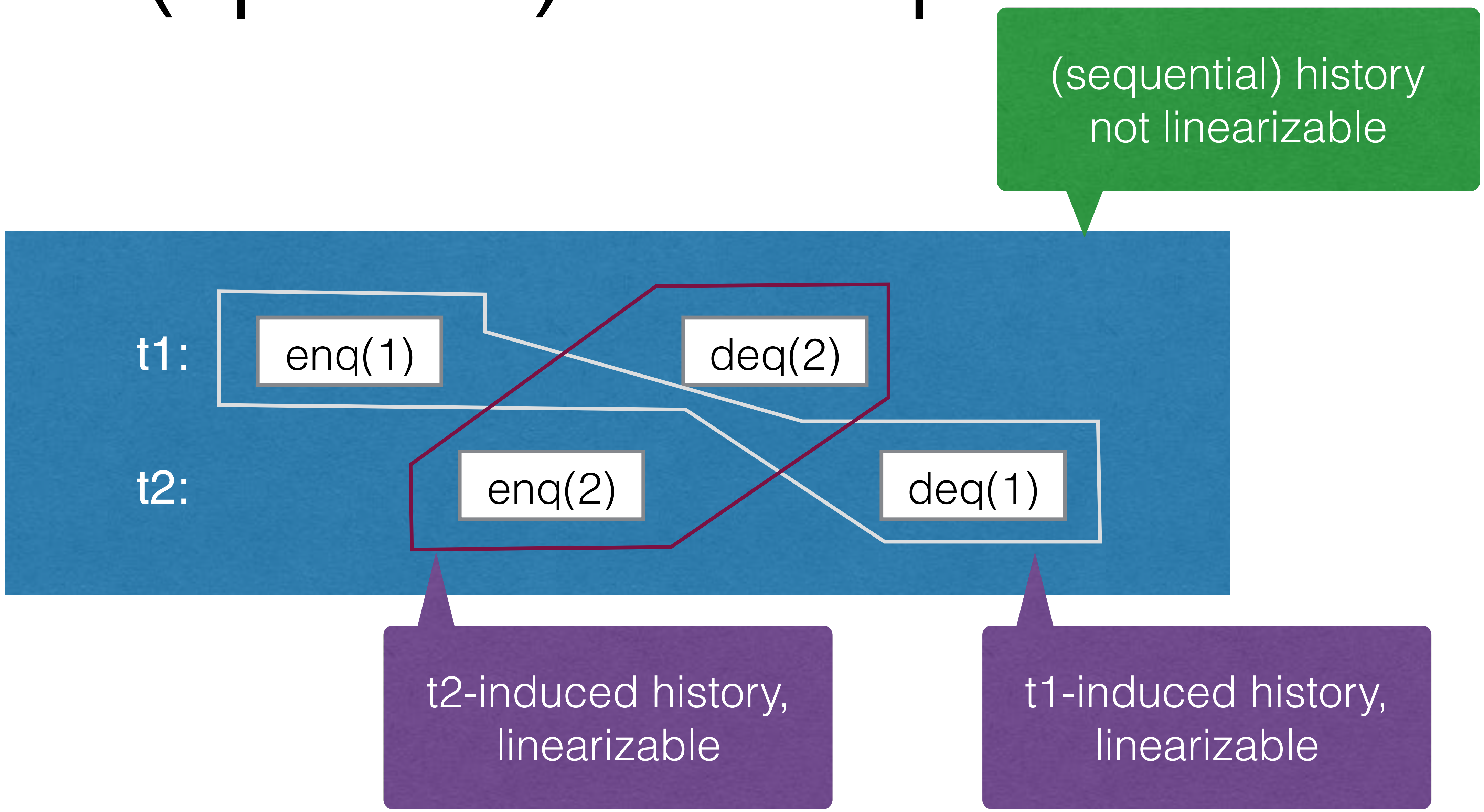

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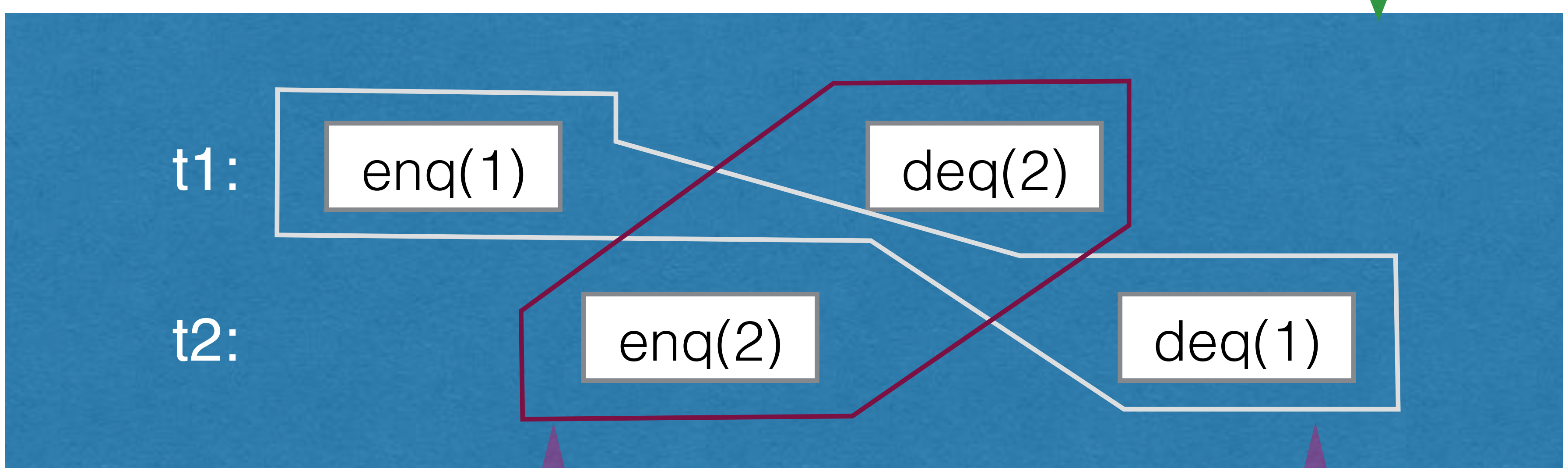

t2-induced history, linearizable

t1-induced history, linearizable

locally linearizable

Local Linearizability (queue) definition

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Queue signature $\Sigma = \{enq(x) \mid x \in V\} \cup \{deq(x) \mid x \in V\} \cup \{deq(empty)\}$

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For a history **h** with a thread T, we put

$$I_T = \{enq(x)^T \in \mathbf{h} \mid x \in V\}$$

$$O_T = \{deq(x)^{T'} \in \mathbf{h} \mid enq(x)^T \in I_T\} \cup \{deq(empty)\}$$

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in-methods of thread T are enqueues performed by thread T

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h is locally linearizable iff every thread-induced history

$$\mathbf{h}_T = \mathbf{h} \mid (I_T \cup O_T)$$

is linearizable.

Generalizations of Local Linearizability

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Signature Σ

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For a history **h** with n threads, choose

$In_{\mathbf{h}}(i)$

$Out_{\mathbf{h}}(i)$

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in-methods of thread i,
methods that go in $\mathbf{h}_i$

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Generalizations of Local Linearizability

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Generalizations of Local Linearizability

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in-methods of thread i, methods that go in $\mathbf{h}_i$

$Out_{\mathbf{h}}(i)$

by increasing the in-methods, LL gradually moves to linearizability

out-methods of thread i, dependent methods on the methods in $In_{\mathbf{h}}(i)$ (performed by any thread)

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Where do we stand?

In general

Linearizability

Local Linearizability

Sequential Consistency

Where do we stand?

For queues (and most container-type data structures)

Linearizability

Local Linearizability

Sequential Consistency

Properties

Properties

Local linearizability is compositional

Properties

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like linearizability
unlike sequential consistency

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h (over multiple objects) is locally linearizable
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uses decomposition into smaller histories, by definition

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Local linearizability is modular / "decompositional"

uses decomposition into smaller histories, by definition

↓

may allow for modular verification

Verification (queue)

Queue sequential specification (axiomatic)

$\mathbf{s}$ is a legal queue sequence

iff

1. $\mathbf{s}$ is a legal pool sequence, and
2. $enq(x) <_{\mathbf{s}} enq(y) \;\wedge\; deq(y) \in \mathbf{s} \quad \Rightarrow \quad deq(x) \in \mathbf{s} \;\wedge\; deq(x) <_{\mathbf{s}} deq(y)$

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Queue linearizability (axiomatic)

Henzinger, Sezgin, Vafeiadis CONCUR13

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precedence order

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thread-local precedence order

Generic Implementations

Generic Implementations

Your favorite linearizable data structure implementation

Generic Implementations

Your favorite linearizable data structure implementation

Φ

Generic Implementations

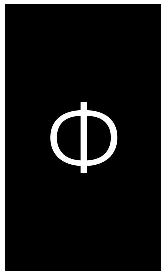
Your favorite linearizable data structure implementation

Φ

turns into a locally linearizable implementation by:

Generic Implementations

Your favorite linearizable data structure implementation



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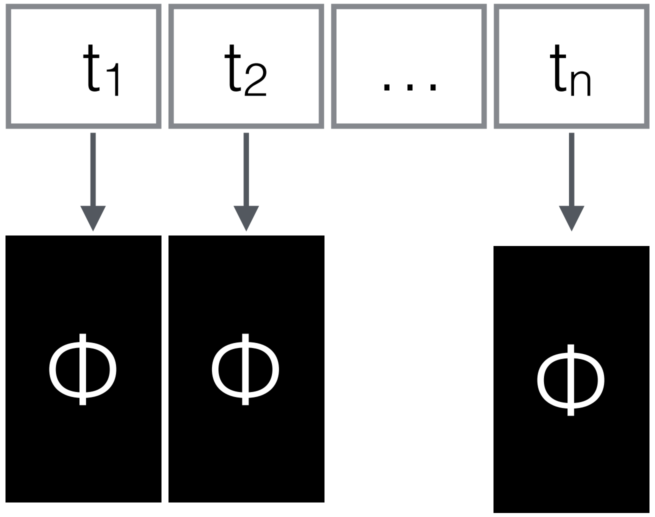
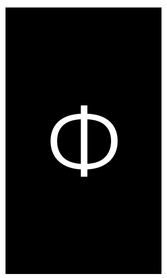
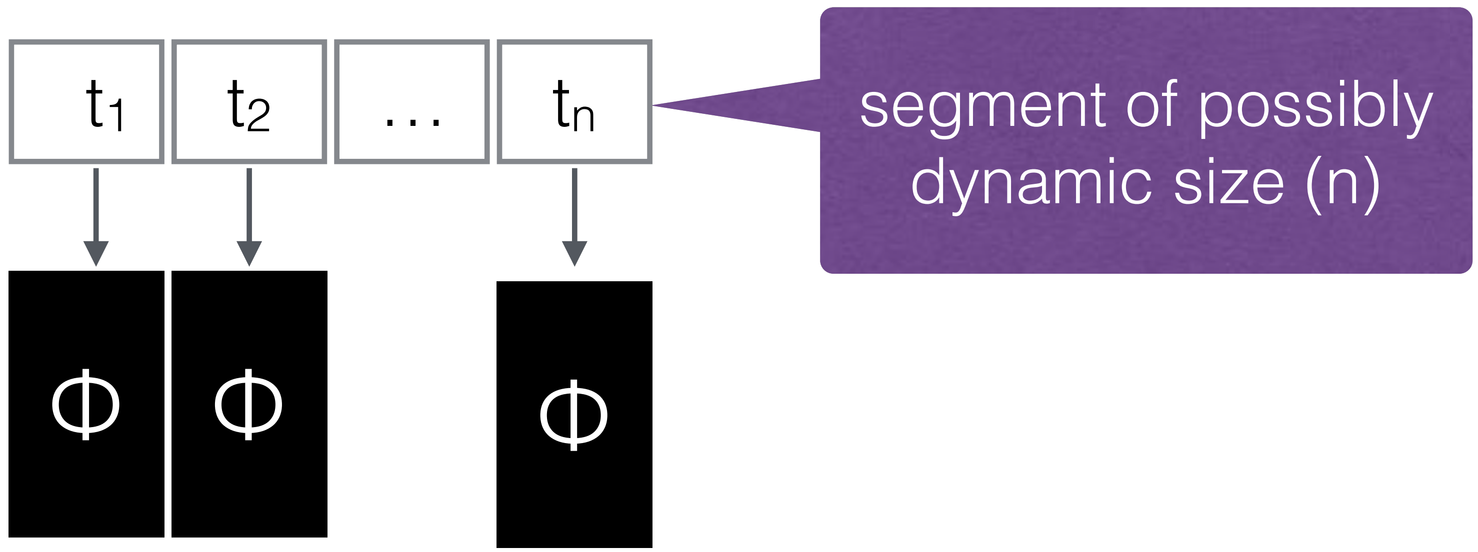


Generic Implementations

Your favorite linearizable data structure implementation



turns into a locally linearizable implementation by:



Generic Implementations

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Φ

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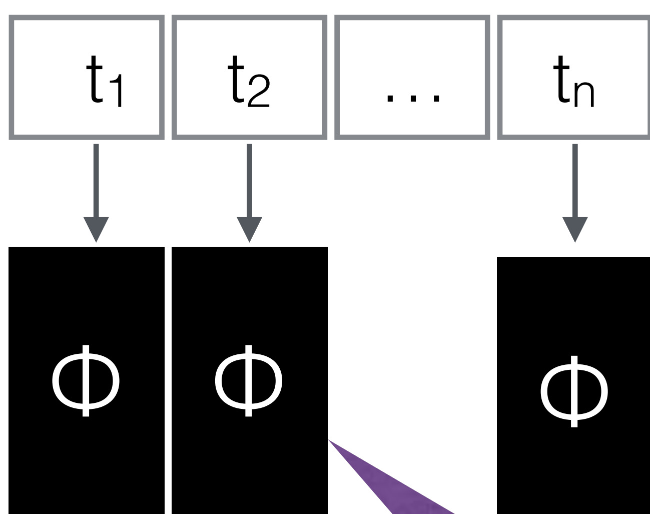

segment of possibly dynamic size (n)

local inserts / global (randomly distributed) removes

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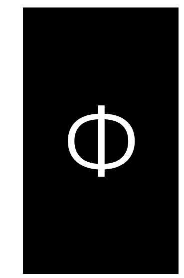

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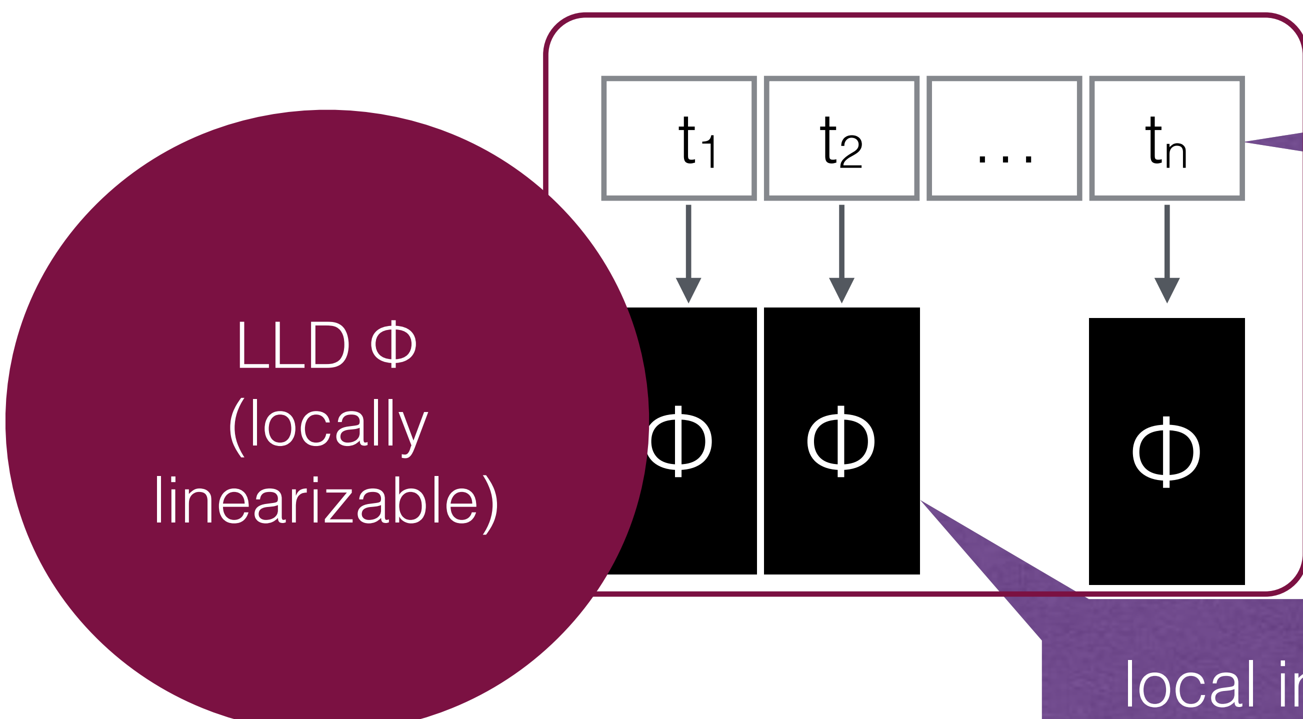

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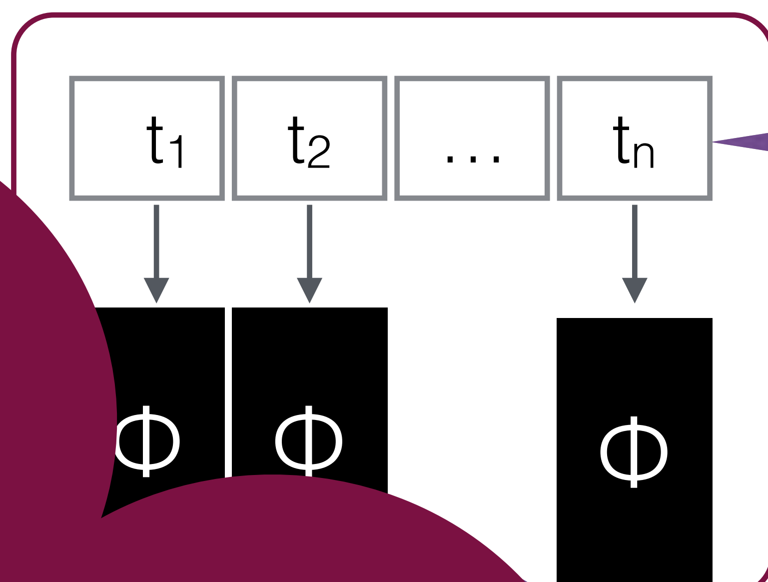


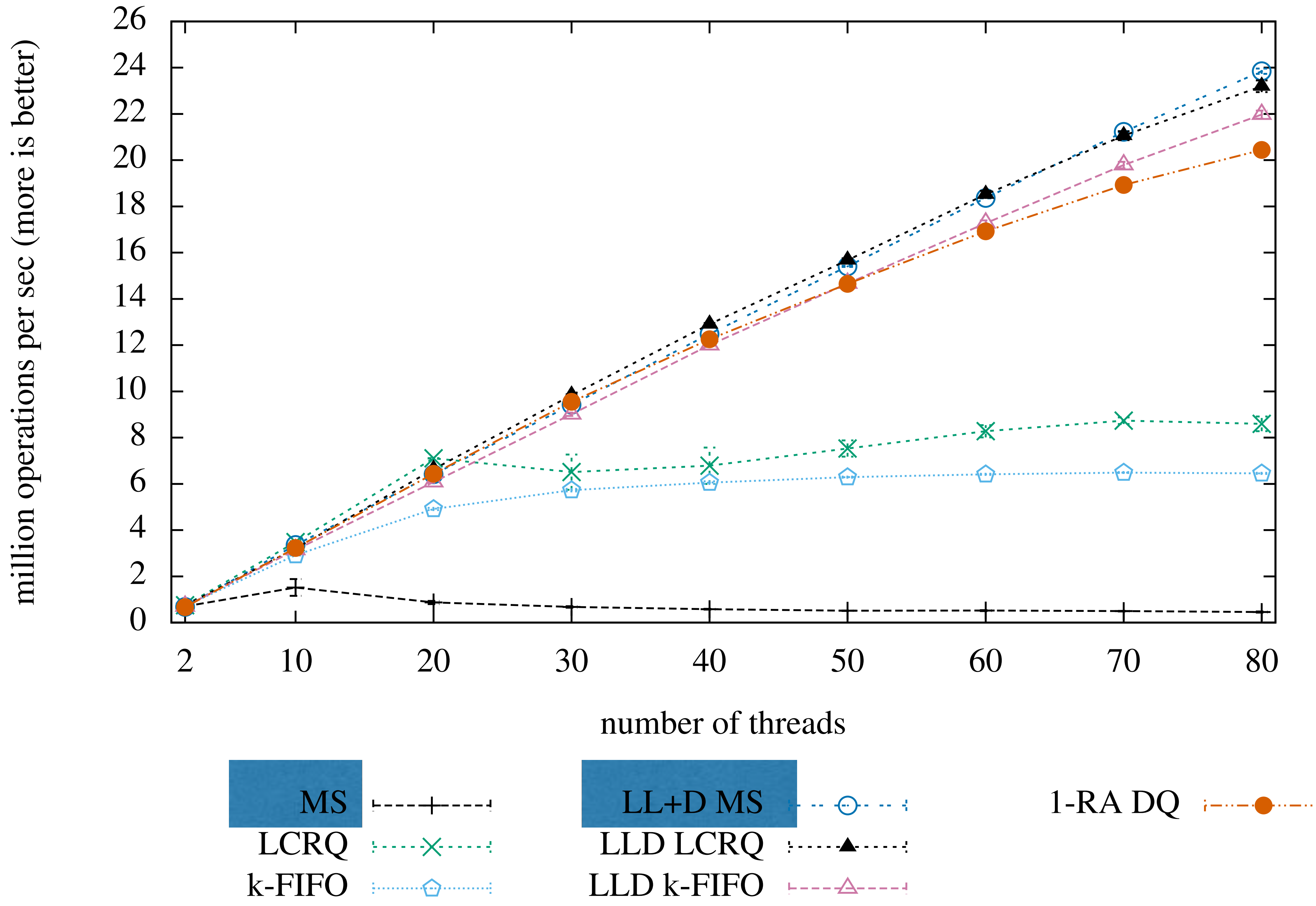
segment of possibly dynamic size (n)

local inserts / global (randomly distributed) removes

LLD Φ (locally linearizable)

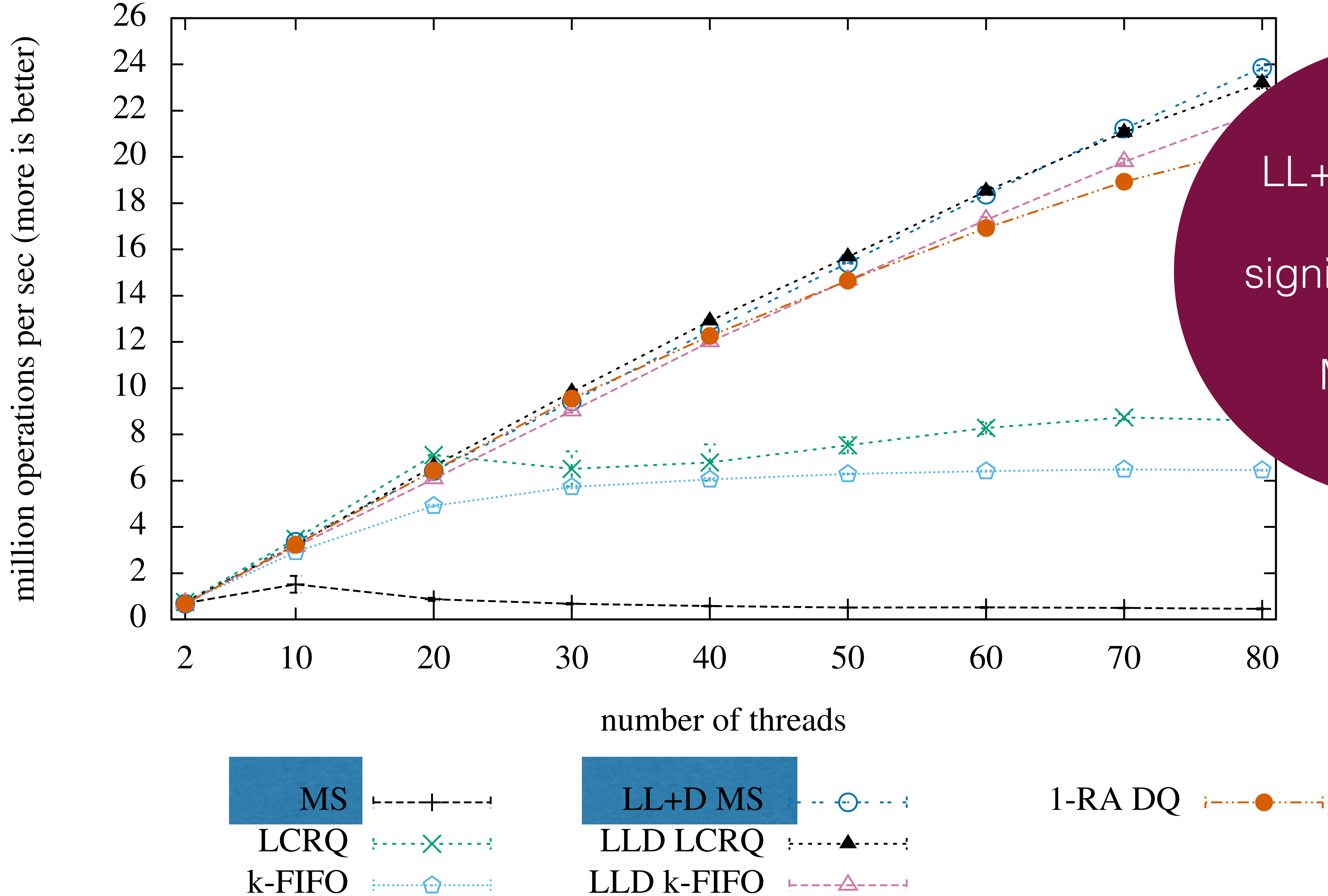
LL+D Φ (also pool linearizable)

Performance



(a) Queues, LL queues, and "queue-like" pools

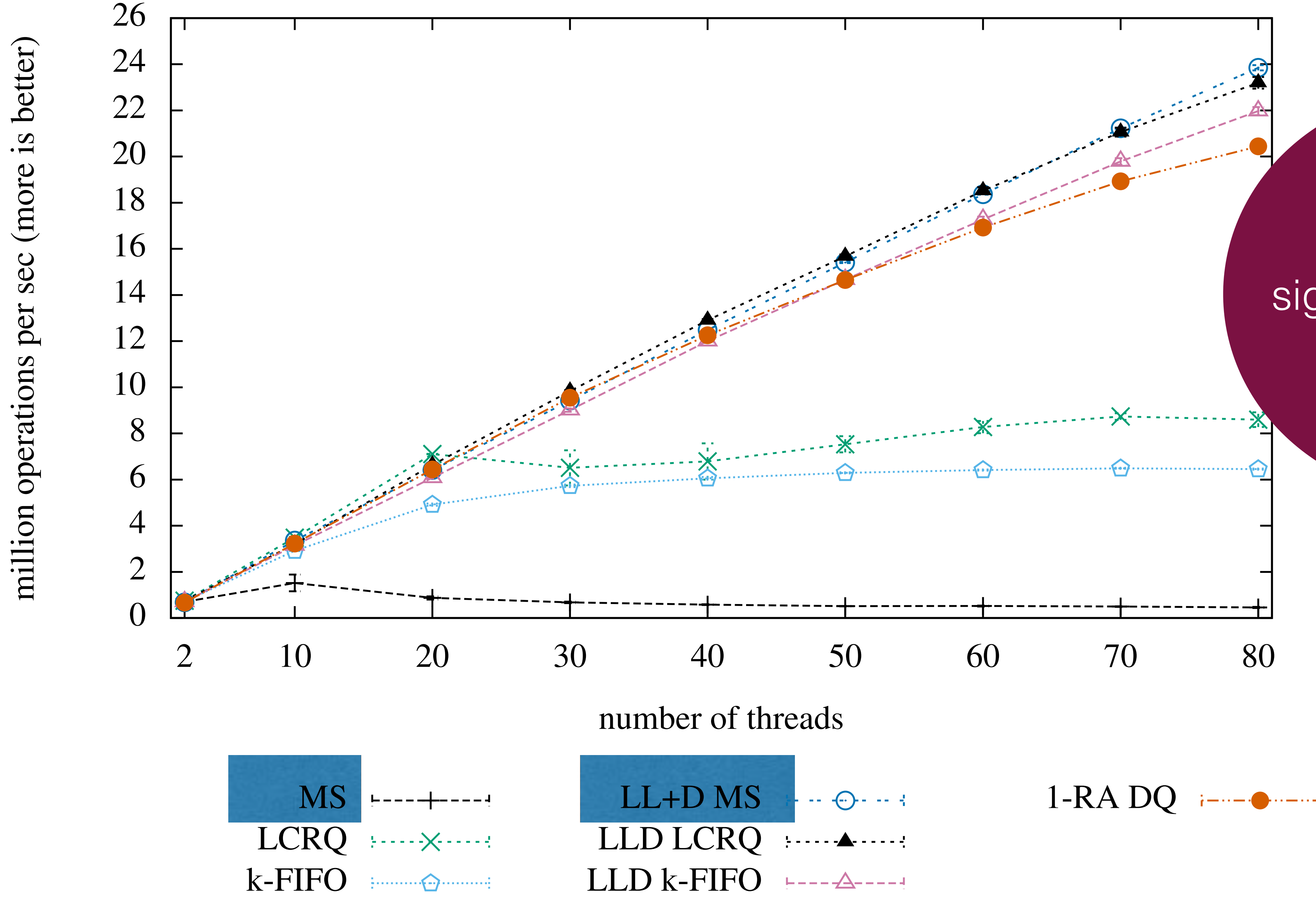
Performance



(a) Queues, LL queues, and "queue-like" pools

LL+D MS queue performs significantly better than MS queue

Performance



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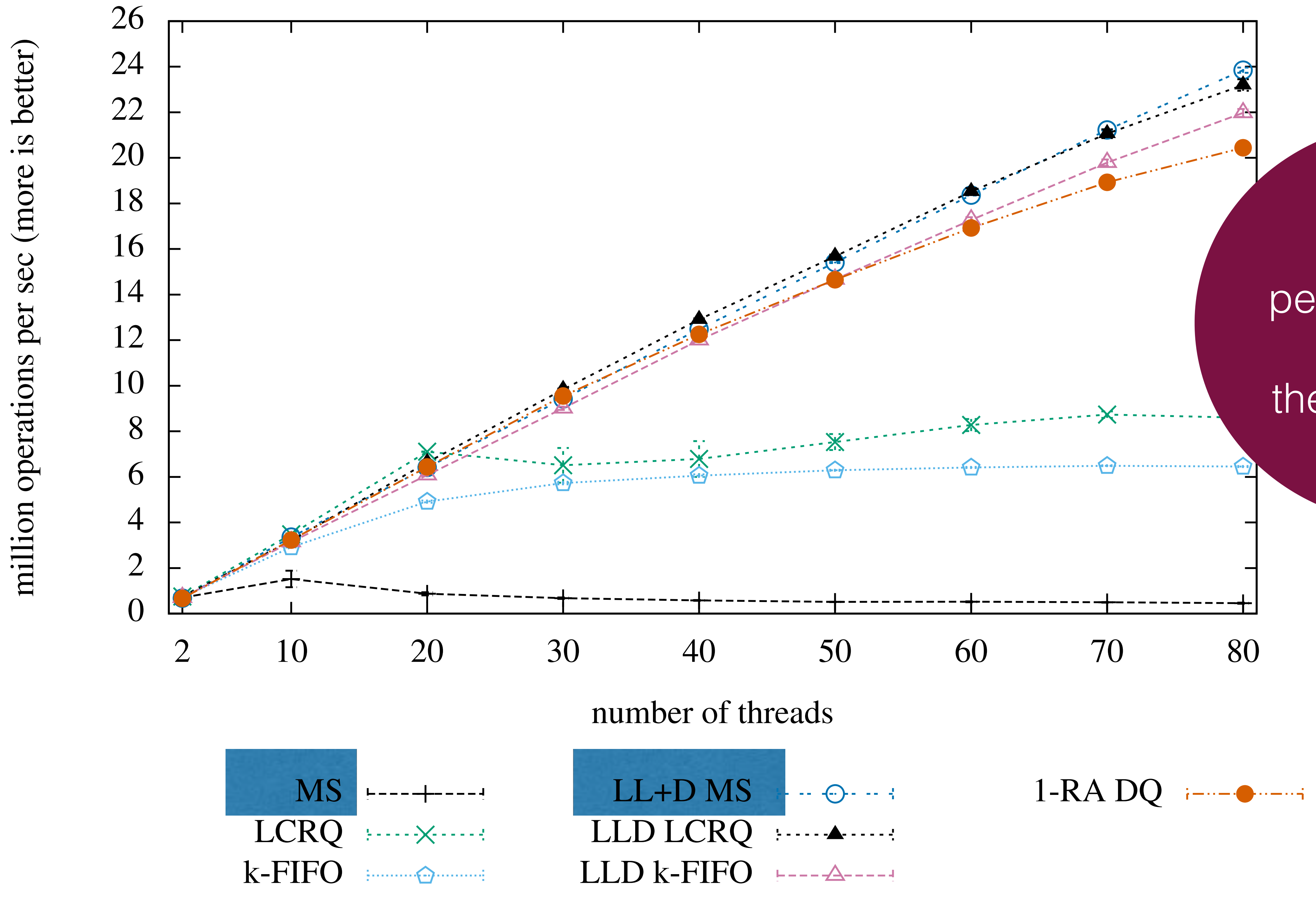
Φ

Performance



(a) Queues, LL queues, and "queue-like" pools

LL+D MS queue performs better than the best known pools

Thank You!

Local Linearizability

Ana Sokolova UNIVERSITY of SALZBURG

joint work with:

Andreas Haas Google

Andreas Holzer UNIVERSITY OF TORONTO

Michael Lippautz Google

Ali Sezgin UNIVERSITY OF CAMBRIDGE

Tom Henzinger IST AUSTRIA

Christoph Kirsch UNIVERSITY of SALZBURG

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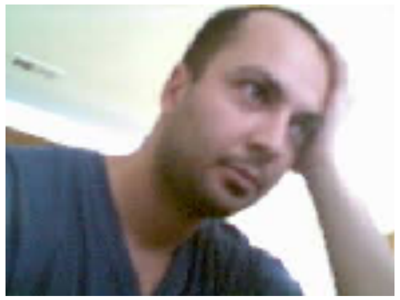

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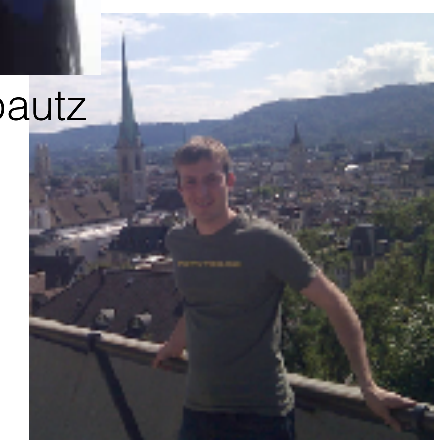

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